AUFMANN • BARKER • LOCKWOOD

BASIC MATH

SECOND EDITION

SECOND CUSTOM EDITION FOR
CHALK DUST COMPANY

CHALK DUST COMPANY

BASIC MATH

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Chapter 1

Whole Numbers

When a contractor prepares an estimate for a job, many factors are considered: the number of workers needed, their hourly wages, how many hours they will work, the cost of the materials. **Exercises 103-105 on page 32** illustrate how a contractor would use operations on whole numbers to determine costs associated with a construction job.

Objectives

Section 1.1

A To identify the order relation between two numbers
B To write whole numbers in words and in standard form
C To write whole numbers in expanded form
D To round a whole number to a given place value

Section 1.2

A To add whole numbers
B To solve application problems

Section 1.3

A To subtract whole numbers without borrowing
B To subtract whole numbers with borrowing
C To solve application problems

Section 1.4

A To multiply a number by a single digit
B To multiply larger whole numbers
C To solve application problems

Section 1.5

A To divide by a single digit with no remainder in the quotient
B To divide by a single digit with a remainder in the quotient
C To divide by larger whole numbers
D To solve application problems

Section 1.6

A To simplify expressions that contain exponents
B To use the Order of Operations Agreement to simplify expressions

Section 1.7

A To factor numbers
B To find the prime factorization of a number

Need help? For on-line student resources, such as section quizzes, visit this textbook's web site at **math.college.hmco.com/students.**

Prep Test

1. Name the number of ♦s shown below.

♦ ♦ ♦ ♦ ♦ ♦ ♦ ♦

2. Write the numbers from 1 to 10.

1 ___ ___ ___ ___ ___ ___ ___ ___ 10

3. Match the number with its word form.

a. 4	**A.** five
b. 2	**B.** one
c. 5	**C.** zero
d. 1	**D.** four
e. 3	**E.** two
f. 0	**F.** three

Go Figure

Five adults and two children want to cross a river in a rowboat. The boat can hold one adult or two children or one child. Everyone is able to row the boat. What is the minimum number of trips that will be necessary for everyone to get to the other side?

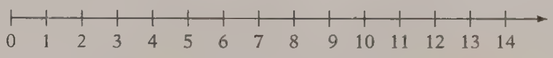
1.1 Introduction to Whole Numbers

Objective A To identify the order relation between two numbers

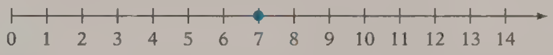
The **whole numbers** are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, ...

The three dots mean that the list continues on and on and that there is no largest whole number.

Just as distances are associated with the markings on the edge of a ruler, the whole numbers can be associated with points on a line. This line is called the **number line.** The arrow on the number line below indicates that there is no largest whole number.

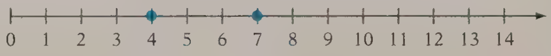


The **graph of a whole number** is shown by placing a heavy dot on the number line directly above the number. Here is the graph of 7 on the number line:

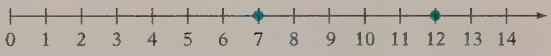


The number line can be used to show the order of whole numbers. A number that appears to the left of a given number is **less than** the given number. The symbol for "is less than" is <. A number that appears to the right of a given number is **greater than** the given number. The symbol for "is greater than" is >.

Four is less than seven.
$4 < 7$

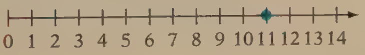


Twelve is greater than seven.
$12 > 7$



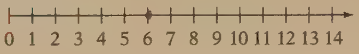
Example 1 Graph 11 on the number line.

Solution



You Try It 1 Graph 6 on the number line.

Your solution



Example 2 Place the correct symbol, < or >, between the two numbers.

a. 39 24
b. 0 51

Solution

a. $39 > 24$
b. $0 < 51$

You Try It 2 Place the correct symbol, < or >, between the two numbers.

a. 45 29
b. 27 0

Your solution

a.
b.

Solutions on p. S1

Objective B

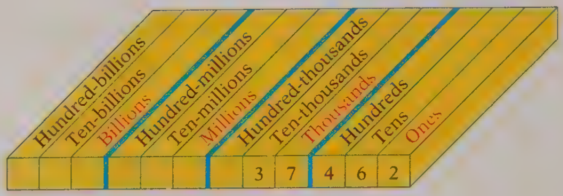
To write whole numbers in words and in standard form

Point of Interest

The Babylonians had a place-value system based on 60. Its influence is still with us in angle measurement and time: 60 seconds in 1 minute, 60 minutes in 1 hour. It appears that the earliest record of a base-10 place-value system for natural numbers dates from the 8th century.

When a whole number is written using the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9, it is said to be in **standard form.** The position of each digit in the number determines the digit's **place value.** The diagram below shows a **place-value chart** naming the first twelve place values. The number 37,462 is in standard form and has been entered in the chart.

In the number 37,462, the position of the digit 3 determines that its place value is ten-thousands.



When a number is written in standard form, each group of digits separated by a comma is called a **period.** The number 3,786,451,294 has four periods. The period names are shown in red in the place-value chart above.

To write a number in words, start from the left. Name the number in each period. Then write the period name in place of the comma.

3,786,451,294 is read "three billion seven hundred eighty-six million four hundred fifty-one thousand two hundred ninety-four."

To write a whole number in standard form, write the number named in each period, and replace each period name with a comma.

Four million sixty-two thousand five hundred eighty-four is written 4,062,584. The zero is used as a place holder for the hundred-thousands place.

Example 3 Write 25,478,083 in words.

Solution Twenty-five million four hundred seventy-eight thousand eighty-three

You Try It 3 Write 36,462,075 in words.

Your solution

Example 4 Write three hundred three thousand three in standard form.

Solution 303,003

You Try It 4 Write four hundred fifty-two thousand seven in standard form.

Your solution

Solutions on p. S1

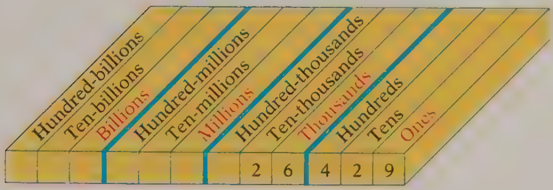
Objective C

To write whole numbers in expanded form

The whole number 26,429 can be written in **expanded form** as

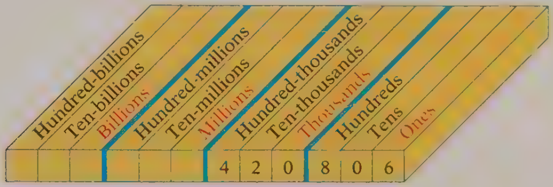
$20{,}000 + 6000 + 400 + 20 + 9$

The place-value chart can be used to find the expanded form of a number.


Hundred-billions
Ten-billions
Billions
Hundred-millions
Ten-millions
Millions
Hundred-thousands
Ten-thousands
Thousands
Hundreds
Tens
Ones
2 6 4 2 9

2 Ten-thousands	+	6 Thousands	+	4 Hundreds	+	2 Tens	+	9 Ones
20,000	+	6000	+	400	+	20	+	9

The number 420,806 is written in expanded form below. Note the effect of having zeros in the number.


Hundred-billions
Ten-billions
Billions
Hundred-millions
Ten-millions
Millions
Hundred-thousands
Ten-thousands
Thousands
Hundreds
Tens
Ones
4 2 0 8 0 6

4 Hundred-thousands	+	2 Ten-thousands	+	0 Thousands	+	8 Hundreds	+	0 Tens	+	6 Ones
400,000	+	20,000	+	0	+	800	+	0	+	6

or simply 400,000 + 20,000 + 800 + 6.

Example 5 Write 23,859 in expanded form.

Solution 20,000 + 3000 + 800 + 50 + 9

You Try It 5 Write 68,281 in expanded form.

Your solution

Example 6 Write 709,542 in expanded form.

Solution 700,000 + 9000 + 500 + 40 + 2

You Try It 6 Write 109,207 in expanded form.

Your solution

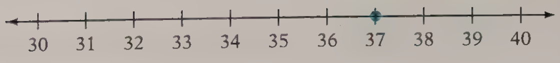
Solutions on p. S1

Objective B

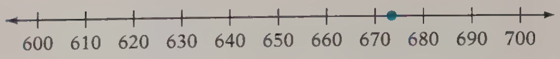
To round a whole number to a given place value

When the distance to the moon is given as 240,000 miles, the number represents an approximation to the true distance. Giving an approximate value for an exact number is called **rounding.** A rounded number is always rounded to a given place value.

37 is closer to 40 than it is to 30. 37 rounded to the nearest ten is 40.



673 rounded to the nearest ten is 670. 673 rounded to the nearest hundred is 700.



A whole number is rounded to a given place value without using the number line by looking at the first digit to the right of the given place value.

If the digit to the right of the given place value is less than 5, that digit and all digits to the right are replaced by zeros.

⇨ Round 13,834 to the nearest hundred.

Given place value
13,834
3 < 5

13,834 rounded to the nearest hundred is 13,800.

If the digit to the right of the given place value is greater than or equal to 5, increase the digit in the given place value by 1, and replace all other digits to the right by zeros.

⇨ Round 386,217 to the nearest ten-thousand.

Given place value
386,217
6 > 5

386,217 rounded to the nearest ten-thousand is 390,000.

Example 7 Round 525,453 to the nearest ten-thousand.

Solution

Given place value
525,453
5 = 5

525,453 rounded to the nearest ten-thousand is 530,000.

You Try It 7 Round 368,492 to the nearest ten-thousand.

Your solution

Example 8 Round 1972 to the nearest hundred.

Solution

Given place value
1972
7 > 5

1972 rounded to the nearest hundred is 2000.

You Try It 8 Round 3962 to the nearest hundred.

Your solution

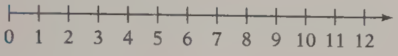
Solutions on p. S1

1.1 Exercises

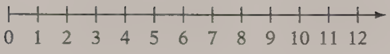
Objective A

Graph the number on the number line.

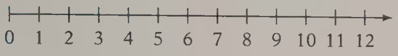
1. 3


0 1 2 3 4 5 6 7 8 9 10 11 12

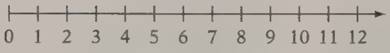
2. 5


0 1 2 3 4 5 6 7 8 9 10 11 12

3. 9


0 1 2 3 4 5 6 7 8 9 10 11 12

4. 0


0 1 2 3 4 5 6 7 8 9 10 11 12

Place the correct symbol, < or >, between the two numbers.

5. 37 49

6. 58 21

7. 101 87

8. 16 5

9. 245 158

10. 2701 2071

11. 0 45

12. 107 0

13. 815 928

Objective B

Name the place value of the digit 3.

14. 83,479

15. 3,491,507

16. 2,634,958

17. 76,319,204

Write the number in words.

18. 2675

19. 3790

20. 42,928

21. 58,473

22. 356,943

23. 498,512

24. 3,697,483

25. 6,842,715

Write the number in standard form.

26. Eighty-five

27. Three hundred fifty-seven

28. Three thousand four hundred fifty-six

29. Sixty-three thousand seven hundred eighty

30. Six hundred nine thousand nine hundred forty-eight

31. Seven million twenty-four thousand seven hundred nine

Objective C

Write the number in expanded form.

32. 5287

33. 6295

34. 58,943

35. 453,921

36. 200,583

37. 301,809

38. 403,705

39. 3,000,642

Objective D

Round the number to the given place value.

40. 926 Tens

41. 845 Tens

42. 1439 Hundreds

43. 3973 Hundreds

44. 43,607 Thousands

45. 52,715 Thousands

46. 647,989 Ten-thousands

47. 253,678 Ten-thousands

APPLYING THE CONCEPTS

Answer true or false for Exercise 48a and 48b. If the answer is false, give an example to show that it is false.

48. **a.** If you are given two distinct whole numbers, then one of the numbers is always greater than the other number.
b. A rounded-off number is always less than its exact value.

49. What is the largest three-digit whole number? What is the smallest five-digit whole number?

50. If 3846 is rounded to the nearest ten and then that number is rounded to the nearest hundred, is the result the same as what you get when you round 3846 to the nearest hundred? If not, which of the two methods is correct for rounding to the nearest hundred?

1.2

Addition of Whole Numbers

Objective A **To add whole numbers**

Addition is the process of finding the total of two or more numbers.

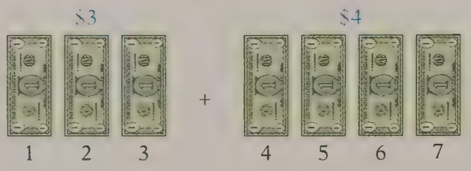
Point of Interest

The first use of the plus sign appeared in 1489 in *Mercantile Arithmetic*. It was used to indicate a surplus and not as the symbol for addition. That use did not appear until around 1515.

By counting, we see that the total of \$3 and \$4 is \$7.

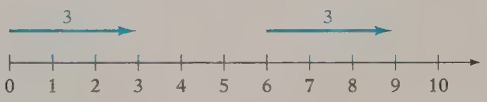
$3 + $4 = $7

Addend **Addend** **Sum**

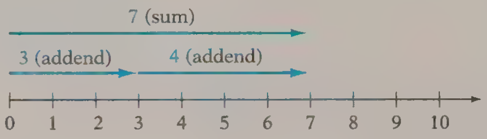


Addition can be illustrated on the number line by using arrows to represent the addends. The size, or magnitude, of a number can be represented on the number line by an arrow.

The number 3 can be represented anywhere on the number line by an arrow that is 3 units in length.



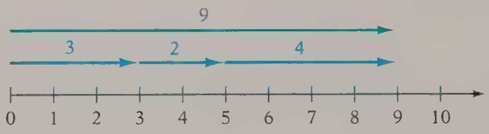
To add on the number line, place the arrows representing the addends head to tail, with the first arrow starting at zero. The sum is represented by an arrow starting at zero and stopping at the tip of the last arrow.



$3 + 4 = 7$

More than two numbers can be added on the number line.

$3 + 2 + 4 = 9$



TAKE NOTE

This is the same addition problem shown on the number line above.

Some special properties of addition that are used frequently are given below.

Addition Property of Zero

Zero added to a number does not change the number.

$4 + 0 = 4$

$0 + 7 = 7$

Commutative Property of Addition

Two numbers can be added in either order; the sum will be the same.

$4 + 8 = 8 + 4$

$12 = 12$

Associative Property of Addition

Grouping the addition in any order gives the same result. The parentheses are grouping symbols and have the meaning "Do the operations inside the parentheses first."

$$\underbrace{(3 + 2)}_{5} + 4 = 3 + \underbrace{(2 + 4)}_{6}$$

$$9 = 9$$

The number line is not useful for adding large numbers. The basic addition facts for adding one digit to one digit should be memorized. Addition of larger numbers requires the repeated use of the basic addition facts.

To add large numbers, begin by arranging the numbers vertically, keeping the digits of the same place value in the same column.

⇨ Add: 321 + 6472

$$\begin{array}{ccccc}
 & \text{THOUSANDS} & \text{HUNDREDS} & \text{TENS} & \text{ONES}\\
 & & 3 & 2 & 1\\
+ & 6 & 4 & 7 & 2\\ \hline
 & 6 & 7 & 9 & 3
\end{array}$$

- Add the digits in each column.

There are several words or phrases in English that indicate the operation of addition. Here are some examples:

added to	3 added to 5	5 + 3
more than	7 more than 5	5 + 7
the sum of	the sum of 3 and 9	3 + 9
increased by	4 increased by 6	4 + 6
the total of	the total of 8 and 3	8 + 3
plus	5 plus 10	5 + 10

[illegible]

A scientific calculator is a useful tool in mathematical computation. To add 24 + 71 with your calculator, enter the following:

24 [+] 71 [=]

⇨ What is the sum of 24 and 71?

$$\begin{array}{r}
24\\
+\ 71\\ \hline
95
\end{array}$$

- The phrase *the sum of* means to add.

The sum of 24 and 71 is 95.

When the sum of the digits in a column exceeds 9, the addition will involve "carrying."

⇨ Add: 487 + 369

$$\begin{array}{cccc}
 & \text{HUNDREDS} & \text{TENS} & \text{ONES}\\
 & & 1 & \\
 & 4 & 8 & 7\\
+ & 3 & 6 & 9\\ \hline
 & & & 6
\end{array}$$

- Add the ones column.
  7 + 9 = 16 (1 ten + 6 ones).
  Write the 6 in the ones column and carry the 1 ten to the tens column.

$$\begin{array}{cccc}
 & 1 & 1 & \\
 & 4 & 8 & 7\\
+ & 3 & 6 & 9\\ \hline
 & & 5 & 6
\end{array}$$

- Add the tens column.
  1 + 8 + 6 = 15 (1 hundred + 5 tens).
  Write the 5 in the tens column and carry the 1 hundred to the hundreds column.

$$\begin{array}{cccc}
 & 1 & 1 & \\
 & 4 & 8 & 7\\
+ & 3 & 6 & 9\\ \hline
 & 8 & 5 & 6
\end{array}$$

- Add the hundreds column.
  1 + 4 + 3 = 8 (8 hundreds).
  Write the 8 in the hundreds column.

Example 1 Find the total of 17, 103, and 8.

Solution

$$\begin{array}{r} {\scriptstyle 1} \\ 17 \\ 103 \\ +\ \ \ 8 \\ \hline 128 \end{array}$$

You Try It 1 What is 347 increased by 12,453?

Your solution

Example 2 Add: 89 + 36 + 98

Solution

$$\begin{array}{r} {\scriptstyle 2} \\ 89 \\ 36 \\ +\ 98 \\ \hline 223 \end{array}$$

You Try It 2 Add: 95 + 88 + 67

Your solution

Example 3 Add:

$$\begin{array}{r} 41{,}395 \\ 4{,}327 \\ 497{,}625 \\ +\ \ 32{,}991 \\ \hline \end{array}$$

Solution

$$\begin{array}{r} {\scriptstyle 1\,1\,2\;\;2\,1} \\ 41{,}395 \\ 4{,}327 \\ 497{,}625 \\ +\ \ 32{,}991 \\ \hline 576{,}338 \end{array}$$

You Try It 3 Add:

$$\begin{array}{r} 392 \\ 4{,}079 \\ 89{,}035 \\ +\ \ 4{,}992 \\ \hline \end{array}$$

Your solution

Solutions on p. S1

ESTIMATION

Estimation and Calculators

At some places in the text, you will be asked to use your calculator. Effective use of a calculator requires that you estimate the answer to the problem. This helps ensure that you have entered the numbers correctly and pressed the correct keys.

For example, if you use your calculator to find 22,347 + 5896 and the answer in the calculator's display is 131,757,912, you should realize that you have entered some part of the calculation incorrectly. In this case, you pressed $\boxed{\times}$ instead of $\boxed{+}$. By estimating the answer to a problem, you can help ensure the accuracy of your calculations. The symbol ≈ is used to denote **is approximately equal to.**

For example, to estimate the answer to 22,347 + 5896, round each number to the same place value. In this case, we will round to the nearest thousand. Then add.

$$\begin{array}{rcr} 22{,}347 & \approx & 22{,}000 \\ +\ \ \ 5{,}896 & \approx & +\ \ \ 6{,}000 \\ \hline & & 28{,}000 \end{array}$$

The sum 22,347 + 5896 is approximately 28,000. Knowing this, you would know that 131,757,912 is much too large and is therefore incorrect.

To estimate the sum of two numbers, first round each whole number to the same place value and then add. Compare this answer with the calculator's answer.

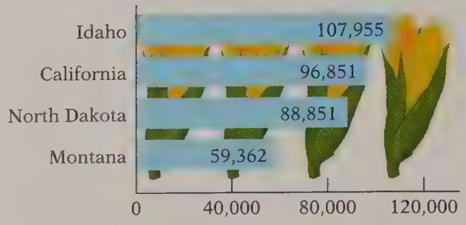
Objective B To solve application problems

To solve an application problem, first read the problem carefully. The **Strategy** involves identifying the quantity to be found and planning the steps that are necessary to find that quantity. The **Solution** involves performing each operation stated in the Strategy and writing the answer.

The table below displays the states with the most organic cropland, in acres.

State	*Number of Acres*
Idaho	107,955
California	96,851
North Dakota	88,851
Montana	59,362

Source: Agriculture Department Agricultural Outlook, April 2000



Number of Acres of Organic Cropland

➡ Find the total number of acres of organic cropland in the four states shown in the table and the graph above.

Strategy To find the total number of acres of organic cropland in the four states, read the table to find the number of acres in each state. Then add the numbers.

Solution

$$\begin{array}{r} 107{,}955 \\ 96{,}851 \\ 88{,}851 \\ +\ 59{,}362 \\ \hline 353{,}019 \end{array}$$

There are 353,019 acres of organic cropland in the four states listed.

Example 4

Your paycheck shows deductions of $225 for savings, $98 for taxes, and $27 for insurance. Find the total of the three deductions.

Strategy

To find the total of the deductions, add the three amounts (225, 98, and 27).

Solution

$$\begin{array}{r} 225 \\ 98 \\ +\ 27 \\ \hline 350 \end{array}$$

The total of the three deductions is $350.

You Try It 4

For Halloween, the average U.S. household spends $30 for candy, $22 for costumes, $16 for pumpkins, and $11 for decorations. (Source: 1999 American Express Retail Index) Find the total amount that the average household spends on these items.

Your strategy

Your solution

Solution on p. S1

1.2 Exercises

Objective A

Add.

1. $\begin{array}{r} 17 \\ +\ 11 \\ \hline \end{array}$

2. $\begin{array}{r} 25 \\ +\ 63 \\ \hline \end{array}$

3. $\begin{array}{r} 83 \\ +\ 42 \\ \hline \end{array}$

4. $\begin{array}{r} 63 \\ +\ 94 \\ \hline \end{array}$

5. $\begin{array}{r} 77 \\ +\ 25 \\ \hline \end{array}$

6. $\begin{array}{r} 63 \\ +\ 49 \\ \hline \end{array}$

7. $\begin{array}{r} 56 \\ +\ 98 \\ \hline \end{array}$

8. $\begin{array}{r} 86 \\ +\ 68 \\ \hline \end{array}$

9. $\begin{array}{r} 658 \\ +\ 831 \\ \hline \end{array}$

10. $\begin{array}{r} 842 \\ +\ 936 \\ \hline \end{array}$

11. $\begin{array}{r} 735 \\ +\ \ 93 \\ \hline \end{array}$

12. $\begin{array}{r} 189 \\ +\ \ 50 \\ \hline \end{array}$

13. $\begin{array}{r} 859 \\ +\ 725 \\ \hline \end{array}$

14. $\begin{array}{r} 637 \\ +\ 829 \\ \hline \end{array}$

15. $\begin{array}{r} 470 \\ +\ 749 \\ \hline \end{array}$

16. $\begin{array}{r} 427 \\ +\ 690 \\ \hline \end{array}$

17. $\begin{array}{r} 36{,}925 \\ +\ 65{,}392 \\ \hline \end{array}$

18. $\begin{array}{r} 56{,}772 \\ +\ 51{,}239 \\ \hline \end{array}$

19. $\begin{array}{r} 50{,}873 \\ +\ 28{,}453 \\ \hline \end{array}$

20. $\begin{array}{r} 34{,}872 \\ +\ 46{,}079 \\ \hline \end{array}$

21. $\begin{array}{r} 878 \\ 737 \\ +\ 189 \\ \hline \end{array}$

22. $\begin{array}{r} 768 \\ 461 \\ +\ 669 \\ \hline \end{array}$

23. $\begin{array}{r} 319 \\ 348 \\ +\ 912 \\ \hline \end{array}$

24. $\begin{array}{r} 292 \\ 579 \\ +\ 315 \\ \hline \end{array}$

25. $\begin{array}{r} 9409 \\ 3253 \\ +\ 7078 \\ \hline \end{array}$

26. $\begin{array}{r} 8188 \\ 8020 \\ +\ 7104 \\ \hline \end{array}$

27. $\begin{array}{r} 2038 \\ 2243 \\ +\ 3139 \\ \hline \end{array}$

28. $\begin{array}{r} 4252 \\ 6882 \\ +\ 5235 \\ \hline \end{array}$

29. $\begin{array}{r} 67{,}428 \\ 32{,}171 \\ +\ 20{,}971 \\ \hline \end{array}$

30. $\begin{array}{r} 52{,}801 \\ 11{,}664 \\ +\ 89{,}638 \\ \hline \end{array}$

31. $\begin{array}{r} 76{,}290 \\ 43{,}761 \\ +\ 87{,}402 \\ \hline \end{array}$

32. $\begin{array}{r} 43{,}901 \\ 98{,}301 \\ +\ 67{,}943 \\ \hline \end{array}$

Add.

33. 20,958 + 3218 + 42

34. 80,973 + 5168 + 29

35. 392 + 37 + 10,924 + 621

36. 694 + 62 + 70,129 + 217

37. 294 + 1029 + 7935 + 65

38. 692 + 2107 + 3196 + 92

39. 97 + 7234 + 69,532 + 276

40. 87 + 1698 + 27,317 + 727

41. What is 9874 plus 4509?

42. What is 7988 plus 5678?

43. What is 3487 increased by 5986?

44. What is 99,567 added to 126,863?

45. What is 23,569 more than 9678?

46. What is 7894 more than 45,872?

47. What is 479 added to 4579?

48. What is 23,902 added to 23,885?

49. Find the total of 659, 55, and 1278.

50. Find the total of 4561, 56, and 2309.

51. Find the sum of 34, 329, 8, and 67,892.

52. Find the sum of 45, 1289, 7, and 32,876.

Estimate by rounding to the nearest hundred. Then use your calculator to add.

53. 1234 + 9780 + 6740

54. 919 + 3642 + 8796

55. 241 + 569 + 390 + 1672

56. 107 + 984 + 1035 + 2904

Estimate by rounding to the nearest thousand. Then use your calculator to add.

57.
$$\begin{array}{r} 32,461 \\ 9,844 \\ +\ 59,407 \\ \hline \end{array}$$

58.
$$\begin{array}{r} 29,036 \\ 22,904 \\ +\ \ 7,903 \\ \hline \end{array}$$

59.
$$\begin{array}{r} 25,432 \\ 62,941 \\ +\ 70,390 \\ \hline \end{array}$$

60.
$$\begin{array}{r} 66,541 \\ 29,365 \\ +\ 98,742 \\ \hline \end{array}$$

Estimate by rounding to the nearest ten-thousand. Then use your calculator to add.

61.
67,421
82,984
66,361
10,792
+ 34,037

62.
21,896
4,235
62,544
21,892
+ 1,334

63.
281,421
9,874
34,394
526,398
+ 94,631

64.
542,698
97,327
7,235
73,667
+ 173,201

Estimate by rounding to the nearest million. Then use your calculator to add.

65.
28,627,052
983,073
+ 3,081,496

66.
1,792,085
29,919,301
+ 3,406,882

67.
12,377,491
3,409,723
7,928,026
+ 10,705,682

68.
46,751,070
6,095,832
280,011
+ 1,563,897

Objective B *Application Problems*

69. In a recent year, according to the U.S. Department of Health and Human Services, there were 110,670 twin births in this country, 6919 triplet births, 627 quadruplet deliveries, and 79 quintuplet and other higher-order multiple births. Find the total number of multiple births during the year.

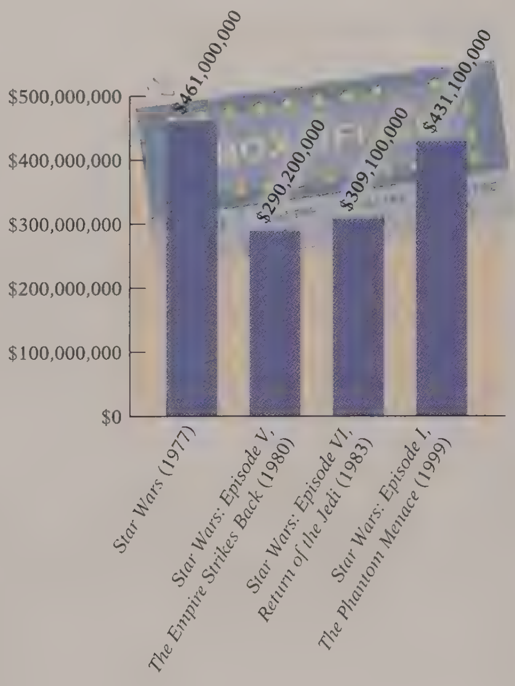
70. The Census Bureau estimates that the U.S. population will grow by 296 million people from 2000 to 2100. Given that the U.S. population in 2000 was 281 million, find the Census Bureau's estimate of the U.S. population in 2100.

The graph at the right shows the domestic box-office income from the *Star Wars* movies. Use this information for Exercises 71 to 73.

71. Estimate the total income from the four *Star Wars* movies.

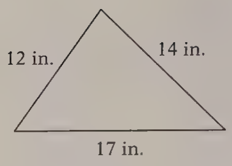
72. Find the total income from the four *Star Wars* movies.

73. **a.** Find the total income from the two movies with the lowest box-office incomes.
b. Does the total income from the two movies with the lowest box-office incomes exceed the income from the 1977 *Star Wars* production?



Source: www.worldwideboxoffice.com

74. The perimeter of a triangle is the sum of the lengths of the three sides of the triangle. Find the perimeter of a triangle that has sides that measure 12 inches, 14 inches, and 17 inches.



75. The odometer on a moving van reads 68,692. The driver plans to drive 515 miles the first day, 492 miles the second day, and 278 miles the third day.
- **a.** How many miles will be driven during the three days?
- **b.** What will the odometer reading be at the end of the trip?

The accompanying table shows the average amounts of money that all Americans have invested in selected assets and the average amounts invested for Americans between the ages of 16 and 34.

	All Americans	*Ages 16 to 34*
Checking accounts	$487	$375
Savings accounts	$3,494	$1,155
U.S. Savings Bonds	$546	$266
Money market	$10,911	$4,427
Stock/mutual funds	$4,510	$1,615
Home equity	$43,070	$17,184
Retirement	$9,016	$4,298

76. What is the total average amount for all Americans in checking accounts, savings accounts, and U.S. Savings Bonds?

77. What is the total average amount for Americans aged 16 to 34 in checking accounts, savings accounts, and U.S. Savings Bonds?

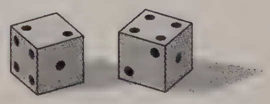
78. What is the total average amount for Americans aged 16 to 34 in all categories except home equity and retirement?

79. Is the sum of the average amounts invested in home equity and retirement for all Americans greater than or less than that same sum for Americans between the ages of 16 and 34?

APPLYING THE CONCEPTS

80. How many two-digit numbers are there? How many three-digit numbers are there?

81. If you roll two ordinary six-sided dice and add the two numbers that appear on top, how many different sums are possible?



82. If you add two *different* whole numbers, is the sum always greater than either one of the numbers? If not, give an example.

83. If you add two whole numbers, is the sum always greater than either one of the numbers? If not, give an example. (Compare this with the previous exercise.)

84. Make up a word problem for which the answer is the sum of 34 and 28.

85. Call a number "lucky" if it ends in a 7. How many lucky numbers are less than 100?

1.3 Subtraction of Whole Numbers

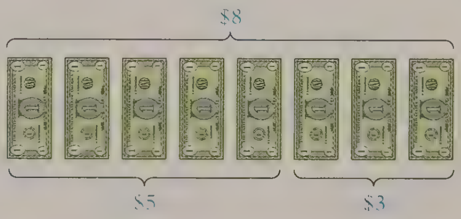
Objective A To subtract whole numbers without borrowing

Point of Interest

The use of the minus sign dates from the same period as the plus sign, around 1515.

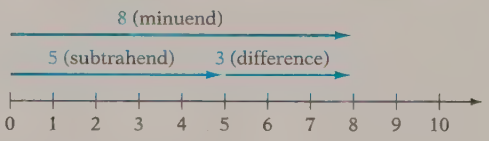
Subtraction is the process of finding the difference between two numbers.

By counting, we see that the difference between \$8 and \$5 is \$3.



\$8	−	\$5	=	\$3
Minuend		**Subtrahend**		**Difference**

The difference 8 − 5 can be shown on the number line.



Note from the number line that addition and subtraction are related.

$$\begin{array}{lr} \text{Subtrahend} & 5 \\ +\ \text{Difference} & \underline{+\ 3} \\ =\ \text{Minuend} & 8 \end{array}$$

The fact that the sum of the subtrahend and the difference equals the minuend can be used to check subtraction.

To subtract large numbers, begin by arranging the numbers vertically, keeping the digits that have the same place value in the same column. Then subtract the digits in each column.

➡ Subtract 8955 − 2432 and check.

$$\begin{array}{rcccc} & \text{THOUSANDS} & \text{HUNDREDS} & \text{TENS} & \text{ONES} \\ & 8 & 9 & 5 & 5 \\ - & 2 & 4 & 3 & 2 \\ \hline & 6 & 5 & 2 & 3 \end{array} \qquad \textit{Check:} \quad \begin{array}{lr} \text{Subtrahend} & 2432 \\ +\ \text{Difference} & \underline{+\ 6523} \\ =\ \text{Minuend} & 8955 \end{array}$$

Example 1 Subtract 6594 − 3271 and check.

Solution

$$\begin{array}{r} 6594 \\ \underline{-\ 3271} \\ 3323 \end{array} \qquad \textit{Check:} \quad \begin{array}{r} 3271 \\ \underline{+\ 3323} \\ 6594 \end{array}$$

You Try It 1 Subtract 8925 − 6413 and check.

Your solution

Example 2 Subtract 15,762 − 7541 and check.

Solution

$$\begin{array}{r} 15{,}762 \\ \underline{-\ \ 7{,}541} \\ 8{,}221 \end{array} \qquad \textit{Check:} \quad \begin{array}{r} 7{,}541 \\ \underline{+\ \ 8{,}221} \\ 15{,}762 \end{array}$$

You Try It 2 Subtract 17,504 − 9302 and check.

Your solution

Solutions on p. S1

Objective B To subtract whole numbers with borrowing

In all the subtraction problems in the previous objective, for each place value the lower digit was not larger than the upper digit. When the lower digit *is* larger than the upper digit, subtraction will involve "borrowing."

⇨ Subtract: 692 − 378

$$\begin{array}{rccc} & \text{HUNDREDS} & \text{TENS} & \text{ONES}\\ & & 8+1 & \\ & 6 & \cancel{9} & 2\\ - & 3 & 7 & 8\\ \hline \end{array}$$

Because 8 > 2, borrowing is necessary.
9 tens = 8 tens + 1 ten.

$$\begin{array}{rccc} & \text{HUNDREDS} & \text{TENS} & \text{ONES}\\ & & 8+\textcircled{1} & \curvearrowright 10\\ & 6 & \cancel{9} & 2\\ - & 3 & 7 & 8\\ \hline \end{array}$$

Borrow 1 ten from the tens column and write 10 in the ones column.

$$\begin{array}{rccc} & \text{HUNDREDS} & \text{TENS} & \text{ONES}\\ & & 8 & 12\\ & 6 & \cancel{9} & \cancel{2}\\ - & 3 & 7 & 8\\ \hline \end{array}$$

Add the borrowed 10 to 2.

$$\begin{array}{rccc} & \text{HUNDREDS} & \text{TENS} & \text{ONES}\\ & & 8 & 12\\ & 6 & \cancel{9} & \cancel{2}\\ - & 3 & 7 & 8\\ \hline & 3 & 1 & 4 \end{array}$$

Subtract the digits in each column.

The phrases below are used to indicate the operation of subtraction. An example is shown at the right of each phrase.

minus	8 minus 5	8 − 5
less	9 less 3	9 − 3
less than	2 less than 7	7 − 2
the difference between	the difference between 8 and 2	8 − 2
decreased by	5 decreased by 1	5 − 1

⇨ Find the difference between 1234 and 485 and check.

"The difference between 1234 and 485" means 1234 − 485.

$$\begin{array}{rcccc} & & & 2 & 14\\ & 1 & 2 & \cancel{3} & \cancel{4}\\ - & & 4 & 8 & 5\\ \hline & & & & 9 \end{array} \qquad \begin{array}{rcccc} & & 1 & 12 & 14\\ & 1 & \cancel{2} & \cancel{3} & \cancel{4}\\ - & & 4 & 8 & 5\\ \hline & & & 4 & 9 \end{array} \qquad \begin{array}{rcccc} & 0 & 11 & 12 & 14\\ & \cancel{1} & 2 & \cancel{3} & \cancel{4}\\ - & & 4 & 8 & 5\\ \hline & & 7 & 4 & 9 \end{array}$$

Check:

$$\begin{array}{r} {\scriptstyle 1\,1}\\ 485\\ +\ 749\\ \hline 1234 \end{array}$$

Subtraction with a zero in the minuend involves repeated borrowing.

⇨ Subtract: 3904 − 1775

$$\begin{array}{rcccc} & & 8 & 10 & \\ & 3 & \cancel{9} & \cancel{0} & 4\\ - & 1 & 7 & 7 & 5\\ \hline \end{array}$$

5 > 4
There is a 0 in the tens column. Borrow 1 hundred (= 10 tens) from the hundreds column and write 10 in the tens column.

$$\begin{array}{rcccc} & & & 9 & \\ & & 8 & \cancel{10} & 14\\ & 3 & \cancel{9} & \cancel{0} & \cancel{4}\\ - & 1 & 7 & 7 & 5\\ \hline \end{array}$$

Borrow 1 ten from the tens column and add 10 to the 4 in the ones column.

$$\begin{array}{rcccc} & & & 9 & \\ & & 8 & \cancel{10} & 14\\ & 3 & \cancel{9} & \cancel{0} & \cancel{4}\\ - & 1 & 7 & 7 & 5\\ \hline & 2 & 1 & 2 & 9 \end{array}$$

Subtract the digits in each column.

Example 3 Subtract 4392 − 678 and check.

Solution

$$\begin{array}{rrrrr} & \overset{3}{\cancel{4}} & \overset{13}{\cancel{3}} & \overset{8}{\cancel{9}} & \overset{12}{\cancel{2}} \\ - & & 6 & 7 & 8 \\ \hline & 3 & 7 & 1 & 4 \end{array} \qquad \textit{Check:} \quad \begin{array}{r} 678 \\ +\,3714 \\ \hline 4392 \end{array}$$

You Try It 3 Subtract 3481 − 865 and check.

Your solution

Example 4 Find 23,954 less than 63,221 and check.

Solution

$$\begin{array}{rrrrrr} & \overset{5}{\cancel{6}} & \overset{12}{\cancel{3}}, & \overset{11}{\cancel{2}} & \overset{11}{\cancel{2}} & \overset{11}{\cancel{1}} \\ - & 2 & 3, & 9 & 5 & 4 \\ \hline & 3 & 9, & 2 & 6 & 7 \end{array} \qquad \textit{Check:} \quad \begin{array}{r} 23{,}954 \\ +39{,}267 \\ \hline 63{,}221 \end{array}$$

You Try It 4 Find 54,562 decreased by 14,485 and check.

Your solution

Example 5 Subtract 46,005 − 32,167 and check.

Solution

$$\begin{array}{r} 4\overset{5}{\cancel{6}},\overset{10}{\cancel{0}}05 \\ -32{,}167 \\ \hline \end{array}$$

- There are two zeros in the minuend. Borrow 1 thousand from the thousands column and write 10 in the hundreds column.

$$\begin{array}{r} 4\overset{5}{\cancel{6}},\overset{\overset{9}{\cancel{10}}}{\cancel{0}}\overset{10}{\cancel{0}}5 \\ -32{,}167 \\ \hline \end{array}$$

- Borrow 1 hundred from the hundreds column and write 10 in the tens column.

$$\begin{array}{r} 4\overset{5}{\cancel{6}},\overset{\overset{9}{\cancel{10}}}{\cancel{0}}\overset{\overset{9}{\cancel{10}}}{\cancel{0}}\overset{15}{\cancel{5}} \\ -32{,}167 \\ \hline 13{,}838 \end{array}$$

- Borrow 1 ten from the tens column and add 10 to the 5 in the ones column.

$$\textit{Check:} \quad \begin{array}{r} 32{,}167 \\ +\,13{,}838 \\ \hline 46{,}005 \end{array}$$

You Try It 5 Subtract 64,003 − 54,936 and check.

Your solution

Solutions on p. S1

ESTIMATION

Estimating the Difference Between Two Whole Numbers

Estimate and then use your calculator to find 323,502 − 28,912.

To estimate the difference between two numbers, round each number to the same place value. In this case we will round to the nearest ten-thousand. Then subtract. The estimated answer is 290,000.

$$\begin{array}{rcr} 323{,}502 & \approx & 320{,}000 \\ -\ \ 28{,}912 & \approx & -\ \ 30{,}000 \\ \cline{3-3} & & 290{,}000 \end{array}$$

Now use your calculator to find the exact difference. The exact answer is 294,590.

323502 [−] 28912 [=] 294590

Objective C To solve application problems

The table at the right shows the number of personnel on active duty in the U.S. branches of the military in 1940 and 1945. Use this table for Example 6 and You Try It 6.

Branch	*1940*	*1945*
U.S. Army	267,767	8,266,373
U.S. Navy	160,997	3,380,817
U.S. Air Force	51,165	2,282,259
U.S. Marine Corps	28,345	474,680

Source: Dept. of the Army, Dept. of the Navy, Air Force Dept., Dept. of the Marines, U.S. Dept. of Defense

Example 6

Find the difference between the number of U.S. Army personnel on active duty in 1940 and in 1945.

Strategy

To find the difference, subtract the number of U.S. Army personnel on active duty in 1940 (267,767) from the number on active duty in 1945 (8,266,373).

Solution

$$\begin{array}{r} 8{,}266{,}373 \\ -\quad 267{,}767 \\ \hline 7{,}998{,}606 \end{array}$$

There were 7,998,606 more personnel on active duty in the U.S. Army in 1945 than in 1940.

You Try It 6

Find the difference between the number of personnel on active duty in the Navy and in the Air Force in 1945.

Your strategy

Your solution

Example 7

You had a balance of $415 on your student debit card. You then used the card, deducting $197 for books, $48 for art supplies, and $24 for theater tickets. What is your new student debit card balance?

Strategy

To find your new debit card balance:

- Add to find the total of the three deductions (197 + 48 + 24).
- Subtract the total of the three deductions from the old balance (415).

Solution

$$\begin{array}{r} 197 \\ 48 \\ +\ 24 \\ \hline 269 \end{array} \text{ total deductions} \qquad \begin{array}{r} 415 \\ -269 \\ \hline 146 \end{array}$$

Your new debit card balance is $146.

You Try It 7

Your total weekly salary is $638. Deductions of $127 for taxes, $18 for insurance, and $35 for savings are taken from your pay. Find your weekly take-home pay.

Your strategy

Your solution

Solutions on pp. S1–S2

1.3 Exercises

Objective A

Subtract.

1. $\begin{array}{r} 9 \\ -\ 5 \\ \hline \end{array}$

2. $\begin{array}{r} 8 \\ -\ 7 \\ \hline \end{array}$

3. $\begin{array}{r} 8 \\ -\ 4 \\ \hline \end{array}$

4. $\begin{array}{r} 7 \\ -\ 3 \\ \hline \end{array}$

5. $\begin{array}{r} 10 \\ -\ 0 \\ \hline \end{array}$

6. $\begin{array}{r} 11 \\ -\ 4 \\ \hline \end{array}$

7. $\begin{array}{r} 12 \\ -\ 8 \\ \hline \end{array}$

8. $\begin{array}{r} 19 \\ -\ 8 \\ \hline \end{array}$

9. $\begin{array}{r} 15 \\ -\ 6 \\ \hline \end{array}$

10. $\begin{array}{r} 16 \\ -\ 7 \\ \hline \end{array}$

11. $\begin{array}{r} 25 \\ -\ 3 \\ \hline \end{array}$

12. $\begin{array}{r} 55 \\ -\ 4 \\ \hline \end{array}$

13. $\begin{array}{r} 68 \\ -\ 8 \\ \hline \end{array}$

14. $\begin{array}{r} 77 \\ -\ 3 \\ \hline \end{array}$

15. $\begin{array}{r} 89 \\ -\ 23 \\ \hline \end{array}$

16. $\begin{array}{r} 54 \\ -\ 21 \\ \hline \end{array}$

17. $\begin{array}{r} 88 \\ -\ 57 \\ \hline \end{array}$

18. $\begin{array}{r} 1202 \\ -\ 701 \\ \hline \end{array}$

19. $\begin{array}{r} 1305 \\ -\ 404 \\ \hline \end{array}$

20. $\begin{array}{r} 1763 \\ -\ 801 \\ \hline \end{array}$

21. $\begin{array}{r} 1497 \\ -\ 706 \\ \hline \end{array}$

22. $\begin{array}{r} 8974 \\ -\ 3972 \\ \hline \end{array}$

23. $\begin{array}{r} 2836 \\ -\ 1711 \\ \hline \end{array}$

24. $\begin{array}{r} 8976 \\ -\ 7463 \\ \hline \end{array}$

25. $\begin{array}{r} 9273 \\ -\ 6142 \\ \hline \end{array}$

26. 77 − 36

27. 129 − 82

28. 132 − 61

29. 969 − 44

30. 1347 − 103

31. 4865 − 304

32. 1525 − 702

33. 9999 − 6794

34. 7806 − 3405

35. 8843 − 7621

36. What is 3795 minus 1092?

37. What is 9071 minus 6050?

38. Find the difference between 9763 and 541.

39. Find the difference between 6094 and 3072.

40. What is 3701 less than 6932?

41. What is 2031 less than 5071?

42. Find 6509 decreased by 3102.

43. Find 7994 decreased by 7782.

44. Find 23,907 less 12,705.

45. Find 65,986 less 5741.

Objective B

Subtract.

46. $\begin{array}{r} 71 \\ -\ 18 \\ \hline \end{array}$

47. $\begin{array}{r} 93 \\ -\ 28 \\ \hline \end{array}$

48. $\begin{array}{r} 47 \\ -\ 18 \\ \hline \end{array}$

49. $\begin{array}{r} 44 \\ -\ 27 \\ \hline \end{array}$

50. $\begin{array}{r} 37 \\ -\ 29 \\ \hline \end{array}$

51. $\begin{array}{r} 50 \\ -\ 27 \\ \hline \end{array}$

52. $\begin{array}{r} 70 \\ -\ 33 \\ \hline \end{array}$

53. $\begin{array}{r} 993 \\ -\ 537 \\ \hline \end{array}$

54. $\begin{array}{r} 250 \\ -\ 192 \\ \hline \end{array}$

55. $\begin{array}{r} 840 \\ -\ 783 \\ \hline \end{array}$

56. $\begin{array}{r} 768 \\ -\ 194 \\ \hline \end{array}$

57. $\begin{array}{r} 770 \\ -\ 395 \\ \hline \end{array}$

58. 674 − 337

59. 3526 − 387

60. 1712 − 289

61. 4350 − 729

62. 1702 − 948

63. 1607 − 869

64. 5933 − 3754

65. 7293 − 3748

66. 9407 − 2918

67. 3706 − 2957

68. 8605 − 7716

69. 8052 − 2709

70. 80,305 − 9176

71. 70,702 − 4239

72. 10,004 − 9306

73. 80,009 − 63,419

74. 70,618 − 41,213

75. 80,053 − 27,649

76. 70,700 − 21,076

77. 80,800 − 42,023

78. $\begin{array}{r} 2600 \\ -\ 1972 \\ \hline \end{array}$

79. $\begin{array}{r} 8400 \\ -\ 3762 \\ \hline \end{array}$

80. $\begin{array}{r} 9003 \\ -\ 2471 \\ \hline \end{array}$

81. $\begin{array}{r} 6004 \\ -\ 2392 \\ \hline \end{array}$

82. $\begin{array}{r} 8202 \\ -\ 3916 \\ \hline \end{array}$

83. $\begin{array}{r} 7050 \\ -\ 4137 \\ \hline \end{array}$

84. $\begin{array}{r} 7015 \\ -\ 2973 \\ \hline \end{array}$

85. $\begin{array}{r} 4207 \\ -\ 1624 \\ \hline \end{array}$

86. $\begin{array}{r} 7005 \\ -\ 1796 \\ \hline \end{array}$

87. $\begin{array}{r} 8003 \\ -\ 2735 \\ \hline \end{array}$

88. $\begin{array}{r} 20{,}005 \\ -\ \ 9{,}627 \\ \hline \end{array}$

89. $\begin{array}{r} 80{,}004 \\ -\ \ 8{,}237 \\ \hline \end{array}$

90. Find 10,051 less 9027.

91. Find 17,031 less 5792.

92. Find the difference between 1003 and 447.

93. What is 29,874 minus 21,392?

94. What is 29,797 less than 68,005?

95. What is 69,379 less than 70,004?

96. What is 25,432 decreased by 7994?

97. What is 86,701 decreased by 9976?

Use the relationship between addition and subtraction to complete the statement.

98. ____ + 39 = 104

99. 67 + ____ = 90

100. ____ + 497 = 862

101. 253 + ____ = 4901

Estimate by rounding to the nearest ten-thousand. Then use your calculator to subtract.

102. $\begin{array}{r} 80,032 \\ -\ 19,605 \\ \hline \end{array}$

103. $\begin{array}{r} 90,765 \\ -\ 60,928 \\ \hline \end{array}$

104. $\begin{array}{r} 32,574 \\ -\ 10,961 \\ \hline \end{array}$

105. $\begin{array}{r} 96,430 \\ -\ 59,762 \\ \hline \end{array}$

106. $\begin{array}{r} 567,423 \\ -\ 208,444 \\ \hline \end{array}$

107. $\begin{array}{r} 300,712 \\ -\ 198,714 \\ \hline \end{array}$

Objective C *Application Problems*

108. You have $304 in your checking account. If you write a check for $139, how much is left in your checking account?

109. In a recent year, the U.S. Patent and Trademark Office granted 151,024 patents. IBM received 2682 of those patents, more than any other company. Canon, with 1934 patents, received the second highest number. (Source: IFI/Plenum Data) How many more patents was IBM granted than Canon?

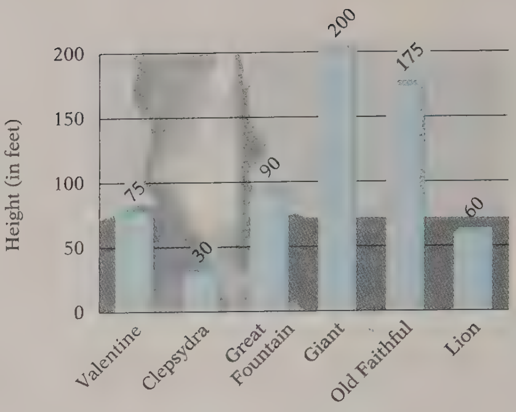
110. The tennis coach at a high school purchased a video camera that costs $1079 and made a down payment of $180. Find the amount that remains to be paid.

111. Rod Guerra, an engineer, purchased a used car that cost $11,225 and made a down payment of $950. Find the amount that remains to be paid.

112. The leader in pass receiving for the 1990s in the National Football League was Jerry Rice with 12,078 yards. His closest competitor was Michael Irvin with 10,872 yards. (Source: NFL) How many more yards did Jerry Rice gain than Michael Irvin?

113. Use the graph at the right to find the difference between the maximum height to which Great Fountain geyser erupts and the maximum height to which Valentine erupts.

114. According to the graph at the right, how much higher is the eruption of the Giant than that of Old Faithful?



The Maximum Heights of the Eruptions of Six Geysers at Yellowstone National Park

115. The top six live acts of the 1990s are shown in the table at the right.

a. How much more did the Rolling Stones gross from their concerts in the 1990s than the Grateful Dead?

b. How much more did Neil Diamond gross from his concerts in the 1990s than Jimmy Buffett?

Live Acts	*Gross*
Rolling Stones	$751,000,000
Grateful Dead	$285,000,000
U2	$283,000,000
Eagles	$197,000,000
Neil Diamond	$183,000,000
Jimmy Buffett	$167,000,000

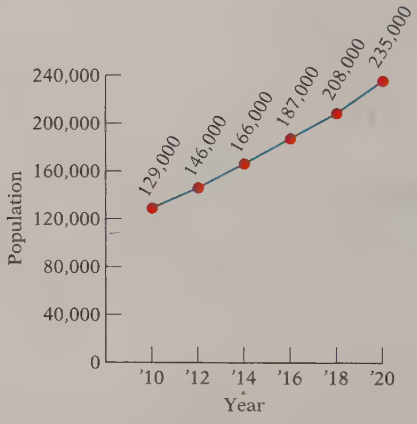
Source: Amusement Business BOXSCORE

The graph at the right shows the expected U.S. population aged 100 and over, every two years from 2010 to 2020. Use this information for Exercises 116 and 117.

116. What is the expected growth in the population aged 100 and over during the 10-year period?

117. **a.** Which 2-year period has the smallest increase in the number of people aged 100 and over?

b. Which 2-year period has the greatest increase?



Expected U.S. Population Aged 100 and Over

Source: Census Bureau

APPLYING THE CONCEPTS

118. Answer true or false.

a. The phrases "the difference between 9 and 5" and "5 less than 9" mean the same thing.

b. $9 - (5 - 3) = (9 - 5) - 3$.

c. Subtraction is an associative operation. *Hint*: See part b of this exercise.

119. Make up a word problem for which the difference between 15 and 8 is the answer.

1.4 Multiplication of Whole Numbers

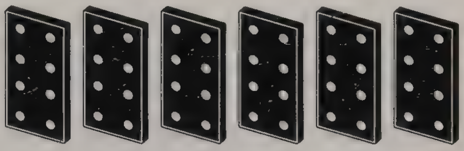
Objective A To multiply a number by a single digit

Six boxes of toasters are ordered. Each box contains eight toasters. How many toasters are ordered?

This problem can be worked by adding 6 eights.

$8 + 8 + 8 + 8 + 8 + 8 = 48$

This problem involves repeated addition of the same number and can be worked by a shorter process called **multiplication.** Multiplication is the repeated addition of the same number.



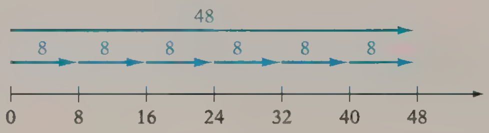
$8 + 8 + 8 + 8 + 8 + 8 = 48$

The numbers that are multiplied are called **factors.** The answer is called the **product.**

or

6	$\times$	8	$= 48$
Factor		**Factor**	**Product**

The product of 6×8 can be represented on the number line. The arrow representing the whole number 8 is repeated 6 times. The result is the arrow representing 48.



The times sign "×" is one symbol that is used to mean multiplication. Another common symbol used for this purpose is a dot placed between the numbers.

$$7 \times 8 = 56 \qquad 7 \cdot 8 = 56$$

As with addition, there are some useful properties of multiplication.

Multiplication Property of Zero

The product of a number and zero is zero.

$0 \times 4 = 0$
$7 \times 0 = 0$

Multiplication Property of One

The product of a number and one is the number.

$1 \times 6 = 6$
$8 \times 1 = 8$

Commutative Property of Multiplication

Two numbers can be multiplied in either order. The product will be the same.

$4 \times 3 = 3 \times 4$
$12 = 12$

Associative Property of Multiplication

Grouping the numbers to be multiplied in any order gives the same result. Do the multiplication inside the parentheses first.

$$\underbrace{(4 \times 2)}_{8} \times 3 = 4 \times \underbrace{(2 \times 3)}_{6}$$
$$24 = 24$$

The basic facts for multiplying one-digit numbers should be memorized. Multiplication of larger numbers requires the repeated use of the basic multiplication facts.

⇨ Multiply: 37 × 4

$$\begin{array}{rr} & \overset{2}{3}\ 7 \\ \times & 4 \\ \hline & 8 \end{array}$$

- $4 \times 7 = 28$ (2 tens + 8 ones). Write the 8 in the ones column and carry the 2 to the tens column.

$$\begin{array}{rr} & \overset{2}{3}\ 7 \\ \times & 4 \\ \hline & 14\ 8 \end{array}$$

- The 3 in 37 is 3 tens.

$$\begin{array}{lr} 4 \times 3 \text{ tens} = & 12 \text{ tens} \\ \text{Add the carry digit.} & +\ 2 \text{ tens} \\ \hline & 14 \text{ tens} \end{array}$$

- Write the 14.

The phrases below are used to indicate the operation of multiplication. An example is shown at the right of each phrase.

times	7 times 3	$7 \cdot 3$
the product of	the product of 6 and 9	$6 \cdot 9$
multiplied by	8 multiplied by 2	$2 \cdot 8$

Example 1 Multiply: 735 × 9

Solution

$$\begin{array}{rr} & \overset{3}{7}\overset{4}{3}5 \\ \times & 9 \\ \hline & 6615 \end{array}$$

You Try It 1 Multiply: 648 × 7

Your solution

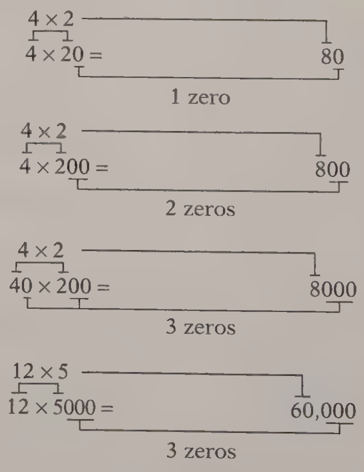
Solution on p. S2

Objective B To multiply larger whole numbers

Note the pattern when the following numbers are multiplied.

Multiply the nonzero part of the factors.

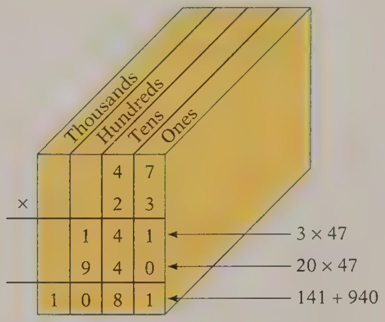
Now attach the same number of zeros to the product as the total number of zeros in the factors.



➡ Find the product of 47 and 23.

Multiply by the ones digit.	Multiply by the tens digit.	Add.
$\begin{array}{r} 47 \\ \times\ 23 \\ \hline 141 \end{array}$ (= 47 × 3)	$\begin{array}{r} 47 \\ \times\ 23 \\ \hline 141 \\ 940 \\ \hline \end{array}$ (= 47 × 20)	$\begin{array}{r} 47 \\ \times\ 23 \\ \hline 141 \\ 940 \\ \hline 1081 \end{array}$

Writing the 0 is optional.



The place-value chart illustrates the placement of the products.

Note the placement of the products when we are multiplying by a factor that contains a zero.

➡ Multiply: 439 × 206

$$\begin{array}{r} 439 \\ \times\ 206 \\ \hline 2634 \\ 000 \\ 878 \\ \hline 90{,}434 \end{array} \qquad 0 \times 439$$

When working the problem, we usually write only one zero. Writing this zero ensures the proper placement of the products.

$$\begin{array}{r} 439 \\ \times\ 206 \\ \hline 2634 \\ 8780 \\ \hline 90{,}434 \end{array}$$

Example 2 Find 829 multiplied by 603.

Solution

$$\begin{array}{r} 829 \\ \times\ 603 \\ \hline 2487 \\ 49740 \\ \hline 499{,}887 \end{array}$$

You Try It 2 Multiply: 756 × 305

Your solution

Solution on p. S2

ESTIMATION

Estimating the Product of Two Whole Numbers

Estimate and then use your calculator to find 3267 × 389.

To estimate the product, round each number so that all the digits are zero except the first digit. Then multiply. The estimated answer is 1,200,000.

$$\begin{array}{r} 3267 \approx \\ \times\ 389 \approx \\ \hline \end{array} \quad \begin{array}{r} 3000 \\ \times\ 400 \\ \hline 1{,}200{,}000 \end{array}$$

Now use your calculator to find the exact answer. The exact answer is 1,270,863.

3267 ⊠ 389 ⊟ 1270863

Objective C To solve application problems

Example 3

An auto mechanic receives a salary of $1050 each week. How much does the auto mechanic earn in 4 weeks?

Strategy

To find the mechanic's earnings for 4 weeks, multiply the weekly salary (1050) by the number of weeks (4).

Solution

$$\begin{array}{r} 1050 \\ \times \quad 4 \\ \hline 4200 \end{array}$$

The mechanic earns $4200 in 4 weeks.

You Try It 3

A new-car dealer receives a shipment of 37 cars each month. Find the number of cars the dealer will receive in 12 months.

Your strategy

Your solution

Example 4

A press operator earns $640 for working a 40-hour week. This week the press operator also worked 7 hours of overtime at $26 an hour. Find the press operator's total pay for the week.

Strategy

To find the press operator's total pay for the week:

- Find the overtime pay by multiplying the hours of overtime (7) by the overtime rate of pay (26).
- Add the weekly salary (640) to the overtime pay.

Solution

$$\begin{array}{r} 26 \\ \times \quad 7 \\ \hline 182 \end{array} \text{ overtime pay} \qquad \begin{array}{r} 640 \\ +182 \\ \hline 822 \end{array}$$

The press operator earned $822 this week.

You Try It 4

The buyer for Ross Department Store can buy 80 men's suits for $4800. Each sports jacket will cost the store $23. The manager orders 80 men's suits and 25 sports jackets. What is the total cost of the order?

Your strategy

Your solution

Solutions on p. S2

1.4 Exercises

Objective A

Write each of the following as a product.

1. 2 + 2 + 2 + 2 + 2 + 2

2. 4 + 4 + 4 + 4 + 4

3. 7 + 7 + 7 + 7

4. 18 + 18 + 18

Multiply.

5. $\begin{array}{r} 3 \\ \times 4 \\ \hline \end{array}$

6. $\begin{array}{r} 2 \\ \times 8 \\ \hline \end{array}$

7. $\begin{array}{r} 5 \\ \times 7 \\ \hline \end{array}$

8. $\begin{array}{r} 6 \\ \times 4 \\ \hline \end{array}$

9. $\begin{array}{r} 5 \\ \times 5 \\ \hline \end{array}$

10. $\begin{array}{r} 7 \\ \times 7 \\ \hline \end{array}$

11. $\begin{array}{r} 0 \\ \times 7 \\ \hline \end{array}$

12. $\begin{array}{r} 8 \\ \times 0 \\ \hline \end{array}$

13. $\begin{array}{r} 8 \\ \times 9 \\ \hline \end{array}$

14. $\begin{array}{r} 7 \\ \times 6 \\ \hline \end{array}$

15. $\begin{array}{r} 66 \\ \times\ \ 3 \\ \hline \end{array}$

16. $\begin{array}{r} 70 \\ \times\ \ 4 \\ \hline \end{array}$

17. $\begin{array}{r} 67 \\ \times\ \ 5 \\ \hline \end{array}$

18. $\begin{array}{r} 127 \\ \times\ \ \ 9 \\ \hline \end{array}$

19. $\begin{array}{r} 623 \\ \times\ \ \ 4 \\ \hline \end{array}$

20. $\begin{array}{r} 802 \\ \times\ \ \ 5 \\ \hline \end{array}$

21. $\begin{array}{r} 607 \\ \times\ \ \ 9 \\ \hline \end{array}$

22. $\begin{array}{r} 300 \\ \times\ \ \ 5 \\ \hline \end{array}$

23. $\begin{array}{r} 600 \\ \times\ \ \ 7 \\ \hline \end{array}$

24. $\begin{array}{r} 906 \\ \times\ \ \ 8 \\ \hline \end{array}$

25. $\begin{array}{r} 703 \\ \times\ \ \ 9 \\ \hline \end{array}$

26. $\begin{array}{r} 127 \\ \times\ \ \ 5 \\ \hline \end{array}$

27. $\begin{array}{r} 632 \\ \times\ \ \ 3 \\ \hline \end{array}$

28. $\begin{array}{r} 559 \\ \times\ \ \ 4 \\ \hline \end{array}$

29. $\begin{array}{r} 632 \\ \times\ \ \ 8 \\ \hline \end{array}$

30. $\begin{array}{r} 524 \\ \times\ \ \ 4 \\ \hline \end{array}$

31. $\begin{array}{r} 337 \\ \times\ \ \ 5 \\ \hline \end{array}$

32. $\begin{array}{r} 841 \\ \times\ \ \ 6 \\ \hline \end{array}$

33. $\begin{array}{r} 6709 \\ \times\ \ \ \ 7 \\ \hline \end{array}$

34. $\begin{array}{r} 3608 \\ \times\ \ \ \ 5 \\ \hline \end{array}$

35. $\begin{array}{r} 8568 \\ \times\ \ \ \ 7 \\ \hline \end{array}$

36. $\begin{array}{r} 5495 \\ \times\ \ \ \ 4 \\ \hline \end{array}$

37. $\begin{array}{r} 4780 \\ \times\ \ \ \ 4 \\ \hline \end{array}$

38. $\begin{array}{r} 3690 \\ \times\ \ \ \ 5 \\ \hline \end{array}$

39. $\begin{array}{r} 9895 \\ \times\ \ \ \ 2 \\ \hline \end{array}$

40. Find the product of 5, 7, and 4.

41. Find the product of 6, 2, and 9.

42. Find the product of 5304 and 9.

43. Find the product of 458 and 8.

44. What is 3208 multiplied by 7?

45. What is 5009 multiplied by 4?

46. What is 3105 times 6?

47. What is 8957 times 8?

Objective B

Multiply.

48. $\begin{array}{r} 16 \\ \times\ 21 \\ \hline \end{array}$

49. $\begin{array}{r} 18 \\ \times\ 24 \\ \hline \end{array}$

50. $\begin{array}{r} 35 \\ \times\ 26 \\ \hline \end{array}$

51. $\begin{array}{r} 27 \\ \times\ 72 \\ \hline \end{array}$

52. $\begin{array}{r} 693 \\ \times\ 91 \\ \hline \end{array}$

53. $\begin{array}{r} 581 \\ \times\ 72 \\ \hline \end{array}$

54. $\begin{array}{r} 419 \\ \times\ 80 \\ \hline \end{array}$

55. $\begin{array}{r} 727 \\ \times\ 60 \\ \hline \end{array}$

56. $\begin{array}{r} 8279 \\ \times\ 46 \\ \hline \end{array}$

57. $\begin{array}{r} 9577 \\ \times\ 35 \\ \hline \end{array}$

58. $\begin{array}{r} 6938 \\ \times\ 78 \\ \hline \end{array}$

59. $\begin{array}{r} 8875 \\ \times\ 67 \\ \hline \end{array}$

60. $\begin{array}{r} 7035 \\ \times\ 57 \\ \hline \end{array}$

61. $\begin{array}{r} 6702 \\ \times\ 48 \\ \hline \end{array}$

62. $\begin{array}{r} 3009 \\ \times\ 35 \\ \hline \end{array}$

63. $\begin{array}{r} 6003 \\ \times\ 57 \\ \hline \end{array}$

64. $\begin{array}{r} 809 \\ \times\ 530 \\ \hline \end{array}$

65. $\begin{array}{r} 607 \\ \times\ 460 \\ \hline \end{array}$

66. $\begin{array}{r} 800 \\ \times\ 325 \\ \hline \end{array}$

67. $\begin{array}{r} 700 \\ \times\ 274 \\ \hline \end{array}$

68. $\begin{array}{r} 987 \\ \times\ 349 \\ \hline \end{array}$

69. $\begin{array}{r} 688 \\ \times\ 674 \\ \hline \end{array}$

70. $\begin{array}{r} 312 \\ \times\ 134 \\ \hline \end{array}$

71. $\begin{array}{r} 423 \\ \times\ 427 \\ \hline \end{array}$

72. $\begin{array}{r} 379 \\ \times\ 500 \\ \hline \end{array}$

73. $\begin{array}{r} 684 \\ \times\ 700 \\ \hline \end{array}$

74. $\begin{array}{r} 985 \\ \times\ 408 \\ \hline \end{array}$

75. $\begin{array}{r} 758 \\ \times\ 209 \\ \hline \end{array}$

76. $\begin{array}{r} 3407 \\ \times\ 309 \\ \hline \end{array}$

77. $\begin{array}{r} 5207 \\ \times\ 902 \\ \hline \end{array}$

78. $\begin{array}{r} 4258 \\ \times\ 986 \\ \hline \end{array}$

79. $\begin{array}{r} 6327 \\ \times\ 876 \\ \hline \end{array}$

80. What is 5763 times 45?

81. What is 7349 times 27?

82. Find the product of 2, 19, and 34.

83. Find the product of 6, 73, and 43.

84. What is 376 multiplied by 402?

85. What is 842 multiplied by 309?

86. Find the product of 3005 and 233,489.

87. Find the product of 9007 and 34,985.

Estimate and then use your calculator to multiply.

88. $\begin{array}{r} 8745 \\ \times \quad 63 \\ \hline \end{array}$

89. $\begin{array}{r} 4732 \\ \times \quad 93 \\ \hline \end{array}$

90. $\begin{array}{r} 2937 \\ \times \; 206 \\ \hline \end{array}$

91. $\begin{array}{r} 8941 \\ \times \; 726 \\ \hline \end{array}$

92. $\begin{array}{r} 3097 \\ \times 1025 \\ \hline \end{array}$

93. $\begin{array}{r} 6379 \\ \times 2936 \\ \hline \end{array}$

94. $\begin{array}{r} 32{,}508 \\ \times \quad 591 \\ \hline \end{array}$

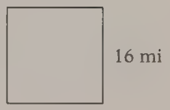
95. $\begin{array}{r} 62{,}504 \\ \times \quad 923 \\ \hline \end{array}$

Objective C Application Problems

96. Rob Hill owns a compact car that averages 43 miles on 1 gallon of gas. How many miles could the car travel on 12 gallons of gas?

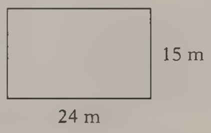
97. A plane flying from Los Angeles to Boston uses 865 gallons of jet fuel each hour. How many gallons of jet fuel were used on a 6-hour flight?

98. The perimeter of a square is equal to four times the length of a side of the square. Find the perimeter of a square whose side measures 16 miles.



99. To find the area of a square, multiply the length of one side of the square times itself. What is the area of a square whose side measures 16 miles? The area of the square will be in square miles.

100. The area of a rectangle is equal to the product of the length of the rectangle times its width. Find the area of a rectangle that has a length of 24 meters and a width of 15 meters. The area will be in square meters.



A lighting consultant to a bank suggests that the bank lobby contain 43 can lights, 15 high-intensity lights, 20 fire safety lights, and one chandelier. The table at the right gives the costs for each type of light from two companies. Use this table for Exercises 101 and 102.

	Company A	*Company B*
Can lights	$2 each	$3 each
High-intensity	$6 each	$4 each
Fire safety	$12 each	$11 each
Chandelier	$998 each	$1089 each

101. Which company offers the lights for the lower total price?

102. How much can the lighting consultant save by purchasing the lights from the company that offers the lower total price?

The table at the right shows the hourly wages of four different job classifications at a small construction company. Use this table for Exercises 103 to 105.

Type of Work	*Wage per Hour*
Electrician	$34
Plumber	$30
Clerk	$16
Bookkeeper	$20

103. The owner of this company wants to provide the electrical installation for a new house. On the basis of the architectural plans for the house, it is estimated that it will require 3 electricians each working 50 hours to complete the job. What is the estimated cost for the electricians' labor?

104. Carlos Vasquez, a plumbing contractor, hires 4 plumbers from this company at the hourly wage given in the table. If each plumber works 23 hours, what are the total wages paid by Carlos?

105. The owner of this company estimates that remodeling a kitchen will require 1 electrician working 30 hours and 1 plumber working 33 hours. This project also requires 3 hours of clerical work and 4 hours of bookkeeping. What is the total cost for these four components of this remodeling?

APPLYING THE CONCEPTS

106. Determine whether each of the following statements is always true, sometimes true, or never true.

a. A whole number times zero is zero.
b. A whole number times one is the whole number.
c. The product of two whole numbers is greater than either one of the whole numbers.

107. According to the National Safety Council, in a recent year a death resulting from an accident occurred at the rate of 1 every 5 minutes. At this rate, how many accidental deaths occurred each hour? Each day? Throughout the year? Explain how you arrived at your answers.

108. According to the Population Reference Bureau, in the world today, 261 people are born every minute and 101 people die every minute. Using this statistic, what is the increase in the world's population every hour? Every day? Every week? Every year? Use a 365-day year. Explain how you arrived at your answers.

109. Pick your favorite number between 1 and 9. Multiply the number by 3. Multiply that product by 37,037. How is the product related to your favorite number? Explain why this works. (*Suggestion*: Multiply 3 and 37,037 first.)

110. There are quite a few tricks based on whole numbers. Here's one about birthdays. Write down the month in which you were born. Multiply by 5. Add 7. Multiply by 20. Subtract 100. Add the day of the month on which you were born. Multiply by 4. Subtract 100. Multiply by 25. Add the year you were born. Subtract 3400. The answer is the month/day/year of your birthday.

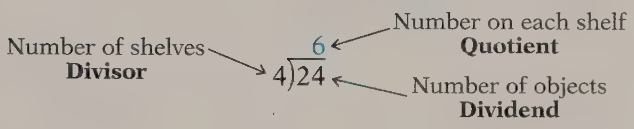
1.5 Division of Whole Numbers

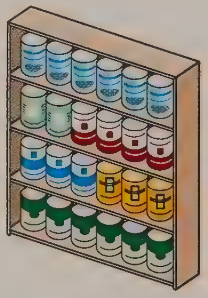
Objective A **To divide by a single digit with no remainder in the quotient**

Division is used to separate objects into equal groups.

A store manager wants to display 24 new objects equally on 4 shelves. From the diagram, we see that the manager would place 6 objects on each shelf.

The manager's division problem can be written as follows:





Note that the quotient multiplied by the divisor equals the dividend.

$\begin{array}{r}6\\4\overline{)24}\end{array}$ because $\boxed{\begin{array}{c}6\\\text{Quotient}\end{array}} \times \boxed{\begin{array}{c}4\\\text{Divisor}\end{array}} = \boxed{\begin{array}{c}24\\\text{Dividend}\end{array}}$

$\begin{array}{r}6\\9\overline{)54}\end{array}$ because $6 \times 9 = 54$

$\begin{array}{r}5\\8\overline{)40}\end{array}$ because $5 \times 8 = 40$

Here are some important quotients and the properties of zero in division:

Important Quotients

Any whole number, except zero, divided by itself is 1. $\begin{array}{r}1\\8\overline{)8}\end{array}$ $\begin{array}{r}1\\14\overline{)14}\end{array}$ $\begin{array}{r}1\\10\overline{)10}\end{array}$

Any whole number divided by 1 is the whole number. $\begin{array}{r}9\\1\overline{)9}\end{array}$ $\begin{array}{r}27\\1\overline{)27}\end{array}$ $\begin{array}{r}10\\1\overline{)10}\end{array}$

Properties of Zero in Division

Zero divided by any other whole number is zero. $\begin{array}{r}0\\7\overline{)0}\end{array}$ $\begin{array}{r}0\\13\overline{)0}\end{array}$ $\begin{array}{r}0\\10\overline{)0}\end{array}$

Division by zero is not allowed. $\begin{array}{r}\boxed{?}\\0\overline{)8}\end{array}$ There is no number whose product with 0 is 8.

When the dividend is a larger whole number, the digits in the quotient are found in steps.

➡ Divide $4\overline{)3192}$ and check.

$$\begin{array}{r} 7 \\ 4\overline{)\,3192} \\ -28 \\ \hline 39 \end{array}$$

- **Think $4\overline{)31}$.**
- **Subtract 7 × 4.**
- **Bring down the 9.**

$$\begin{array}{r} 79 \\ 4\overline{)\,3192} \\ -28 \\ \hline 39 \\ -36 \\ \hline 32 \end{array}$$

- **Think $4\overline{)39}$.**
- **Subtract 9 × 4.**
- **Bring down the 2.**

$$\begin{array}{r} 798 \\ 4\overline{)\,3192} \\ -28 \\ \hline 39 \\ -36 \\ \hline 32 \\ -32 \\ \hline 0 \end{array}$$

- **Think $4\overline{)32}$.**
- **Subtract 8 × 4.**

Check:

$$\begin{array}{r} 798 \\ \times\quad 4 \\ \hline 3192 \end{array}$$

The place-value chart can be used to show why this method works.

$$\begin{array}{rcccl} & \text{HUNDREDS} & \text{TENS} & \text{ONES} & \\ & 7 & 9 & 8 & \\ 4\overline{)\,3} & 1 & 9 & 2 & \\ -2 & 8 & 0 & 0 & \text{7 hundreds} \times 4 \\ \hline & 3 & 9 & 2 & \\ & -3 & 6 & 0 & \text{9 tens} \times 4 \\ \hline & & 3 & 2 & \\ & & -3 & 2 & \text{8 ones} \times 4 \\ \hline & & & 0 & \end{array}$$

There are other ways of expressing division.

54 divided by 9 equals 6.

54 ÷ 9 equals 6.

$\frac{54}{9}$ equals 6.

Example 1 Divide $7\overline{)56}$ and check.

Solution

$$\begin{array}{r} 8 \\ 7\overline{)56} \end{array}$$

Check: $8 \times 7 = 56$

You Try It 1 Divide $9\overline{)63}$ and check.

Your solution

Example 2 Divide $2808 \div 8$ and check.

Solution

$$\begin{array}{r} 351 \\ 8\overline{)\ 2808} \\ \underline{-24} \\ 40 \\ \underline{-40} \\ 08 \\ \underline{-\ 8} \\ 0 \end{array}$$

Check: $351 \times 8 = 2808$

You Try It 2 Divide $4077 \div 9$ and check.

Your solution

Example 3 Divide $7\overline{)2856}$ and check.

Solution

$$\begin{array}{r} 408 \\ 7\overline{)\ 2856} \\ \underline{-28} \\ 05 \\ \underline{-0} \\ 56 \\ \underline{-56} \\ 0 \end{array}$$

- Think $7\overline{)5}$. Place 0 in quotient. Subtract 0×7.
- Bring down the 6.

Check: $408 \times 7 = 2856$

You Try It 3 Divide $9\overline{)6345}$ and check.

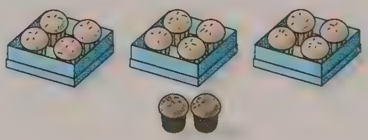
Your solution

Solutions on p. S2

Objective B To divide by a single digit with a remainder in the quotient

Sometimes it is not possible to separate objects into a whole number of equal groups.

A baker has 14 muffins to pack into 3 boxes. Each box holds 4 muffins. From the diagram, we see that after the baker places 4 muffins in each box, there are 2 left over. The 2 is called the **remainder**.



The clerk's division problem could be written

$$\begin{array}{rrl}
 & 4 & \swarrow \ \textbf{Quotient} \\
 & & \text{(Number in each box)} \\
\textbf{Divisor} \longrightarrow & 3\overline{)\,14} & \longleftarrow \ \textbf{Dividend} \\
\text{(Number of boxes)} & \underline{-12} & \text{(Total number of objects)} \\
 & 2 & \longleftarrow \ \textbf{Remainder} \\
 & & \text{(Number left over)}
\end{array}$$

The answer to a division problem with a remainder is frequently written

$$\begin{array}{r} 4\text{ r}2 \\ 3\overline{)14} \end{array}$$

Note that $\boxed{\underset{\text{Quotient}}{4} \times \underset{\text{Divisor}}{3}} + \boxed{\underset{\text{Remainder}}{2}} = \boxed{\underset{\text{Dividend}}{14}}$

Example 4 Divide $4\overline{)2522}$ and check.

Solution

$$\begin{array}{r}
630\text{ r}2 \\
4\overline{)\,2522} \\
\underline{-24} \\
12 \\
\underline{-12} \\
02 \\
\underline{-0} \\
2
\end{array}$$

- Think $4\overline{)2}$. Place 0 in quotient.
- Subtract 0 × 4.

Check: $(630 \times 4) + 2 =$

$2520 \quad + 2 = 2522$

You Try It 4 Divide $6\overline{)5225}$ and check.

Your solution

Example 5 Divide $9\overline{)27{,}438}$ and check.

Solution

$$\begin{array}{r}
3{,}048\text{ r}6 \\
9\overline{)\,27{,}438} \\
\underline{-27\phantom{{,}438}} \\
0\,4 \\
\underline{-0} \\
43 \\
\underline{-36} \\
78 \\
\underline{-72} \\
6
\end{array}$$

- Think $9\overline{)4}$.
- Subtract 0 × 9.

Check: $(3048 \times 9) + 6 =$

$27{,}432 \quad + 6 = 27{,}438$

You Try It 5 Divide $7\overline{)21{,}409}$ and check.

Your solution

Solutions on p. S2

Objective C To divide by larger whole numbers

When the divisor has more than one digit, estimate at each step by using the first digit of the divisor. If that product is too large, lower the guess by 1 and try again.

Divide $34\overline{)1598}$ and check.

$$\begin{array}{r} 5\\ 34\overline{)\,1598}\\ -170\\ \hline \end{array}$$

- Think $3\overline{)15}$.
- Subtract 5 × 34.

170 is too large. Lower the guess by 1 and try again.

$$\begin{array}{r} 4\\ 34\overline{)\,1598}\\ -136\\ \hline 238 \end{array}$$

- Subtract 4 × 34.

$$\begin{array}{r} 47\\ 34\overline{)\,1598}\\ -136\\ \hline 238\\ -238\\ \hline 0 \end{array}$$

- Think $3\overline{)23}$.
- Subtract 7 × 34.

Check:

$$\begin{array}{r} 47\\ \times 34\\ \hline 188\\ 141\\ \hline 1598 \end{array}$$

The phrases below are used to indicate the operation of division. An example is shown at the right of each phrase.

the quotient of	the quotient of 9 and 3	9 ÷ 3
divided by	6 divided by 2	6 ÷ 2

Example 6 Find 7077 divided by 34 and check.

Solution

$$\begin{array}{r} 208\text{ r5}\\ 34\overline{)\,7077}\phantom{\text{ r5}}\\ -68\phantom{\text{ r5}}\\ \hline 27\phantom{\text{ r5}}\\ -\ 0\phantom{\text{ r5}}\\ \hline 277\phantom{\text{ r5}}\\ -272\phantom{\text{ r5}}\\ \hline 5\phantom{\text{ r5}} \end{array}$$

- Think $34\overline{)27}$.
- Place 0 in the quotient.
- Subtract 0 × 34.

Check: (208 × 34) + 5 =
7072 + 5 = 7077

You Try It 6 Divide 4578 ÷ 42 and check.

Your solution

Solution on p. S3

Example 7 Find the quotient of 21,312 and 56 and check.

Solution

$$\begin{array}{r} 380 \text{ r}32 \\ 56\overline{)21,312} \\ -16\,8 \\ \hline 4\,51 \\ -4\,48 \\ \hline 32 \\ -0 \\ \hline 32 \end{array}$$

Think $5\overline{)21}$. 4×56 is too large. Try 3.

Check: $(380 \times 56) + 32 =$
$21,280 + 32 = 21,312$

You Try It 7 Divide 18,359 ÷ 39 and check.

Your solution

Example 8 Divide $427\overline{)24,782}$ and check.

Solution

$$\begin{array}{r} 58 \text{ r}16 \\ 427\overline{)24,782} \\ -21\,35 \\ \hline 3\,432 \\ -3\,416 \\ \hline 16 \end{array}$$

Check: $(58 \times 427) + 16 =$
$24,766 + 16 = 24,782$

You Try It 8 Divide $534\overline{)33,219}$ and check.

Your solution

Example 9 Divide $386\overline{)206,149}$ and check.

Solution

$$\begin{array}{r} 534 \text{ r}25 \\ 386\overline{)206,149} \\ -193\,0 \\ \hline 13\,14 \\ -11\,58 \\ \hline 1\,569 \\ -1\,544 \\ \hline 25 \end{array}$$

Check: $(534 \times 386) + 25 =$
$206,124 + 25 = 206,149$

You Try It 9 Divide $515\overline{)216,848}$ and check.

Your solution

Solutions on p. S3

ESTIMATION

Estimating the Quotient of Two Whole Numbers

Estimate and then use your calculator to find 36,936 ÷ 54.

To estimate a quotient, round each number so that all the digits are zero except the first digit. Then divide.

$$36{,}936 \div 54 \approx$$
$$40{,}000 \div 50 = 800$$

The estimated answer is 800.

Now use your calculator to find the exact quotient. The exact quotient is 684.

36936 $\boxed{\div}$ 54 $\boxed{=}$ 684

Objective D **To solve application problems** 1

The *average* of several numbers is the sum of all the numbers divided by the number of numbers.

$$\text{Average test score} = \frac{81 + 87 + 80 + 85 + 79 + 86}{6} = \frac{498}{6} = 83$$

The table at the right shows the seating capacity of the 8 stadiums used for major-league soccer games. Find the average seating capacity of these stadiums. Round to the nearest whole number.

Stadium	*Seating Capacity*
Columbus Crew Stadium	30,000
Giants Stadium	79,649
Mile High Stadium	76,098
Raymond James Stadium	66,321
Ericsson Stadium	73,250
Legion Field	83,000
Peoria Stadium	10,123
The Rose Bowl	92,542

Source: www.stadianet.com

Strategy
To find the average seating capacity:

- Add to find the sum of the seating capacities of the 8 stadiums.
- Divide the sum by 8.

Solution

$$\begin{array}{r} 30{,}000 \\ 79{,}649 \\ 76{,}098 \\ 66{,}321 \\ 73{,}250 \\ 83{,}000 \\ 10{,}123 \\ +\ 92{,}542 \\ \hline 510{,}983 \end{array}$$

Sum of all the seating capacities

$$\begin{array}{r} 63{,}872 \\ 8\overline{)\,510{,}983} \\ -48\phantom{0{,}983} \\ \hline 30\phantom{{,}983} \\ -24\phantom{{,}983} \\ \hline 6\,9 \\ -6\,4 \\ \hline 58 \\ -56 \\ \hline 23 \\ -16 \\ \hline 7 \end{array}$$

- When rounding to the nearest whole number, compare twice the remainder to the divisor. If twice the remainder is less than the divisor, drop the remainder. If twice the remainder is greater than or equal to the divisor, add 1 to the units digit of the quotient.

- Twice the remainder is 2 × 7 = 14. Because 14 > 8, add 1 to the units digit of the quotient.

The average seating capacity is 63,873 seats.

Example 10

Ngan Hui, a freight supervisor, shipped 192,600 bushels of wheat in 9 railroad cars. Find the amount of wheat shipped in each car.

Strategy

To find the amount of wheat shipped in each car, divide the number of bushels (192,600) by the number of cars (9).

Solution

$$\begin{array}{r}
21{,}400\\
9\overline{)\,192{,}600}\\
\underline{-18}\phantom{2{,}600}\\
12\phantom{{,}600}\\
\underline{-9}\phantom{{,}600}\\
36\\
\underline{-36}\\
0
\end{array}$$

Each car carried 21,400 bushels of wheat.

You Try It 10

Suppose a Michelin retail outlet can store 270 tires on 15 shelves. How many tires can be stored on each shelf?

Your strategy

Your solution

Example 11

The used car you are buying costs $11,216. A down payment of $2000 is required. The remaining balance is paid in 48 equal monthly payments. What is the monthly payment?

Strategy

To find the monthly payment:

- Find the remaining balance by subtracting the down payment (2000) from the total cost of the car (11,216).
- Divide the remaining balance by the number of equal monthly payments (48).

Solution

$$\begin{array}{r}
11{,}216\\
\underline{-\;\;2{,}000}\\
9{,}216
\end{array}
\qquad\qquad
\begin{array}{r}
192\\
48\overline{)\,9216}\\
\underline{-48}\\
441\\
\underline{-432}\\
96\\
\underline{-96}\\
0
\end{array}$$

Remaining balance

The monthly payment is $192.

You Try It 11

A soft-drink manufacturer produces 12,600 cans of soft drink each hour. Cans are packed 24 to a case. How many cases of soft drink are produced in 8 hours?

Your strategy

Your solution

Solutions on p. S3

1.5 Exercises

Objective A

Divide.

1. $4\overline{)8}$

2. $3\overline{)9}$

3. $6\overline{)36}$

4. $9\overline{)81}$

5. $7\overline{)49}$

6. $5\overline{)80}$

7. $6\overline{)96}$

8. $6\overline{)480}$

9. $4\overline{)840}$

10. $3\overline{)690}$

11. $7\overline{)308}$

12. $7\overline{)203}$

13. $9\overline{)6327}$

14. $4\overline{)2120}$

15. $8\overline{)7280}$

16. $9\overline{)8118}$

17. $3\overline{)64,680}$

18. $4\overline{)50,760}$

19. $6\overline{)21,480}$

20. $5\overline{)18,050}$

21. Find the quotient of 1446 and 3.

22. Find the quotient of 4123 and 7.

23. What is 7525 divided by 7?

24. What is 32,364 divided by 4?

Use the relationship between multiplication and division to complete the multiplication problem.

25. ____ × 7 = 364

26. 8 × ____ = 376

27. 34 × ____ = 170

28. ____ × 14 = 322

Objective B

Divide.

29. $4\overline{)9}$

30. $2\overline{)7}$

31. $5\overline{)27}$

32. $9\overline{)88}$

33. $3\overline{)40}$

34. $6\overline{)97}$

35. $8\overline{)83}$

36. $5\overline{)54}$

37. $7\overline{)632}$

38. $4\overline{)363}$

39. $4\overline{)921}$

40. $7\overline{)845}$

41. $8\overline{)1635}$

42. $5\overline{)1548}$

43. $7\overline{)9432}$

44. $7\overline{)8124}$

45. $3\overline{)5162}$

46. $5\overline{)3542}$

47. $8\overline{)3274}$

48. $4\overline{)15{,}301}$

49. $7\overline{)43{,}500}$

50. $8\overline{)72{,}354}$

51. $5\overline{)43{,}542}$

52. Find the quotient of 3107 and 8.

53. Find the quotient of 8642 and 8.

54. What is 45,738 divided by 4? Round to the nearest ten.

55. What is 37,896 divided by 9? Round to the nearest hundred.

56. What is 3572 divided by 7? Round to the nearest ten.

57. What is 78,345 divided by 4? Round to the nearest hundred.

Objective C

Divide.

58. $27\overline{)96}$

59. $44\overline{)82}$

60. $42\overline{)87}$

61. $67\overline{)93}$

62. $41\overline{)897}$

63. $32\overline{)693}$

64. $23\overline{)784}$

65. $25\overline{)772}$

66. $74\overline{)600}$

67. $92\overline{)500}$

68. $70\overline{)329}$

69. $50\overline{)467}$

70. $36\overline{)7225}$

71. $44\overline{)8821}$

72. $19\overline{)3859}$

73. $32\overline{)9697}$

74. $88\overline{)3127}$

75. $92\overline{)6177}$

76. $33\overline{)8943}$

77. $27\overline{)4765}$

78. $22\overline{)98{,}654}$

79. $77\overline{)83{,}629}$

80. $64\overline{)38{,}912}$

81. $78\overline{)31{,}434}$

82. $206\overline{)3097}$ **83.** $504\overline{)6504}$ **84.** $654\overline{)1217}$ **85.** $546\overline{)2344}$

86. Find the quotient of 5432 and 21.

87. Find the quotient of 8507 and 53.

88. What is 37,294 divided by 72?

89. What is 76,788 divided by 46?

90. Find 23,457 divided by 43. Round to the nearest hundred.

91. Find 341,781 divided by 43. Round to the nearest ten.

Estimate and then use your calculator to divide.

92. $76\overline{)389{,}804}$ **93.** $53\overline{)117{,}925}$ **94.** $29\overline{)637{,}072}$ **95.** $67\overline{)738{,}072}$

96. $38\overline{)934{,}648}$ **97.** $34\overline{)906{,}304}$ **98.** $309\overline{)876{,}324}$ **99.** $642\overline{)323{,}568}$

100. $209\overline{)632{,}016}$ **101.** $614\overline{)332{,}174}$ **102.** $179\overline{)5{,}734{,}444}$ **103.** $374\overline{)7{,}712{,}254}$

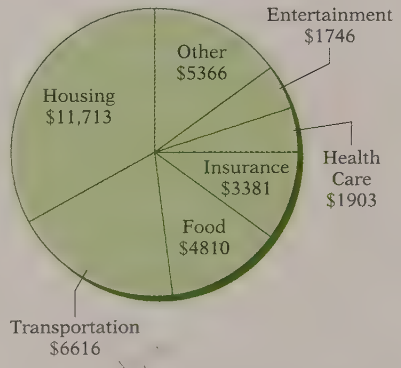
Objective D *Application Problems*

The graph at the right shows the annual expenditures, in a recent year, of the average household in the United States. Use this information for Exercises 104 to 106. Round answers to the nearest whole number. ?

104. What is the total amount spent annually by the average household in the United States?

105. What is the average monthly expense for housing?

106. What is the difference between the average monthly expense for food and the average monthly expense for health care?



Average Annual Household Expenses

Source: Bureau of Labor Statistics Consumer Expenditure Survey

The table at the right shows the sources of laptop insurance claims in a recent year. Claims have been rounded to the nearest ten thousand dollars. Use this information for Exercises 107 and 108.

Source	*Claims*
Accidents	$560,000
Theft	$300,000
Power surge	$80,000
Lightning	$50,000
Transit	$20,000
Water/flood	$20,000
Other	$110,000

Source: Safeware, The Insurance Company

107. What was the average monthly claim for theft?

108. For all sources combined, find the average claims per month.

The table at the right shows, for different countries, the average number of hours per year that employees work. Use this information for Exercises 109 and 110. Use a 50-week year. Round answers to the nearest whole number.

Country	*Annual Number of Hours Worked*
Britain	1731
France	1656
Japan	1889
Norway	1399
United States	1966

Source: International Labor Organization

109. What is the average number of hours worked per week by employees in Britain?

110. On average, how many more hours per week do employees in the United States work than employees in France?

111. The U.S. Mint estimates that about 114,000,000,000 of the 312,000,000,000 pennies it has minted over the last 30 years are in active circulation. That works out to how many pennies in circulation for each of the 280,000,000 people living in the United States?

APPLYING THE CONCEPTS

112. A palindromic number is a whole number that remains unchanged when its digits are written in reverse order. For instance, 292 is a palindromic number. Find the smallest three-digit palindromic number that is divisible by 4.

113. Find the smallest four-digit palindromic number (see Exercise 112) that is divisible by 8. (*Helpful fact:* A whole number whose last three digits form a number that is divisible by 8 is itself divisible by 8. For example, because 408 is divisible by 8, the number 3408 is divisible by 8.)

114. Determine whether each of the following statements is true or false.

a. Any whole number divided by zero is zero.

b. $\frac{0}{0} = 1$

c. Zero divided by any whole number, except zero, is zero.

1.6 Exponential Notation and the Order of Operations Agreement

Objective A

To simplify expressions that contain exponents

Repeated multiplication of the same factor can be written in two ways:

$$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \quad \text{or} \quad 3^5 \leftarrow \textbf{Exponent}$$

The exponent indicates how many times the factor occurs in the multiplication. The expression 3^5 is in **exponential notation.**

It is important to be able to read numbers written in exponential notation.

$6 = 6^1$ is read "six to the first power" or just "six." Usually the exponent 1 is not written.

$6 \cdot 6 = 6^2$ is read "six squared" or "six to the second power."

$6 \cdot 6 \cdot 6 = 6^3$ is read "six cubed" or "six to the third power."

$6 \cdot 6 \cdot 6 \cdot 6 = 6^4$ is read "six to the fourth power."

$6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 = 6^5$ is read "six to the fifth power."

Each place value in the place-value chart can be expressed as a power of 10.

$$\begin{aligned}
\text{Ten} &= 10 = 10 = 10^1\\
\text{Hundred} &= 100 = 10 \cdot 10 = 10^2\\
\text{Thousand} &= 1000 = 10 \cdot 10 \cdot 10 = 10^3\\
\text{Ten-Thousand} &= 10{,}000 = 10 \cdot 10 \cdot 10 \cdot 10 = 10^4\\
\text{Hundred-Thousand} &= 100{,}000 = 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 10^5\\
\text{Million} &= 1{,}000{,}000 = 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 10^6
\end{aligned}$$

To simplify a numerical expression containing exponents, write each factor as many times as indicated by the exponent and carry out the indicated multiplication.

$$4^3 = 4 \cdot 4 \cdot 4 = 64$$
$$2^2 \cdot 3^4 = (2 \cdot 2) \cdot (3 \cdot 3 \cdot 3 \cdot 3) = 4 \cdot 81 = 324$$

Example 1 Write $3 \cdot 3 \cdot 3 \cdot 5 \cdot 5$ in exponential notation.

Solution $3 \cdot 3 \cdot 3 \cdot 5 \cdot 5 = 3^3 \cdot 5^2$

You Try It 1 Write $2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$ in exponential notation.

Your solution

Example 2 Write as a power of 10: $10 \cdot 10 \cdot 10 \cdot 10$

Solution $10 \cdot 10 \cdot 10 \cdot 10 = 10^4$

You Try It 2 Write as a power of 10: $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$

Your solution

Example 3 Simplify $3^2 \cdot 5^3$.

Solution
$$\begin{aligned} 3^2 \cdot 5^3 &= (3 \cdot 3) \cdot (5 \cdot 5 \cdot 5)\\ &= 9 \cdot 125 = 1125 \end{aligned}$$

You Try It 3 Simplify $2^3 \cdot 5^2$.

Your solution

Solutions on p. S3

Objective B

To use the Order of Operations Agreement to simplify expressions

More than one operation may occur in a numerical expression. The answer may be different, depending on the order in which the operations are performed. For example, consider $3 + 4 \times 5$.

Multiply first, then add.

$$3 + \underbrace{4 \times 5}$$
$$\underbrace{3 + 20}$$
$$23$$

Add first, then multiply.

$$\underbrace{3 + 4} \times 5$$
$$\underbrace{7 \times 5}$$
$$35$$

An Order of Operations Agreement is used so that only one answer is possible.

The Order of Operations Agreement

Step 1 Do all the operations inside parentheses.
Step 2 Simplify any number expressions containing exponents.
Step 3 Do multiplication and division as they occur from left to right.
Step 4 Do addition and subtraction as they occur from left to right.

➡ Simplify $3 \times (2 + 1) - 2^2 + 4 \div 2$ by using the Order of Operations Agreement.

$$3 \times \underbrace{(2 + 1)} - 2^2 + 4 \div 2$$
$$3 \times 3 - \underbrace{2^2} + 4 \div 2$$
$$\underbrace{3 \times 3} - 4 + 4 \div 2$$
$$9 - 4 + \underbrace{4 \div 2}$$
$$\underbrace{9 - 4} + 2$$
$$\underbrace{5 + 2}$$
$$7$$

1. Perform operations in parentheses.
2. Simplify expressions with exponents.
3. Do multiplications and divisions as they occur from left to right.
4. Do additions and subtractions as they occur from left to right.

P
E
MD
AS

One or more of the above steps may not be needed to simplify an expression. In that case, proceed to the next step in the Order of Operations Agreement.

➡ Simplify $5 + 8 \div 2$.

There are no parentheses or exponents. Proceed to Step 3 of the Agreement.

$$5 + \underbrace{8 \div 2}$$
$$\underbrace{5 + 4}$$
$$9$$

3. Do multiplication or division.
4. Do addition or subtraction.

Example 4 Simplify: $64 \div (8 - 4)^2 \cdot 9 - 5^2$

Solution

$$\begin{aligned} &64 \div (8 - 4)^2 \cdot 9 - 5^2 \\ &= 64 \div 4^2 \cdot 9 - 5^2 \\ &= 64 \div 16 \cdot 9 - 25 \\ &= 4 \cdot 9 - 25 \\ &= 36 - 25 \\ &= 11 \end{aligned}$$

You Try It 4 Simplify: $5 \cdot (8 - 4)^2 \div 4 - 2$

Your solution

Solution on p. S3

1.6 Exercises

Objective A

Write the number in exponential notation.

1. $2 \cdot 2 \cdot 2$

2. $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$

3. $6 \cdot 6 \cdot 6 \cdot 7 \cdot 7 \cdot 7 \cdot 7$

4. $6 \cdot 6 \cdot 9 \cdot 9 \cdot 9 \cdot 9$

5. $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$

6. $3 \cdot 3 \cdot 10 \cdot 10$

7. $5 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$

8. $4 \cdot 4 \cdot 4 \cdot 5 \cdot 5 \cdot 5$

9. $3 \cdot 3 \cdot 3 \cdot 6 \cdot 6 \cdot 6 \cdot 6$

10. $2 \cdot 2 \cdot 5 \cdot 5 \cdot 5 \cdot 8$

11. $3 \cdot 3 \cdot 3 \cdot 5 \cdot 9 \cdot 9 \cdot 9$

12. $2 \cdot 2 \cdot 2 \cdot 4 \cdot 7 \cdot 7 \cdot 7$

Simplify.

13. 2^3

14. 2^6

15. $2^4 \cdot 5^2$

16. $2^6 \cdot 3^2$

17. $3^2 \cdot 10^2$

18. $2^3 \cdot 10^4$

19. $6^2 \cdot 3^3$

20. $4^3 \cdot 5^2$

21. $5 \cdot 2^3 \cdot 3$

22. $6 \cdot 3^2 \cdot 4$

23. $2^2 \cdot 3^2 \cdot 10$

24. $3^2 \cdot 5^2 \cdot 10$

25. $0^2 \cdot 4^3$

26. $6^2 \cdot 0^3$

27. $3^2 \cdot 10^4$

28. $5^3 \cdot 10^3$

29. $2^2 \cdot 3^3 \cdot 5$

30. $5^2 \cdot 7^3 \cdot 2$

31. $2 \cdot 3^4 \cdot 5^2$

32. $6 \cdot 2^6 \cdot 7^2$

33. $5^2 \cdot 3^2 \cdot 7^2$

34. $4^2 \cdot 9^2 \cdot 6^2$

35. $3^4 \cdot 2^6 \cdot 5$

36. $4^3 \cdot 6^3 \cdot 7$

37. $4^2 \cdot 3^3 \cdot 10^4$

Objective B

Simplify by using the Order of Operations Agreement.

38. $4 - 2 + 3$

39. $6 - 3 + 2$

40. $6 \div 3 + 2$

41. $8 \div 4 + 8$

42. $6 \cdot 3 + 5$

43. $5 \cdot 9 + 2$

44. $3^2 - 4$

45. $5^2 - 17$

46. $4 \cdot (5 - 3) + 2$

47. $3 + (4 + 2) \div 3$

48. $5 + (8 + 4) \div 6$

49. $8 - 2^2 + 4$

50. $16 \cdot (3 + 2) \div 10$

51. $12 \cdot (1 + 5) \div 12$

52. $10 - 2^3 + 4$

53. $5 \cdot 3^2 + 8$

54. $16 + 4 \cdot 3^2$

55. $12 + 4 \cdot 2^3$

56. $16 + (8 - 3) \cdot 2$

57. $7 + (9 - 5) \cdot 3$

58. $2^2 + 3 \cdot (6 - 2)^2$

59. $3^3 + 5 \cdot (8 - 6)^3$

60. $2^2 \cdot 3^2 + 2 \cdot 3$

61. $4 \cdot 6 + 3^2 \cdot 4^2$

62. $16 - 2 \cdot 4$

63. $12 + 3 \cdot 5$

64. $3 \cdot (6 - 2) + 4$

65. $5 \cdot (8 - 4) - 6$

66. $8 - (8 - 2) \div 3$

67. $12 - (12 - 4) \div 4$

68. $8 + 2 - 3 \cdot 2 \div 3$

69. $10 + 1 - 5 \cdot 2 \div 5$

70. $3 \cdot (4 + 2) \div 6$

71. $(7 - 3)^2 \div 2 - 4 + 8$

72. $20 - 4 \div 2 \cdot (3 - 1)^3$

73. $12 \div 3 \cdot 2^2 + (7 - 3)^2$

74. $(4 - 2) \cdot 6 \div 3 + (5 - 2)^2$

75. $18 - 2 \cdot 3 + (4 - 1)^3$

76. $100 \div (2 + 3)^2 - 8 \div 2$

APPLYING THE CONCEPTS

77. Memory in computers is measured in bytes. One kilobyte (1 K) is 2^{10} bytes. Write this number in standard form.

78. Explain the difference in the order of operations for **a.** $\frac{14 - 2}{2} \div 2 \cdot 3$ and **b.** $\frac{14 - 2}{2} \div (2 \cdot 3)$. Work the two problems. What is the difference between the larger and the smaller answer?

79. If a number is divisible by 6 and 10, is the number divisible by 30? If so, explain why. If not, give an example.

1.7 Prime Numbers and Factoring

Objective A **To factor numbers**

Whole-number **factors of a number** divide that number evenly (there is no remainder).

1, 2, 3, and 6 are whole-number factors of 6 because they divide 6 evenly.

$$\begin{array}{r}6\\1\overline{)6}\end{array}\quad\begin{array}{r}3\\2\overline{)6}\end{array}\quad\begin{array}{r}2\\3\overline{)6}\end{array}\quad\begin{array}{r}1\\6\overline{)6}\end{array}$$

Note that both the divisor and the quotient are factors of the dividend.

To find the factors of a number, try dividing the number by 1, 2, 3, 4, 5, ... Those numbers that divide the number evenly are its factors. Continue this process until the factors start to repeat.

⇨ Find all the factors of 42.

42 ÷ 1 = 42	1 and 42 are factors.	
42 ÷ 2 = 21	2 and 21 are factors.	
42 ÷ 3 = 14	3 and 14 are factors.	
42 ÷ 4	Will not divide evenly	
42 ÷ 5	Will not divide evenly	
42 ÷ 6 = 7	6 and 7 are factors.	Factors are repeating; all the factors of 42 have been found.
42 ÷ 7 = 6	7 and 6 are factors.	

1, 2, 3, 6, 7, 14, 21, and 42 are factors of 42.

The following rules are helpful in finding the factors of a number.

2 is a factor of a number if the last digit of the number is 0, 2, 4, 6, or 8.	436 ends in 6; therefore, 2 is a factor of 436. (436 ÷ 2 = 218)
3 is a factor of a number if the sum of the digits of the number is divisible by 3.	The sum of the digits of 489 is 4 + 8 + 9 = 21. 21 is divisible by 3. Therefore, 3 is a factor of 489. (489 ÷ 3 = 163)
5 is a factor of a number if the last digit of the number is 0 or 5.	520 ends in 0; therefore, 5 is a factor of 520. (520 ÷ 5 = 104)

Example 1 Find all the factors of 30.

Solution

30 ÷ 1 = 30
30 ÷ 2 = 15
30 ÷ 3 = 10
30 ÷ 4 Will not divide evenly
30 ÷ 5 = 6
30 ÷ 6 = 5

1 2, 3, 5, 6, 10, 15, and 30 are factors of 30.

You Try It 1 Find all the factors of 40.

Your solution

Solution on p. S3

Objective B To find the prime factorization of a number

Point of Interest

Prime numbers are an important part of cryptology, the study of secret codes. To make it less likely that codes can be broken, cryptologists use prime numbers that have hundreds of digits.

A number is a **prime number** if its only whole-number factors are 1 and itself. 7 is prime because its only factors are 1 and 7. If a number is not prime, it is called a **composite number.** Because 6 has factors of 2 and 3, 6 is a composite number. The number 1 is not considered a prime number; therefore it is not included in the following list of prime numbers less than 50.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47

The **prime factorization** of a number is the expression of the number as a product of its prime factors. We use a "T-diagram" to find the prime factors of 60. Begin with the smallest prime number as a trial divisor, and continue with prime numbers as trial divisors until the final quotient is 1.

$$\begin{array}{r|r} \multicolumn{2}{c}{60} \\ \hline 2 & 30 \\ 2 & 15 \\ 3 & 5 \\ 5 & 1 \end{array} \qquad \begin{array}{r} 60 \div 2 = 30 \\ 30 \div 2 = 15 \\ 15 \div 3 = 5 \\ 5 \div 5 = 1 \end{array}$$

The prime factorization of 60 is $2 \cdot 2 \cdot 3 \cdot 5$.

Finding the prime factorization of larger numbers can be more difficult. Try each prime number as a trial divisor. Stop when the square of the trial divisor is greater than the number being factored.

⇨ Find the prime factorization of 106.

$$\begin{array}{r|r} \multicolumn{2}{c}{106} \\ \hline 2 & 53 \\ 53 & 1 \end{array}$$

- 53 cannot be divided evenly by 2, 3, 5, 7, or 11. Prime numbers greater than 11 need not be tested because 11^2 is greater than 53.

The prime factorization of 106 is $2 \cdot 53$.

Example 2 Find the prime factorization of 315.

Solution

$$\begin{array}{r|r} \multicolumn{2}{c}{315} \\ \hline 3 & 105 \\ 3 & 35 \\ 5 & 7 \\ 7 & 1 \end{array}$$

$315 = 3 \cdot 3 \cdot 5 \cdot 7$

You Try It 2 Find the prime factorization of 44.

Your solution

Example 3 Find the prime factorization of 201.

Solution

$$\begin{array}{r|r} \multicolumn{2}{c}{201} \\ \hline 3 & 67 \\ 67 & 1 \end{array}$$

- Try only 2, 3, 5, 7, and 11 because $11^2 > 67$.

$201 = 3 \cdot 67$

You Try It 3 Find the prime factorization of 177.

Your solution

Solutions on p. S4

1.7 Exercises

Objective A

Find all the factors of the number.

1. 4

2. 6

3. 10

4. 20

5. 7

6. 12

7. 9

8. 8

9. 13

10. 17

11. 18

12. 24

13. 56

14. 36

15. 45

16. 28

17. 29

18. 33

19. 22

20. 26

21. 52

22. 49

23. 82

24. 37

25. 57

26. 69

27. 48

28. 64

29. 95

30. 46

31. 54

32. 50

33. 66

34. 77

35. 80

36. 100

37. 96

38. 85

39. 90

40. 101

Objective B

Find the prime factorization.

41. 6

42. 14

43. 17

44. 83

45. 24

46. 12

47. 27

48. 9

49. 36

50. 40

51. 19

52. 37

53. 90

54. 65

55. 115

56. 80

57. 18

58. 26

59. 28

60. 49

61. 31

62. 42

63. 62

64. 81

65. 22

66. 39

67. 101

68. 89

69. 66

70. 86

71. 74

72. 95

73. 67

74. 78

75. 55

76. 46

77. 120

78. 144

79. 160

80. 175

81. 216

82. 400

83. 625

84. 225

APPLYING THE CONCEPTS

85. Twin primes are two prime numbers that differ by 2. For instance, 17 and 19 are twin primes. Find three sets of twin primes, not including 17 and 19.

86. In 1742, Christian Goldbach conjectured that every even number greater than 2 could be expressed as the sum of two prime numbers. Show that this conjecture is true for 8, 24, and 72. (*Note*: Mathematicians have not yet been able to determine whether Goldbach's conjecture is true or false.)

87. Explain why 2 is the only even prime number.

88. Explain the method of finding prime numbers using the Sieve of Erathosthenes.

Focus on Problem Solving

Introduction to Problem Solving

You encounter problem-solving situations every day. Some problems are easy to solve, and you may mentally solve these problems without considering the steps you are taking in order to draw a conclusion. Others may be more challenging and may require more thought and consideration.

Suppose a friend suggests that you both take a trip over spring break. You'd like to go. What questions go through your mind? You might ask yourself some of the following questions:

How much will the trip cost? What will be the cost for travel, hotel rooms, meals, and so on?

Are some costs going to be shared by both me and my friend?

Can I afford it?

How much money do I have in the bank?

How much more money than I have now do I need?

How much time is there to earn that much money?

How much can I earn in that amount of time?

How much money must I keep in the bank in order to pay the next tuition bill (or some other expense)?

These questions require different mathematical skills. Determining the cost of the trip requires **estimation;** for example, you must use your knowledge of air fares or the cost of gasoline to arrive at an estimate of these costs. If some of the costs are going to be shared, you need to **divide** those costs by 2 in order to determine your share of the expense. The question regarding how much more money you need requires **subtraction:** the amount needed minus the amount currently in the bank. To determine how much money you can earn in the given amount of time requires **multiplication**—for example, the amount you earn per week times the number of weeks to be worked. To determine if the amount you can earn in the given amount of time is sufficient, you need to use your knowledge of **order relations** to compare the amount you can earn with the amount needed.

Facing the problem-solving situation described above may not seem difficult to you. The reason may be that you have faced similar situations before and, therefore, know how to work through this one. You may feel better prepared to deal with a circumstance such as this one because you know what questions to ask. An important aspect of learning to solve problems is learning what questions to ask. As you work through application problems in this text, try to become more conscious of the mental process you are going through. You might begin the process by asking yourself the following questions whenever you are solving an application problem.

1. Have I read the problem enough times to be able to understand the situation being described?
2. Will restating the problem in different words help me to understand the problem situation better?
3. What facts are given? (You might make a list of the information contained in the problem.)
4. What information is being asked for?

5. What relationship exists among the given facts? What relationship exists between the given facts and the solution?
6. What mathematical operations are needed in order to solve the problem?

Try to focus on the problem-solving situation, not on the computation or on getting the answer quickly. And remember, the more problems you solve, the better able you will be to solve other problems in the future, partly because you are learning what questions to ask.

Projects and Group Activities

Order of Operations

Does your calculator use the Order of Operations Agreement? To find out, try this problem:

$$2 + 4 \cdot 7$$

If your answer is 30, then the calculator uses the Order of Operations Agreement. If your answer is 42, it does not use that agreement.

Even if your calculator does not use the Order of Operations Agreement, you can still correctly evaluate numerical expressions. The parentheses keys, $\boxed{(}$ and $\boxed{)}$, are used for this purpose.

Remember that $2 + 4 \cdot 7$ means $2 + (4 \cdot 7)$ because the multiplication must be completed before the addition. To evaluate this expression, enter the following:

Enter:	2	$\boxed{+}$	$\boxed{(}$	4	$\boxed{\times}$	7	$\boxed{)}$	$\boxed{=}$
Display:	2	2	$\boxed{(}$	4	4	7	28	30

When using your calculator to evaluate numerical expressions, insert parentheses around multiplications and around divisions. This has the effect of forcing the calculator to do the operations in the order you want.

Evaluate.

1. $3 \cdot 8 - 5$

2. $6 + 8 \div 2$

3. $3 \cdot (8 - 2)^2$

4. $24 - (4 - 2)^2 \div 4$

5. $3 + (6 \div 2 + 4)^2 - 2$

6. $16 \div 2 + 4 \cdot (8 - 12 \div 4)^2 - 50$

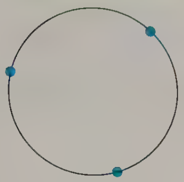
7. $3 \cdot (15 - 2 \cdot 3) - 36 \div 3$

8. $4 \cdot 2^2 - (12 + 24 \div 6) + 5$

9. $16 \div 4 \cdot 3 + (3 \cdot 4 - 5) + 2$

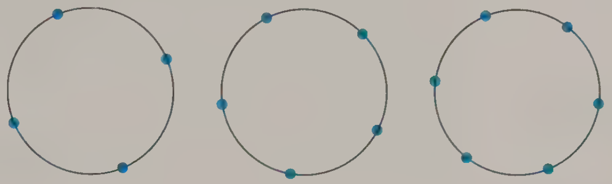
10. $15 \cdot 3 \div 9 + (2 \cdot 6 - 3) + 4$

Patterns in Mathematics



For the circle at the left, use a straight line to connect each dot on the circle with every other dot on the circle. How many different straight lines are there?

Follow the same procedure for each of the circles shown below. How many different straight lines are there in each?



Find a pattern to describe the number of dots on a circle and the corresponding number of different lines drawn. Use the pattern to determine the number of different lines that would be drawn in a circle with 7 dots and in a circle with 8 dots.

Now use the pattern to answer the following question. You are arranging a tennis tournament with 9 players. How many singles matches will be played among the 9 players if each player plays each of the other players only once?

Search the World Wide Web

On the Internet, go to www.census.gov. Find the web page on the U.S. population. Use the data to estimate the following to the nearest thousand. Assume that the same rate of increase in population will continue into the future.

1. What is the population of the United States today?
2. Find the increase in population for a 24-hour period.
3. Using the answer for Exercise 2, find the increase in population for a 30-day period. Find the increase in population for 1 year (365 days).
4. Using the information for Exercise 3, estimate the population of the United States 50 years from now.

Now find the web page on world population. Use the data to estimate the following to the nearest million.

5. How much did the population increase in the last month? In the last year?
6. Using the answer for Exercise 5, estimate what the population of the world will be 10 years from now and 50 years from now.

Chapter Summary

Key Words

The *whole numbers* are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, ... [p. 3]

The symbol for *is less than* is <. The symbol for *is greater than* is >. [p. 3]

When a whole number is written using the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9, it is said to be in *standard form*. The number 58,429 is written in standard form. The position of each digit in the number determines the digit's *place value*. The number 58,429 is written in *expanded form* as 50,000 + 8000 + 400 + 20 + 9. [p. 4]

Addition is the process of finding the total of two or more numbers. The answer to an addition problem is called the *sum*. [p. 9]

Subtraction is the process of finding the *difference* between two numbers. [p. 17]

Multiplication is the repeated addition of the same number. The numbers that are multiplied are called *factors*. The answer is the *product*. [p. 25]

Division is used to separate objects into equal groups. The answer to a division problem is called the *quotient*. [p. 33]

The expression 4^3 is in *exponential notation*. The *exponent*, 3, indicates how many times 4 occurs as a factor in the multiplication. [p. 45]

Factors of a number divide that number evenly (there is no remainder). [p. 49]

A number greater than 1 is *prime* if its only whole-number factors are 1 and itself. [p. 50]

The *prime factorization* of a number is the expression of the number as a product of its prime factors. [p. 50]

Essential Rules

The Addition Property of Zero	Zero added to a number does not change the number. $5 + 0 = 0 + 5 = 5$ [p. 9]
The Commutative Property of Addition	Two numbers can be added in either order. $6 + 5 = 5 + 6 = 11$ [p. 9]
The Associative Property of Addition	Grouping numbers to be added in any order gives the same result. $(2 + 3) + 5 = 2 + (3 + 5) = 10$ [p. 9]
The Multiplication Property of Zero	The product of a number and zero is zero. $0 \times 3 = 3 \times 0 = 0$ [p. 25]
The Multiplication Property of One	The product of a number and one is the number. $1 \times 8 = 8 \times 1 = 8$ [p. 25]
The Commutative Property of Multiplication	Two numbers can be multiplied in any order. $5 \times 2 = 2 \times 5 = 10$ [p. 25]
The Associative Property of Multiplication	Grouping numbers to be multiplied in any order gives the same result. $(3 \times 2) \times 5 = 3 \times (2 \times 5) = 30$ [p. 25]
The Properties of Zero in Division	Zero divided by any other number is zero. Division by zero is not allowed. [p. 33]

Rounding a Number to a Given Place Value
If the digit to the right of the given place value is less than 5, replace that digit and all digits to the right by zeros. If the digit to the right of the given place value is greater than or equal to 5, increase the digit in the given place value by 1, and replace all other digits to the right by zeros. [p. 6]

Order of Operations Agreement [p. 46]

Step 1 Perform operations inside grouping symbols.
Step 2 Simplify expressions with exponents.
Step 3 Do multiplications and divisions as they occur from left to right.
Step 4 Do additions and subtractions as they occur from left to right.

Chapter Review

1. Simplify: $3 \cdot 2^3 \cdot 5^2$

2. Write 10,327 in expanded form.

3. Find all the factors of 18.

4. Find the sum of 5894, 6301, and 298.

5. Subtract: $\begin{array}{r} 4926 \\ -\ 3177 \\ \hline \end{array}$

6. Divide: $7\overline{)14{,}945}$

7. Place the correct symbol, < or >, between the two numbers: 101 87

8. Write $5 \cdot 5 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$ in exponential notation.

9. What is 2019 multiplied by 307?

10. What is 10,134 decreased by 4725?

11. Add: $\begin{array}{r} 298 \\ 461 \\ +\ 322 \\ \hline \end{array}$

12. Simplify: $2^3 - 3 \cdot 2$

13. Round 45,672 to the nearest hundred.

14. Write 276,057 in words.

15. Find the quotient of 109,763 and 84.

16. Write two million eleven thousand forty-four in standard form.

17. What is 3906 divided by 8?

18. Simplify: $3^2 + 2^2 \cdot (5 - 3)$

19. Simplify: $8 \cdot (6 - 2)^2 \div 4$

20. Find the prime factorization of 72.

21. What is 3895 minus 1762?

22. Multiply: $\begin{array}{r} 843 \\ \times\ 27 \\ \hline \end{array}$

23. Vincent Meyers, a sales assistant, earns $480 for working a 40-hour week. Last week Vincent worked an additional 12 hours at $24 an hour. Find Vincent's total pay for last week's work.

24. Louis Reyes, a sales executive, drove a car 351 miles on 13 gallons of gas. Find the number of miles driven per gallon of gasoline.

25. A car is purchased for $17,880, with a down payment of $3000. The balance is paid in 48 equal monthly payments. Find the monthly car payment.

26. An insurance account executive received commissions of $723, $544, $812, and $488 during a 4-week period. Find the total income from commissions for the 4 weeks.

27. You had a balance of $516 in your checking account before making deposits of $88 and $213. Find the total amount deposited, and determine your new account balance.

28. You have a car payment of $246 per month. What is the total of the car payments over a 12-month period?

The table at the right shows the average earnings, by level of education, for people 18 years of age and older. Use this information for Exercises 29 to 31.

Level of Education	*Men*	*Women*
Some college or an associates degree	$33,830	$21,057
Bachelor's degree	$50,056	$30,119
Advanced degree	$86,617	$50,546

Source: Census Bureau, *Current Population Reports*, March 1998

29. What is the difference between the annual salary of a woman with an advanced degree and that of a woman with an associate's degree?

30. Find the difference between the annual salary of a man and that of a woman if both have a bachelor's degree.

31. How much more does a man with an associate's degree earn in 5 years than a woman with the same degree?

Chapter Test

1. Simplify: $3^3 \cdot 4^2$

2. Write 207,068 in words.

3. Subtract: $\begin{array}{r} 17{,}495 \\ -\ 8{,}162 \\ \hline \end{array}$

4. Find all the factors of 20.

5. Multiply: $\begin{array}{r} 9736 \\ \times\ 704 \\ \hline \end{array}$

6. Simplify: $4^2 \cdot (4 - 2) \div 8 + 5$

7. Write 906,378 in expanded form.

8. Round 74,965 to the nearest hundred.

9. Divide: $97\overline{)108{,}764}$

10. Write $3 \cdot 3 \cdot 3 \cdot 7 \cdot 7$ in exponential form.

11. Find the sum of 8756, 9094, and 37,065.

12. Find the prime factorization of 84.

13. Simplify: $16 \div 4 \cdot 2 - (7 - 5)^2$

14. Find the product of 8 and 90,763.

15. Write one million two hundred four thousand six in standard form.

16. Divide: $7\overline{)60{,}972}$

17. Place the correct symbol, < or >, between the two numbers: 21 19

18. Find the quotient of 5624 and 8.

19. Add:

$$\begin{array}{r} 25{,}492 \\ +71{,}306 \\ \hline \end{array}$$

20. Find the difference between 29,736 and 9814.

The table at the right shows the projected enrollment in public and private elementary and secondary schools in the fall of 2000 and the fall of 2005. Use this information for Exercises 21 and 22.

Year	*Kindergarten Through Grade 8*	*Grades 9 Through 12*
2000	39,152,000	15,250,000
2005	39,437,000	16,434,000

Source: The National Center for Education Statistics

21. Find the difference between the total enrollment in 2000 and that projected for 2005.

22. Find the average projected enrollment in each of grades 9 through 12 in 2005.

23. A farmer harvested 48,290 pounds of lemons from one grove and 23,710 pounds of lemons from another grove. The lemons were packed in boxes with 24 pounds of lemons in each box. How many boxes were needed to pack the lemons?

24. An investor receives $237 each month from a corporate bond fund. How much will the investor receive over a 12-month period?

25. A family drives 425 miles the first day, 187 miles the second day, and 243 miles the third day of their vacation. The odometer read 47,626 miles at the start of the vacation.

a. How many miles were driven during the 3 days?

b. What is the odometer reading at the end of the 3 days?

Chapter 2 Fractions

Objectives

Section 2.1

A To find the least common multiple (LCM)
B To find the greatest common factor (GCF)

Section 2.2

A To write a fraction that represents part of a whole
B To write an improper fraction as a mixed number or a whole number, and a mixed number as an improper fraction

Section 2.3

A To find equivalent fractions by raising to higher terms
B To write a fraction in simplest form

Section 2.4

A To add fractions with the same denominator
B To add fractions with unlike denominators
C To add whole numbers, mixed numbers, and fractions
D To solve application problems

Section 2.5

A To subtract fractions with the same denominator
B To subtract fractions with unlike denominators
C To subtract whole numbers, mixed numbers, and fractions
D To solve application problems

Section 2.6

A To multiply fractions
B To multiply whole numbers, mixed numbers, and fractions
C To solve application problems

Section 2.7

A To divide fractions
B To divide whole numbers, mixed numbers, and fractions
C To solve application problems

Section 2.8

A To identify the order relation between two fractions
B To simplify expressions containing exponents
C To use the Order of Operations Agreement to simplify expressions

Members of the Boston Pops follow conductor Keith Lockhart's movements to guarantee synchronized playing. By paying attention to both Lockhart and the time signature for each musical piece, the musicians play in time with each other. The time signature appears as a fraction at the beginning of a piece of music and tells them how many beats to play per measure. The **project on page 112** demonstrates how to interpret the time signature.

Need help? For on-line student resources, such as section quizzes, visit this textbook's web site at **math.college.hmco.com/students.**

Prep Test

For Exercises 1 to 6, add, subtract, multiply, or divide.

1. 4×5

2. $2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$

3. 9×1

4. $6 + 4$

5. $10 - 3$

6. $63 \div 30$

7. Which of the following numbers divide evenly into 12?

1 2 3 4 5 6 7 8 9 10 11 12

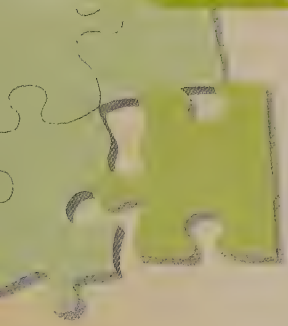
8. Simplify: $8 \times 7 + 3$

9. Complete: $8 = ? + 1$

10. Place the correct symbol, $<$ or $>$, between the two numbers.

44 48

Go Figure



You and a friend are swimming laps in a pool. You swim one lap every 4 minutes. Your friend swims one lap every 5 minutes. If you start at the same time from the same end of the pool, in how many minutes will both of you be at the starting point again? How many times will you have passed each other in the pool prior to that time?

2.1 The Least Common Multiple and Greatest Common Factor

Objective A

To find the least common multiple (LCM)

The **multiples** of a number are the products of that number and the numbers 1, 2, 3, 4, 5, ...

$3 \times 1 = 3$
$3 \times 2 = 6$
$3 \times 3 = 9$
$3 \times 4 = 12$ The multiples of 3 are 3, 6, 9, 12, 15, ...
$3 \times 5 = 15$
.
.
.

A number that is a multiple of two or more numbers is a **common multiple** of those numbers.

The multiples of 4 are 4, 8, 12, 16, 20, 24, 28, 32, 36, ...
The multiples of 6 are 6, 12, 18, 24, 30, 36, 42, ...
Some common multiples of 4 and 6 are 12, 24, and 36.

The **least common multiple** (LCM) is the smallest common multiple of two or more numbers.

The least common multiple of 4 and 6 is 12.

Listing the multiples of each number is one way to find the LCM. Another way to find the LCM uses the prime factorization of each number.

To find the LCM of 450 and 600, find the prime factorization of each number and write the factorization of each number in a table. Circle the largest product in each column. The LCM is the product of the circled numbers.

	2	3	5
450 =	2	$\boxed{3 \cdot 3}$	$\boxed{5 \cdot 5}$
600 =	$\boxed{2 \cdot 2 \cdot 2}$	3	$5 \cdot 5$

In the column headed by 5, the products are equal. Circle just one product.

The LCM is the product of the circled numbers.
The LCM $= 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 5 = 1800$.

Example 1 Find the LCM of 24, 36, and 50.

Solution

	2	3	5
24 =	$\boxed{2 \cdot 2 \cdot 2}$	3	
36 =	$2 \cdot 2$	$\boxed{3 \cdot 3}$	
50 =	2		$\boxed{5 \cdot 5}$

The LCM $= 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 5$
$= 1800$.

You Try It 1 Find the LCM of 50, 84, and 135.

Your solution

Solution on p. S4

Objective B To find the greatest common factor (GCF)

Recall that a number that divides another number evenly is a factor of that number. The number 64 can be evenly divided by 1, 2, 4, 8, 16, 32, and 64, so the numbers 1, 2, 4, 8, 16, 32, and 64 are factors of 64.

A number that is a factor of two or more numbers is a **common factor** of those numbers.

The factors of 30 are 1, 2, 3, 5, 6, 10, 15, and 30.
The factors of 105 are 1, 3, 5, 7, 15, 21, 35, and 105.
The common factors of 30 and 105 are 1, 3, 5, and 15.

The **greatest common factor** (GCF) is the largest common factor of two or more numbers.

The greatest common factor of 30 and 105 is 15.

Listing the factors of each number is one way of finding the GCF. Another way to find the GCF uses the prime factorization of each number.

To find the GCF of 126 and 180, find the prime factorization of each number and write the factorization of each number in a table. Circle the smallest product in each column that does not have a blank. The GCF is the product of the circled numbers.

	2	3	5	7
126 =	②	(3 · 3)		7
180 =	2 · 2	3 · 3	5	

In the column headed by 3, the products are equal. Circle just one product.
Columns 5 and 7 have a blank, so 5 and 7 are not common factors of 126 and 180. Do not circle any number in these columns.

The GCF is the product of the circled numbers.
The GCF = 2 · 3 · 3 = 18.

Example 2 Find the GCF of 90, 168, and 420.

Solution

	2	3	5	7
90 =	②	3 · 3	5	
168 =	2 · 2 · 2	③		7
420 =	2 · 2	3	5	7

The GCF = 2 · 3 = 6.

You Try It 2 Find the GCF of 36, 60, and 72.

Your solution

Example 3 Find the GCF of 7, 12, and 20.

Solution

	2	3	5	7
7 =				7
12 =	2 · 2	3		
20 =	2 · 2		5	

Because no numbers are circled, the GCF = 1.

You Try It 3 Find the GCF of 11, 24, and 30.

Your solution

Solutions on p. S4

2.1 Exercises

Objective A

Find the LCM.

1. 5, 8
2. 3, 6
3. 3, 8
4. 2, 5
5. 5, 6

6. 5, 7
7. 4, 6
8. 6, 8
9. 8, 12
10. 12, 16

11. 5, 12
12. 3, 16
13. 8, 14
14. 6, 18
15. 3, 9

16. 4, 10
17. 8, 32
18. 7, 21
19. 9, 36
20. 14, 42

21. 44, 60
22. 120, 160
23. 102, 184
24. 123, 234
25. 4, 8, 12

26. 5, 10, 15
27. 3, 5, 10
28. 2, 5, 8
29. 3, 8, 12
30. 5, 12, 18

31. 9, 36, 64
32. 18, 54, 63
33. 16, 30, 84
34. 9, 12, 15

Objective B

Find the GCF.

35. 3, 5
36. 5, 7
37. 6, 9
38. 18, 24
39. 15, 25

40. 14, 49
41. 25, 100
42. 16, 80
43. 32, 51
44. 21, 44

45. 12, 80

46. 8, 36

47. 16, 140

48. 12, 76

49. 24, 30

50. 48, 144

51. 44, 96

52. 18, 32

53. 3, 5, 11

54. 6, 8, 10

55. 7, 14, 49

56. 6, 15, 36

57. 10, 15, 20

58. 12, 18, 20

59. 24, 40, 72

60. 3, 17, 51

61. 17, 31, 81

62. 14, 42, 84

63. 25, 125, 625

64. 12, 68, 92

65. 28, 35, 70

66. 1, 49, 153

67. 32, 56, 72

68. 24, 36, 48

APPLYING THE CONCEPTS

69. Define the phrase *relatively prime numbers.* List three pairs of relatively prime numbers.

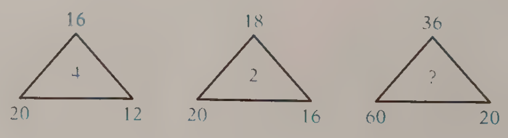
70. Joe Salvo, a clerk, works three days and then has a day off. A friend works five days and then has a day off. How many days after Joe and his friend have a day off together will they have another day off together?

71. Find the LCM of each of the following pairs of numbers: 2 and 3, 5 and 7, and 11 and 19. Can you draw a conclusion about the LCM of two prime numbers? Suggest a way of finding the LCM of three prime numbers.

72. Find the GCF of each of the following pairs of numbers: 3 and 5, 7 and 11, and 29 and 43. Can you draw a conclusion about the GCF of two prime numbers? What is the GCF of three prime numbers?

73. Is the LCM of two numbers always divisible by the GCF of the two numbers? If so, explain why. If not, give an example.

74. Using the pattern for the first two triangles at the right, determine the center number of the last triangle.



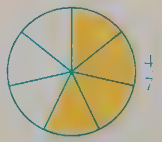
2.2 Introduction to Fractions

Objective A

To write a fraction that represents part of a whole

A **fraction** can represent the number of equal parts of a whole.

The shaded portion of the circle is represented by the fraction $\frac{4}{7}$. Four of the seven equal parts of the circle (that is, four-sevenths of it) are shaded.



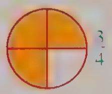
Each part of a fraction has a name.

$$\textbf{Fraction bar} \rightarrow \frac{4 \leftarrow \textbf{Numerator}}{7 \leftarrow \textbf{Denominator}}$$

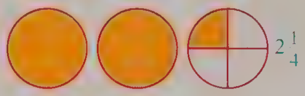
Point of Interest

The fraction bar was first used in 1050 by al-Hassar. It is also called a vinculum.

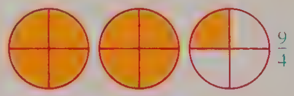
A **proper fraction** is a fraction less than 1. The numerator of a proper fraction is smaller than the denominator. The shaded portion of the circle can be represented by the proper fraction $\frac{3}{4}$.

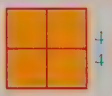


A **mixed number** is a number greater than 1 with a whole-number part and a fractional part. The shaded portion of the circles can be represented by the mixed number $2\frac{1}{4}$.



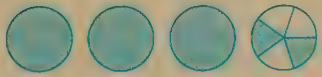
An **improper fraction** is a fraction greater than or equal to 1. The numerator of an improper fraction is greater than or equal to the denominator. The shaded portion of the circles can be represented by the improper fraction $\frac{9}{4}$. The shaded portion of the square can be represented by $\frac{4}{4}$.





Example 1

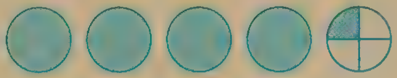
Express the shaded portion of the circles as a mixed number.



Solution $3\frac{2}{5}$

You Try It 1

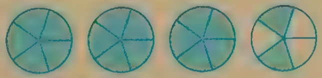
Express the shaded portion of the circles as a mixed number.



Your solution

Example 2

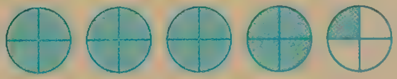
Express the shaded portion of the circles as an improper fraction.



Solution $\frac{17}{5}$

You Try It 2

Express the shaded portion of the circles as an improper fraction.



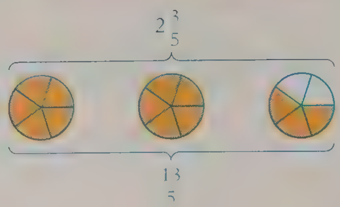
Your solution

Solutions on p. S4

Objective B

To write an improper fraction as a mixed number or a whole number, and a mixed number as an improper fraction

Note from the diagram that the mixed number $2\frac{3}{5}$ and the improper fraction $\frac{13}{5}$ both represent the shaded portion of the circles.



$$2\frac{3}{5} = \frac{13}{5}$$

An improper fraction can be written as a mixed number or a whole number.

⇨ Write $\frac{13}{5}$ as a mixed number.

Divide the numerator by the denominator.	To write the fractional part of the mixed number, write the remainder over the divisor.	Write the answer.
$\begin{array}{r} 2 \\ 5\overline{)\,13} \\ -10 \\ \hline 3 \end{array}$	$\begin{array}{r} 2\frac{3}{5} \\ 5\overline{)\,13} \\ -10 \\ \hline 3 \end{array}$	$\frac{13}{5} = 2\frac{3}{5}$

Point of Interest

Archimedes (c. 287–212 B.C.) is the person who calculated that $\pi \approx 3\frac{1}{7}$. He actually showed that $3\frac{10}{71} < \pi < 3\frac{1}{7}$. The approximation $3\frac{10}{71}$ is more accurate but more difficult to use.

To write a mixed number as an improper fraction, multiply the denominator of the fractional part by the whole-number part. The sum of this product and the numerator of the fractional part is the numerator of the improper fraction. The denominator remains the same.

⇨ Write $7\frac{3}{8}$ as an improper fraction.

$$\overset{+}{7}\frac{3}{\underset{\times}{8}} = \frac{(8 \times 7) + 3}{8} = \frac{56 + 3}{8} = \frac{59}{8} \qquad 7\frac{3}{8} = \frac{59}{8}$$

Example 3

Write $\frac{21}{4}$ as a mixed number.

Solution

$$\begin{array}{r} 5 \\ 4\overline{)\,21} \\ -20 \\ \hline 1 \end{array} \qquad \frac{21}{4} = 5\frac{1}{4}$$

You Try It 3

Write $\frac{22}{5}$ as a mixed number.

Your solution

Example 4

Write $\frac{18}{6}$ as a whole number.

Solution

$$\begin{array}{r} 3 \\ 6\overline{)\,18} \\ -18 \\ \hline 0 \end{array} \qquad \frac{18}{6} = 3$$

Note: The remainder is zero.

You Try It 4

Write $\frac{28}{7}$ as a whole number.

Your solution

Example 5

Write $21\frac{3}{4}$ as an improper fraction.

Solution

$$\overset{+}{21}\frac{3}{\underset{\times}{4}} = \frac{84 + 3}{4} = \frac{87}{4}$$

You Try It 5

Write $14\frac{5}{8}$ as an improper fraction.

Your solution

Solutions on p. S4

2.2 Exercises

Objective A

Identify the fraction as a proper fraction, an improper fraction, or a mixed number.

1. $\frac{12}{7}$

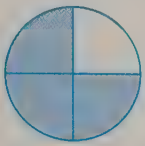
2. $5\frac{2}{11}$

3. $\frac{29}{40}$

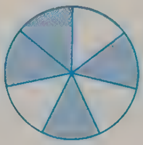
4. $\frac{19}{13}$

Express the shaded portion of the circle as a fraction.

5.



6.



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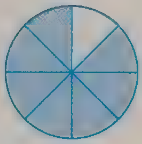

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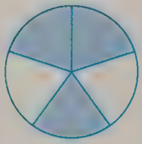
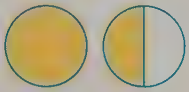


Express the shaded portion of the circles as a mixed number.

9.



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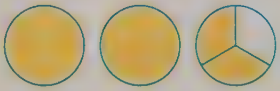

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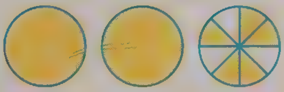

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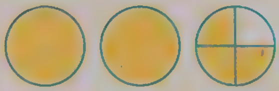

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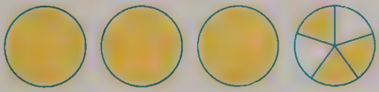

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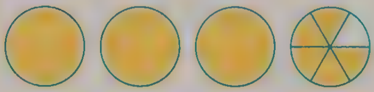
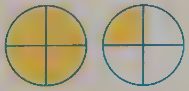


Express the shaded portion of the circles as an improper fraction.

15.



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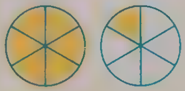

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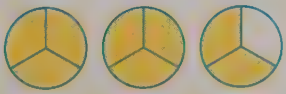

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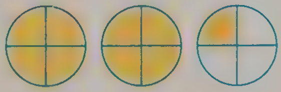

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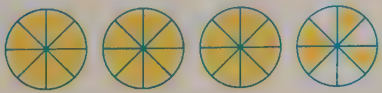

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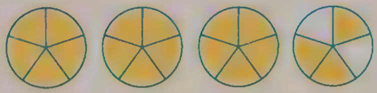
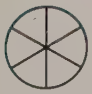
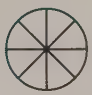


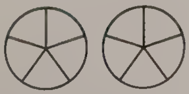
21. Shade $\frac{5}{6}$ of



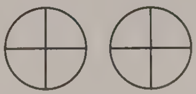
22. Shade $\frac{3}{8}$ of



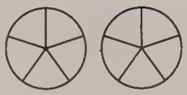
23. Shade $1\frac{2}{5}$ of



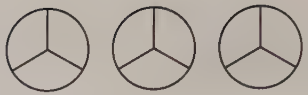
24. Shade $1\frac{3}{4}$ of



25. Shade $\frac{6}{5}$ of



26. Shade $\frac{7}{3}$ of



Objective B

Write the improper fraction as a mixed number or a whole number.

27. $\frac{11}{4}$
28. $\frac{16}{3}$
29. $\frac{20}{4}$
30. $\frac{18}{9}$
31. $\frac{9}{8}$
32. $\frac{13}{4}$
33. $\frac{23}{10}$
34. $\frac{29}{2}$
35. $\frac{48}{16}$
36. $\frac{51}{3}$
37. $\frac{8}{7}$
38. $\frac{16}{9}$
39. $\frac{7}{3}$
40. $\frac{9}{5}$
41. $\frac{16}{1}$
42. $\frac{23}{1}$
43. $\frac{17}{8}$
44. $\frac{31}{16}$
45. $\frac{12}{5}$
46. $\frac{19}{3}$
47. $\frac{9}{9}$
48. $\frac{40}{8}$
49. $\frac{72}{8}$
50. $\frac{3}{3}$

Write the mixed number as an improper fraction.

51. $2\frac{1}{3}$
52. $4\frac{2}{3}$
53. $6\frac{1}{2}$
54. $8\frac{2}{3}$
55. $6\frac{5}{6}$
56. $7\frac{3}{8}$
57. $9\frac{1}{4}$
58. $6\frac{1}{4}$
59. $10\frac{1}{2}$
60. $15\frac{1}{8}$
61. $8\frac{1}{9}$
62. $3\frac{5}{12}$
63. $5\frac{3}{11}$
64. $3\frac{7}{9}$
65. $2\frac{5}{8}$
66. $12\frac{2}{3}$
67. $1\frac{5}{8}$
68. $5\frac{3}{7}$
69. $11\frac{1}{9}$
70. $12\frac{3}{5}$
71. $3\frac{3}{8}$
72. $4\frac{5}{9}$
73. $6\frac{7}{13}$
74. $8\frac{5}{14}$

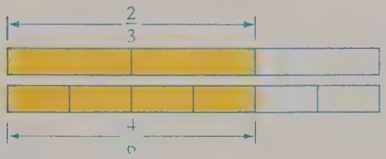
APPLYING THE CONCEPTS

75. Name three situations in which fractions are used. Provide an example of a fraction that is used in each situation.

2.3 Writing Equivalent Fractions

Objective A To find equivalent fractions by raising to higher terms

Equal fractions with different denominators are called **equivalent fractions.**



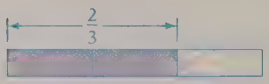
$\frac{4}{6}$ is equivalent to $\frac{2}{3}$.

Remember that the Multiplication Property of One states that the product of a number and one is the number. This is true for fractions as well as whole numbers. This property can be used to write equivalent fractions.

$$\frac{2}{3} \times 1 = \frac{2}{3} \times \frac{1}{1} = \frac{2 \cdot 1}{3 \cdot 1} = \frac{2}{3}$$

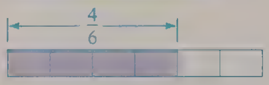
$$\frac{2}{3} \times 1 = \frac{2}{3} \times \boxed{\frac{2}{2}} = \frac{2 \cdot 2}{3 \cdot 2} = \frac{4}{6}$$

$\frac{4}{6}$ is equivalent to $\frac{2}{3}$.

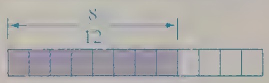


$$\frac{2}{3} \times 1 = \frac{2}{3} \times \boxed{\frac{4}{4}} = \frac{2 \cdot 4}{3 \cdot 4} = \frac{8}{12}$$

$\frac{8}{12}$ is equivalent to $\frac{2}{3}$.



$\frac{2}{3}$ was rewritten as the equivalent fractions $\frac{4}{6}$ and $\frac{8}{12}$.



➡ Write a fraction that is equivalent to $\frac{5}{8}$ and has a denominator of 32.

$32 \div 8 = 4$

$\frac{5}{8} = \frac{5 \cdot 4}{8 \cdot 4} = \frac{20}{32}$

- **Divide the larger denominator by the smaller.**
- **Multiply the numerator and denominator of the given fraction by the quotient (4).**

$\frac{20}{32}$ is equivalent to $\frac{5}{8}$.

Example 1 Write $\frac{2}{3}$ as an equivalent fraction that has a denominator of 42.

Solution $42 \div 3 = 14$ $\quad \frac{2}{3} = \frac{2 \cdot 14}{3 \cdot 14} = \frac{28}{42}$

$\frac{28}{42}$ is equivalent to $\frac{2}{3}$.

You Try It 1 Write $\frac{3}{5}$ as an equivalent fraction that has a denominator of 45.

Your solution

Example 2 Write 4 as a fraction that has a denominator of 12.

Solution Write 4 as $\frac{4}{1}$.

$12 \div 1 = 12$ $\quad 4 = \frac{4 \cdot 12}{1 \cdot 12} = \frac{48}{12}$

$\frac{48}{12}$ is equivalent to 4.

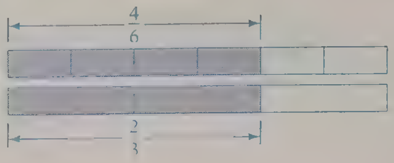
You Try It 2 Write 6 as a fraction that has a denominator of 18.

Your solution

Solutions on p. S4

Objective B To write a fraction in simplest form

A fraction is in **simplest form** when there are no common factors in the numerator and the denominator.



The fractions $\frac{4}{6}$ and $\frac{2}{3}$ are equivalent fractions.

$\frac{4}{6}$ has been written in simplest form as $\frac{2}{3}$.

The Multiplication Property of One can be used to write fractions in simplest form. Write the numerator and denominator of the given fraction as a product of factors. Write factors common to both the numerator and denominator as an improper fraction equivalent to 1.

$$\frac{4}{6} = \frac{2\cdot 2}{2\cdot 3} = \boxed{\frac{2}{2}}\cdot\frac{2}{3} = \boxed{\frac{2}{2}}\cdot\frac{2}{3} = 1\cdot\frac{2}{3} = \frac{2}{3}$$

The process of eliminating common factors is displayed with slashes through the common factors as shown at the right.

$$\frac{4}{6} = \frac{\overset{1}{\cancel{2}}\cdot 2}{\underset{1}{\cancel{2}}\cdot 3} = \frac{2}{3}$$

To write a fraction in simplest form, eliminate the common factors.

$$\frac{18}{30} = \frac{\overset{1}{\cancel{2}}\cdot\overset{1}{\cancel{3}}\cdot 3}{\underset{1}{\cancel{2}}\cdot\underset{1}{\cancel{3}}\cdot 5} = \frac{3}{5}$$

An improper fraction can be changed to a mixed number.

$$\frac{22}{6} = \frac{\overset{1}{\cancel{2}}\cdot 11}{\underset{1}{\cancel{2}}\cdot 3} = \frac{11}{3} = 3\frac{2}{3}$$

Example 3

Write $\frac{15}{40}$ in simplest form.

Solution $\frac{15}{40} = \frac{3\cdot\overset{1}{\cancel{5}}}{2\cdot 2\cdot 2\cdot\underset{1}{\cancel{5}}} = \frac{3}{8}$

You Try It 3

Write $\frac{16}{24}$ in simplest form.

Your solution

Example 4

Write $\frac{6}{42}$ in simplest form.

Solution $\frac{6}{42} = \frac{\overset{1}{\cancel{2}}\cdot\overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}}\cdot\underset{1}{\cancel{3}}\cdot 7} = \frac{1}{7}$

You Try It 4

Write $\frac{8}{56}$ in simplest form.

Your solution

Example 5

Write $\frac{8}{9}$ in simplest form.

Solution $\frac{8}{9} = \frac{2\cdot 2\cdot 2}{3\cdot 3} = \frac{8}{9}$

$\frac{8}{9}$ is already in simplest form because there are no common factors in the numerator and denominator.

You Try It 5

Write $\frac{15}{32}$ in simplest form.

Your solution

Example 6

Write $\frac{30}{12}$ in simplest form.

Solution $\frac{30}{12} = \frac{\overset{1}{\cancel{2}}\cdot\overset{1}{\cancel{3}}\cdot 5}{\underset{1}{\cancel{2}}\cdot 2\cdot\underset{1}{\cancel{3}}} = \frac{5}{2} = 2\frac{1}{2}$

You Try It 6

Write $\frac{48}{36}$ in simplest form.

Your solution

Solutions on p. S4

2.3 Exercises

Objective A

Write an equivalent fraction with the given denominator.

1. $\frac{1}{2} = \frac{}{10}$

2. $\frac{1}{4} = \frac{}{16}$

3. $\frac{3}{16} = \frac{}{48}$

4. $\frac{5}{9} = \frac{}{81}$

5. $\frac{3}{8} = \frac{}{32}$

6. $\frac{7}{11} = \frac{}{33}$

7. $\frac{3}{17} = \frac{}{51}$

8. $\frac{7}{10} = \frac{}{90}$

9. $\frac{3}{4} = \frac{}{16}$

10. $\frac{5}{8} = \frac{}{32}$

11. $3 = \frac{}{9}$

12. $5 = \frac{}{25}$

13. $\frac{1}{3} = \frac{}{60}$

14. $\frac{1}{16} = \frac{}{48}$

15. $\frac{11}{15} = \frac{}{60}$

16. $\frac{3}{50} = \frac{}{300}$

17. $\frac{2}{3} = \frac{}{18}$

18. $\frac{5}{9} = \frac{}{36}$

19. $\frac{5}{7} = \frac{}{49}$

20. $\frac{7}{8} = \frac{}{32}$

21. $\frac{5}{9} = \frac{}{18}$

22. $\frac{11}{12} = \frac{}{36}$

23. $7 = \frac{}{3}$

24. $9 = \frac{}{4}$

25. $\frac{7}{9} = \frac{}{45}$

26. $\frac{5}{6} = \frac{}{42}$

27. $\frac{15}{16} = \frac{}{64}$

28. $\frac{11}{18} = \frac{}{54}$

29. $\frac{3}{14} = \frac{}{98}$

30. $\frac{5}{6} = \frac{}{144}$

31. $\frac{5}{8} = \frac{}{48}$

32. $\frac{7}{12} = \frac{}{96}$

33. $\frac{5}{14} = \frac{}{42}$

34. $\frac{2}{3} = \frac{}{42}$

35. $\frac{17}{24} = \frac{}{144}$

36. $\frac{5}{13} = \frac{}{169}$

37. $\frac{3}{8} = \frac{}{408}$

38. $\frac{9}{16} = \frac{}{272}$

39. $\frac{17}{40} = \frac{}{800}$

40. $\frac{9}{25} = \frac{}{1000}$

Objective B

Write the fraction in simplest form.

41. $\frac{4}{12}$

42. $\frac{8}{22}$

43. $\frac{22}{44}$

44. $\frac{2}{14}$

45. $\frac{2}{12}$

46. $\frac{50}{75}$ **47.** $\frac{40}{36}$ **48.** $\frac{12}{8}$ **49.** $\frac{0}{30}$ **50.** $\frac{10}{10}$

51. $\frac{9}{22}$ **52.** $\frac{14}{35}$ **53.** $\frac{75}{25}$ **54.** $\frac{8}{60}$ **55.** $\frac{16}{84}$

56. $\frac{20}{44}$ **57.** $\frac{12}{35}$ **58.** $\frac{8}{36}$ **59.** $\frac{28}{44}$ **60.** $\frac{12}{16}$

61. $\frac{16}{12}$ **62.** $\frac{24}{18}$ **63.** $\frac{24}{40}$ **64.** $\frac{44}{60}$ **65.** $\frac{8}{88}$

66. $\frac{9}{90}$ **67.** $\frac{144}{36}$ **68.** $\frac{140}{297}$ **69.** $\frac{48}{144}$ **70.** $\frac{32}{120}$

71. $\frac{60}{100}$ **72.** $\frac{33}{110}$ **73.** $\frac{36}{16}$ **74.** $\frac{80}{45}$ **75.** $\frac{32}{160}$

APPLYING THE CONCEPTS

76. Make a list of five different fractions that are equivalent to $\frac{2}{3}$.

77. Make a list of five different fractions that are equivalent to 3.

78. Show that $\frac{15}{24} = \frac{5}{8}$ by using a diagram.

79. **a.** What fraction of the states in the United States of America have names that begin with the letter M?
b. What fraction of the states have names that begin and end with a vowel?

2.4 Addition of Fractions and Mixed Numbers

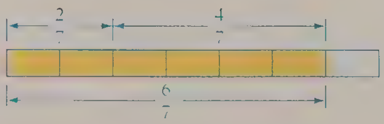
Objective A To add fractions with the same denominator

Fractions with the same denominator are added by adding the numerators and placing the sum over the common denominator. After adding, write the sum in simplest form.

Add: $\frac{2}{7} + \frac{4}{7}$

$$\begin{array}{r} \frac{2}{7} \\ + \frac{4}{7} \\ \hline \frac{6}{7} \end{array}$$

- Add the numerators and place the sum over the common denominator.



$$\frac{2}{7} + \frac{4}{7} = \frac{2+4}{7} = \frac{6}{7}$$

Example 1

Add: $\frac{5}{12} + \frac{11}{12}$

Solution

$$\begin{array}{r} \frac{5}{12} \\ + \frac{11}{12} \\ \hline \frac{16}{12} = \frac{4}{3} = 1\frac{1}{3} \end{array}$$

You Try It 1

Add: $\frac{3}{8} + \frac{7}{8}$

Your solution

Solution on p. S4

Objective B To add fractions with unlike denominators

To add fractions with unlike denominators, first rewrite the fractions as equivalent fractions with a common denominator. The common denominator is the LCM of the denominators of the fractions.

Find the total of $\frac{1}{2}$ and $\frac{1}{3}$.

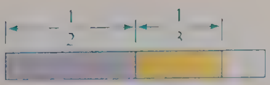
The common denominator is the LCM of 2 and 3. The LCM = 6. The LCM of denominators is sometimes called the **least common denominator** (LCD).

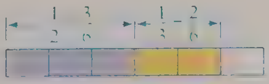
Write equivalent fractions using the LCM.

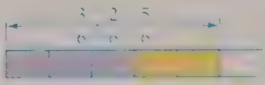
$$\begin{array}{r} \frac{1}{2} = \frac{3}{6} \\ + \frac{1}{3} = \frac{2}{6} \\ \hline \end{array}$$

Add the fractions.

$$\begin{array}{r} \frac{1}{2} = \frac{3}{6} \\ + \frac{1}{3} = \frac{2}{6} \\ \hline \frac{5}{6} \end{array}$$







Example 2

Find $\frac{7}{12}$ more than $\frac{3}{8}$.

Solution

$$\begin{array}{rcl} & \frac{3}{8} = & \frac{9}{24} \\ + & \frac{7}{12} = & \frac{14}{24} \\ \hline & & \frac{23}{24} \end{array}$$

- The LCM of 8 and 12 is 24.

You Try It 2

Find the sum of $\frac{5}{12}$ and $\frac{9}{16}$.

Your solution

Example 3

Add: $\frac{5}{8} + \frac{7}{9}$

Solution

$$\begin{array}{rcl} & \frac{5}{8} = & \frac{45}{72} \\ + & \frac{7}{9} = & \frac{56}{72} \\ \hline & & \frac{101}{72} = 1\frac{29}{72} \end{array}$$

- The LCM of 8 and 9 is 72.

You Try It 3

Add: $\frac{7}{8} + \frac{11}{15}$

Your solution

Example 4

Add: $\frac{2}{3} + \frac{3}{5} + \frac{5}{6}$

Solution

$$\begin{array}{rcl} & \frac{2}{3} = & \frac{20}{30} \\ & \frac{3}{5} = & \frac{18}{30} \\ + & \frac{5}{6} = & \frac{25}{30} \\ \hline & & \frac{63}{30} = 2\frac{3}{30} = 2\frac{1}{10} \end{array}$$

- The LCM of 3, 5, and 6 is 30.

You Try It 4

Add: $\frac{3}{4} + \frac{4}{5} + \frac{5}{8}$

Your solution

Solutions on p. S5

Objective C To add whole numbers, mixed numbers, and fractions

TAKE NOTE

The procedure at the right illustrates why $2 + \frac{2}{3} = 2\frac{2}{3}$. You do not need to show these steps when adding a whole number and a fraction. Here are two more examples:

$$7 + \frac{1}{5} = 7\frac{1}{5}$$

$$6 + \frac{3}{4} = 6\frac{3}{4}$$

The sum of a whole number and a fraction is a mixed number.

➡ Add: $2 + \frac{2}{3}$

$$\boxed{2} + \frac{2}{3} = \boxed{\frac{6}{3}} + \frac{2}{3} = \frac{8}{3} = 2\frac{2}{3}$$

To add a whole number and a mixed number, write the fraction and then add the whole numbers.

➡ Add: $7\frac{2}{5} + 4$

Write the fraction.	Add the whole numbers.
$\begin{array}{r} 7\frac{2}{5} \\ +\ 4 \\ \hline \frac{2}{5} \end{array}$	$\begin{array}{r} 7\frac{2}{5} \\ +\ 4 \\ \hline 11\frac{2}{5} \end{array}$

To add two mixed numbers, add the fractional parts and then add the whole numbers. Remember to reduce the sum to simplest form.

What is $6\frac{14}{15}$ added to $5\frac{4}{9}$?

The LCM of 9 and 15 is 45.

Add the fractional parts.

$$\begin{array}{rl} 5\frac{4}{9} & = 5\frac{20}{45} \\ +\ 6\frac{14}{15} & = 6\frac{42}{45} \\ \hline & \quad\ \frac{62}{45} \end{array}$$

Add the whole numbers.

$$\begin{array}{rl} 5\frac{4}{9} & = 5\frac{20}{45} \\ +\ 6\frac{14}{15} & = 6\frac{42}{45} \\ \hline & 11\frac{62}{45} = 11 + 1\frac{17}{45} = 12\frac{17}{45} \end{array}$$

Example 5

Add: $5 + \frac{3}{8}$

Solution $5 + \frac{3}{8} = 5\frac{3}{8}$

You Try It 5

What is 7 added to $\frac{6}{11}$?

Your solution

Example 6

Find 17 increased by $3\frac{3}{8}$.

Solution

$$\begin{array}{r} 17 \\ +\ 3\frac{3}{8} \\ \hline 20\frac{3}{8} \end{array}$$

You Try It 6

Find the sum of 29 and $17\frac{5}{12}$.

Your solution

Example 7

Add: $5\frac{2}{3} + 11\frac{5}{6} + 12\frac{7}{9}$

Solution

$$\begin{array}{rl} 5\frac{2}{3} & = \ 5\frac{12}{18} \\ 11\frac{5}{6} & = 11\frac{15}{18} \\ +\ 12\frac{7}{9} & = 12\frac{14}{18} \\ \hline & 28\frac{41}{18} = 30\frac{5}{18} \end{array}$$

• LCM = 18

You Try It 7

Add: $7\frac{4}{5} + 6\frac{7}{10} + 13\frac{11}{15}$

Your solution

Example 8

Add: $11\frac{5}{8} + 7\frac{5}{9} + 8\frac{7}{15}$

Solution

$$\begin{array}{rl} 11\frac{5}{8} & = 11\frac{225}{360} \\ 7\frac{5}{9} & = \ 7\frac{200}{360} \\ +\ 8\frac{7}{15} & = \ 8\frac{168}{360} \\ \hline & 26\frac{593}{360} = 27\frac{233}{360} \end{array}$$

• LCM = 360

You Try It 8

Add: $9\frac{3}{8} + 17\frac{7}{12} + 10\frac{14}{15}$

Your solution

Solutions on p. S5

Objective D To solve application problems

Example 9

A rain gauge collected $2\frac{1}{3}$ inches of rain in October, $5\frac{1}{2}$ inches in November, and $3\frac{3}{8}$ inches in December. Find the total rainfall for the 3 months.

Strategy
To find the total rainfall for the 3 months, add the 3 amounts of rainfall $\left(2\frac{1}{3}, 5\frac{1}{2}, \text{and } 3\frac{3}{8}\right)$.

Solution

$$\begin{array}{r}
2\frac{1}{3} = 2\frac{8}{24} \\
5\frac{1}{2} = 5\frac{12}{24} \\
+\ 3\frac{3}{8} = 3\frac{9}{24} \\
\hline
10\frac{29}{24} = 11\frac{5}{24}
\end{array}$$

The total rainfall for the 3 months was $11\frac{5}{24}$ inches.

You Try It 9

On Monday, you spent $4\frac{1}{2}$ hours in class, $3\frac{3}{4}$ hours studying, and $1\frac{1}{3}$ hours driving. Find the number of hours spent on these three activities.

Your strategy

Your solution

Example 10

Barbara Walsh worked 4 hours, $2\frac{1}{3}$ hours, and $5\frac{2}{3}$ hours this week at a part-time job. Barbara is paid \$9 an hour. How much did she earn this week?

Strategy
To find how much Barbara earned:

- Find the total number of hours worked.
- Multiply the total number of hours worked by the hourly wage (9).

Solution

$$\begin{array}{r}
4 \\
2\frac{1}{3} \\
+\ 5\frac{2}{3} \\
\hline
11\frac{3}{3}
\end{array} = 12 \text{ hours worked} \qquad \begin{array}{r}
12 \\
\times\ 9 \\
\hline
108
\end{array}$$

Barbara earned \$108 this week.

You Try It 10

Jeff Sapone, a carpenter, worked $1\frac{2}{3}$ hours of overtime on Monday, $3\frac{1}{3}$ hours of overtime on Tuesday, and 2 hours of overtime on Wednesday. At an overtime hourly rate of \$36, find Jeff's overtime pay for these 3 days.

Your strategy

Your solution

Solutions on p. S5

2.4 Exercises

Objective A

Add.

1. $\frac{2}{7}+\frac{1}{7}$

2. $\frac{3}{11}+\frac{5}{11}$

3. $\frac{1}{2}+\frac{1}{2}$

4. $\frac{1}{3}+\frac{2}{3}$

5. $\frac{8}{11}+\frac{7}{11}$

6. $\frac{9}{13}+\frac{7}{13}$

7. $\frac{8}{5}+\frac{9}{5}$

8. $\frac{5}{3}+\frac{7}{3}$

9. $\frac{3}{5}+\frac{8}{5}+\frac{3}{5}$

10. $\frac{3}{8}+\frac{5}{8}+\frac{7}{8}$

11. $\frac{3}{4}+\frac{1}{4}+\frac{5}{4}$

12. $\frac{2}{7}+\frac{4}{7}+\frac{5}{7}$

13. $\frac{3}{8}+\frac{7}{8}+\frac{1}{8}$

14. $\frac{5}{12}+\frac{7}{12}+\frac{1}{12}$

15. $\frac{4}{15}+\frac{7}{15}+\frac{11}{15}$

16. $\frac{3}{4}+\frac{3}{4}+\frac{1}{4}$

17. $\frac{3}{16}+\frac{5}{16}+\frac{7}{16}$

18. $\frac{5}{18}+\frac{11}{18}+\frac{17}{18}$

19. $\frac{3}{11}+\frac{5}{11}+\frac{7}{11}$

20. $\frac{5}{7}+\frac{4}{7}+\frac{5}{7}$

21. Find the sum of $\frac{4}{9}$ and $\frac{5}{9}$.

22. Find the sum of $\frac{5}{12}$, $\frac{1}{12}$, and $\frac{11}{12}$.

23. Find the total of $\frac{5}{8}$, $\frac{3}{8}$, and $\frac{7}{8}$.

24. Find the total of $\frac{4}{13}$, $\frac{7}{13}$, and $\frac{11}{13}$.

Objective B

Add.

25. $\frac{1}{2}+\frac{2}{3}$

26. $\frac{2}{3}+\frac{1}{4}$

27. $\frac{3}{14}+\frac{5}{7}$

28. $\frac{3}{5}+\frac{7}{10}$

29. $\frac{8}{15}+\frac{7}{20}$

30. $\frac{1}{6}+\frac{7}{9}$

31. $\frac{3}{8}+\frac{9}{14}$

32. $\frac{5}{12}+\frac{5}{16}$

33. $\frac{3}{20}+\frac{7}{30}$

34. $\frac{5}{12}+\frac{7}{30}$

35. $\frac{2}{3}+\frac{6}{19}$

36. $\frac{1}{2}+\frac{3}{29}$

37. $\frac{1}{3} + \frac{5}{6} + \frac{7}{9}$

38. $\frac{2}{3} + \frac{5}{6} + \frac{7}{12}$

39. $\frac{5}{6} + \frac{1}{12} + \frac{5}{16}$

40. $\frac{2}{9} + \frac{7}{15} + \frac{4}{21}$

41. $\frac{2}{3} + \frac{1}{5} + \frac{7}{12}$

42. $\frac{3}{4} + \frac{4}{5} + \frac{7}{12}$

43. $\frac{1}{4} + \frac{4}{5} + \frac{5}{9}$

44. $\frac{2}{3} + \frac{3}{5} + \frac{7}{8}$

45. $\frac{5}{16} + \frac{11}{18} + \frac{17}{24}$

46. $\frac{3}{10} + \frac{14}{15} + \frac{9}{25}$

47. $\frac{2}{3} + \frac{5}{8} + \frac{7}{9}$

48. $\frac{1}{3} + \frac{2}{9} + \frac{7}{8}$

49. What is $\frac{3}{8}$ added to $\frac{3}{5}$?

50. What is $\frac{5}{9}$ added to $\frac{7}{12}$?

51. Find the sum of $\frac{3}{8}$, $\frac{5}{6}$, and $\frac{7}{12}$.

52. Find the sum of $\frac{11}{12}$, $\frac{13}{24}$, and $\frac{4}{15}$.

53. Find the total of $\frac{1}{2}$, $\frac{5}{8}$, and $\frac{7}{9}$.

54. Find the total of $\frac{5}{14}$, $\frac{3}{7}$, and $\frac{5}{21}$.

Objective C

Add.

55. $\begin{array}{r} 1\frac{1}{2} \\ +\ 2\frac{1}{6} \\ \hline \end{array}$

56. $\begin{array}{r} 2\frac{2}{5} \\ +\ 3\frac{3}{10} \\ \hline \end{array}$

57. $\begin{array}{r} 4\frac{1}{2} \\ +\ 5\frac{7}{12} \\ \hline \end{array}$

58. $\begin{array}{r} 3\frac{3}{8} \\ +\ 2\frac{5}{16} \\ \hline \end{array}$

59. $\begin{array}{r} 4 \\ +\ 5\frac{2}{7} \\ \hline \end{array}$

60. $\begin{array}{r} 6\frac{8}{9} \\ +\ 12 \\ \hline \end{array}$

61. $\begin{array}{r} 3\frac{5}{8} \\ +\ 2\frac{11}{20} \\ \hline \end{array}$

62. $\begin{array}{r} 4\frac{5}{12} \\ +\ 6\frac{11}{18} \\ \hline \end{array}$

63. $7\frac{5}{12} + 2\frac{9}{16}$

64. $9\frac{1}{2} + 3\frac{3}{11}$

65. $6\frac{1}{3} + 2\frac{3}{13}$

66. $8\frac{21}{40} + 6\frac{21}{32}$

67. $8\frac{29}{30} + 7\frac{11}{40}$

68. $17\frac{5}{16} + 3\frac{11}{24}$

69. $17\frac{3}{8} + 7\frac{7}{20}$

70. $14\frac{7}{12} + 29\frac{13}{21}$

71. $5\frac{7}{8} + 27\frac{5}{12}$

72. $7\frac{5}{6} + 3\frac{5}{9}$

73. $7\frac{5}{9} + 2\frac{7}{12}$

74. $3\frac{1}{2} + 2\frac{3}{4} + 1\frac{5}{6}$

75. $2\frac{1}{2} + 3\frac{2}{3} + 4\frac{1}{4}$

76. $3\frac{1}{3} + 7\frac{1}{5} + 2\frac{1}{7}$

77. $3\frac{1}{2} + 3\frac{1}{5} + 8\frac{1}{9}$

78. $6\frac{5}{9} + 6\frac{5}{12} + 2\frac{5}{18}$

79. $2\frac{3}{8} + 4\frac{7}{12} + 3\frac{5}{16}$

80. $2\frac{1}{8} + 4\frac{2}{9} + 5\frac{17}{18}$

81. $6\frac{5}{6} + 17\frac{2}{9} + 18\frac{5}{27}$

82. $4\frac{7}{20} + \frac{17}{80} + 25\frac{23}{60}$

83. Find the sum of $2\frac{4}{9}$ and $5\frac{7}{12}$.

84. Find $5\frac{5}{6}$ more than $3\frac{3}{8}$.

85. What is $4\frac{3}{4}$ added to $9\frac{1}{3}$?

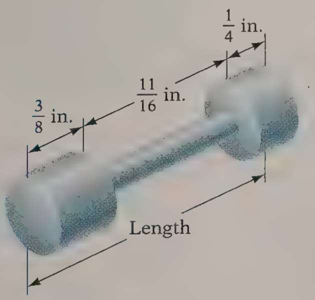
86. What is $4\frac{8}{9}$ added to $9\frac{1}{6}$?

87. Find the total of $2\frac{2}{3}$, $4\frac{5}{8}$, and $2\frac{2}{9}$.

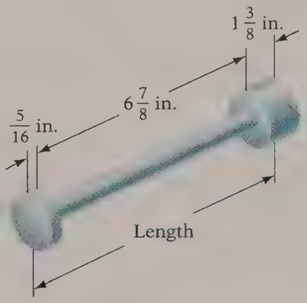
88. Find the total of $1\frac{5}{8}$, $3\frac{5}{6}$, and $7\frac{7}{24}$.

Objective D Application Problems

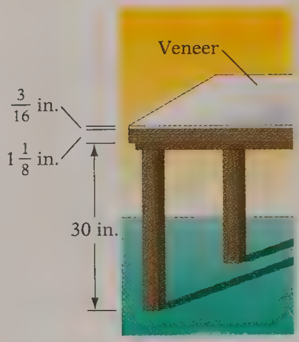
89. Find the length of the shaft.



90. Find the length of the shaft.



91. A table 30 inches high has a top that is $1\frac{1}{8}$ inches thick. Find the total thickness of the table top after a $\frac{3}{16}$-inch veneer is applied.



92. Fred Thomson worked $2\frac{2}{3}$ hours of overtime on Monday, $1\frac{1}{4}$ hours on Wednesday, $1\frac{1}{3}$ hours on Friday, and $6\frac{3}{4}$ hours on Saturday.

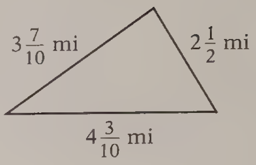
a. Find the total number of overtime hours worked during the week.

b. At an overtime hourly wage of $22 per hour, how much overtime pay does Fred receive?

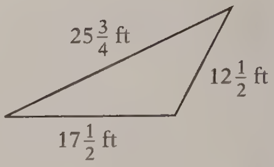
93. You are working a part-time job that pays \$9 an hour. You worked 5, $3\frac{3}{4}$, $2\frac{1}{3}$, $1\frac{1}{4}$, and $7\frac{2}{3}$ hours during the last five days.

a. Find the total number of hours you worked during the last five days.
b. Find your total wages for the five days.

94. The course of a yachting race is in the shape of a triangle with sides that measure $4\frac{3}{10}$ miles, $3\frac{7}{10}$ miles, and $2\frac{1}{2}$ miles. Find the total length of the course.



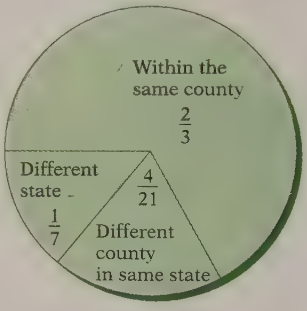
95. A flower garden in the yard of a colonial home is in the shape of a triangle as shown at the right. The wooden beams lining the edge of the garden need to be replaced. Find the total length of wood beams that must be purchased in order to replace the old beams.



96. Alan bought 200 shares of a utility stock for $\$26\frac{3}{16}$ per share. The monthly gains for the next three months were $\$1\frac{1}{2}$, $\$\frac{5}{16}$, and $\$2\frac{5}{8}$ per share. Find the value of one share of the stock at the end of the three months.

97. During a recent year, over 42 million Americans changed homes. The graph at the right shows what fractions of the people moved within the same county, moved to a different county in the same state, and moved to a different state. (Source: Census Bureau; *Geographical Mobility*) What fractional part of those who changed homes moved outside the county they had been living in?

Where Americans Moved



APPLYING THE CONCEPTS

98. What is a unit fraction? Find the sum of the three largest unit fractions. Is there a smallest unit fraction? If so, write it down. If not, explain why.

99. Use a model to illustrate and explain the addition of fractions with unlike denominators.

100. A survey was conducted to determine people's favorite color from among blue, green, red, purple, and other. The surveyor claims that $\frac{1}{3}$ of the people responded blue, $\frac{1}{6}$ responded green, $\frac{1}{8}$ responded red, $\frac{1}{12}$ responded purple, and $\frac{2}{5}$ responded some other color. Is this possible? Explain your answer.

2.5 Subtraction of Fractions and Mixed Numbers

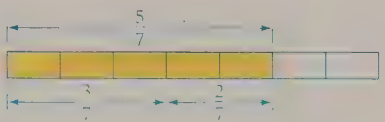
Objective A To subtract fractions with the same denominator

Fractions with the same denominator are subtracted by subtracting the numerators and placing the difference over the common denominator. After subtracting, write the fraction in simplest form.

Subtract: $\frac{5}{7} - \frac{3}{7}$

$$\begin{array}{r} \frac{5}{7} \\ -\frac{3}{7} \\ \hline \frac{2}{7} \end{array}$$

- Subtract the numerators and place the difference over the common denominator.



$$\frac{5}{7} - \frac{3}{7} = \frac{5-3}{7} = \frac{2}{7}$$

Example 1

Find $\frac{17}{30}$ less $\frac{11}{30}$.

Solution

$$\begin{array}{r} \frac{17}{30} \\ -\frac{11}{30} \\ \hline \frac{6}{30} = \frac{1}{5} \end{array}$$

You Try It 1

Subtract: $\frac{16}{27} - \frac{7}{27}$

Your solution

Solution on p. S5

Objective B To subtract fractions with unlike denominators

To subtract fractions with unlike denominators, first rewrite the fractions as equivalent fractions with a common denominator. As with adding fractions, the common denominator is the LCM of the denominators of the fractions.

Subtract: $\frac{5}{6} - \frac{1}{4}$

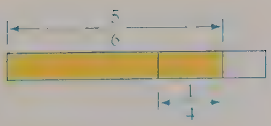
The common denominator is the LCM of 6 and 4. The LCM = 12.

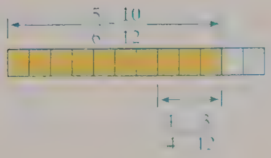
Write equivalent fractions using the LCM.

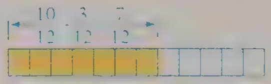
$$\begin{array}{r} \frac{5}{6} = \frac{10}{12} \\ -\frac{1}{4} = \frac{3}{12} \\ \hline \end{array}$$

Subtract the fractions.

$$\begin{array}{r} \frac{5}{6} = \frac{10}{12} \\ -\frac{1}{4} = \frac{3}{12} \\ \hline \frac{7}{12} \end{array}$$







Example 2

Subtract: $\frac{11}{16} - \frac{5}{12}$

Solution

$$\begin{array}{r} \frac{11}{16} = \frac{33}{48} \\ -\ \frac{5}{12} = \frac{20}{48} \\ \hline \frac{13}{48} \end{array}$$

- LCM = 48

You Try It 2

Subtract: $\frac{13}{18} - \frac{7}{24}$

Your solution

Solution on p. S5

Objective C To subtract whole numbers, mixed numbers, and fractions

To subtract mixed numbers without borrowing, subtract the fractional parts and then subtract the whole numbers.

➡ Subtract: $5\frac{5}{6} - 2\frac{3}{4}$

Subtract the fractional parts.

$$\begin{array}{r} 5\frac{5}{6} = 5\frac{10}{12} \\ -\ 2\frac{3}{4} = 2\frac{9}{12} \\ \hline \frac{1}{12} \end{array}$$

- The LCM of 6 and 4 is 12.

Subtract the whole numbers.

$$\begin{array}{r} 5\frac{5}{6} = 5\frac{10}{12} \\ -\ 2\frac{3}{4} = 2\frac{9}{12} \\ \hline 3\frac{1}{12} \end{array}$$

Subtraction of mixed numbers sometimes involves borrowing.

➡ Subtract: $5 - 2\frac{5}{8}$

Borrow 1 from 5.

$$\begin{array}{r} 5 = \overset{4}{\cancel{5}}\ 1 \\ -\ 2\frac{5}{8} = 2\frac{5}{8} \\ \hline \end{array}$$

Write 1 as a fraction so that the fractions have the same denominators.

$$\begin{array}{r} 5 = 4\frac{8}{8} \\ -\ 2\frac{5}{8} = 2\frac{5}{8} \\ \hline \end{array}$$

Subtract the mixed numbers.

$$\begin{array}{r} 5 = 4\frac{8}{8} \\ -\ 2\frac{5}{8} = 2\frac{5}{8} \\ \hline 2\frac{3}{8} \end{array}$$

➡ Subtract: $7\frac{1}{6} - 2\frac{5}{8}$

Write equivalent fractions using the LCM.

$$\begin{array}{r} 7\frac{1}{6} = 7\frac{4}{24} \\ -\ 2\frac{5}{8} = 2\frac{15}{24} \\ \hline \end{array}$$

Borrow 1 from 7. Add the 1 to $\frac{4}{24}$. Write $1\frac{4}{24}$ as $\frac{28}{24}$.

$$\begin{array}{r} 7\frac{1}{6} = \overset{6}{\cancel{7}}1\frac{4}{24} = 6\frac{28}{24} \\ -\ 2\frac{5}{8} = \quad 2\frac{15}{24} = 2\frac{15}{24} \\ \hline \end{array}$$

Subtract the mixed numbers.

$$\begin{array}{r} 7\frac{1}{6} = 6\frac{28}{24} \\ -\ 2\frac{5}{8} = 2\frac{15}{24} \\ \hline 4\frac{13}{24} \end{array}$$

Example 3

Subtract: $15\frac{7}{8} - 12\frac{2}{3}$

Solution

$$\begin{array}{rcl} 15\frac{7}{8} & = & 15\frac{21}{24} \\ -\ 12\frac{2}{3} & = & 12\frac{16}{24} \\ \hline & & 3\frac{5}{24} \end{array}$$

- LCM = 24

You Try It 3

Subtract: $17\frac{5}{9} - 11\frac{5}{12}$

Your solution

Example 4

Subtract: $9 - 4\frac{3}{11}$

Solution

$$\begin{array}{rcl} 9 & = & 8\frac{11}{11} \\ -\ 4\frac{3}{11} & = & 4\frac{3}{11} \\ \hline & & 4\frac{8}{11} \end{array}$$

- LCM = 11

You Try It 4

Subtract: $8 - 2\frac{4}{13}$

Your solution

Example 5

Find $11\frac{5}{12}$ decreased by $2\frac{11}{16}$.

Solution

$$\begin{array}{rcccl} 11\frac{5}{12} & = & 11\frac{20}{48} & = & 10\frac{68}{48} \\ -\ 2\frac{11}{16} & = & 2\frac{33}{48} & = & 2\frac{33}{48} \\ \hline & & & & 8\frac{35}{48} \end{array}$$

- LCM = 48

You Try It 5

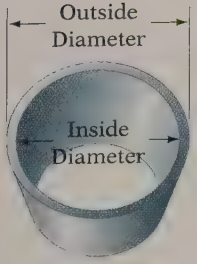
What is $21\frac{7}{9}$ minus $7\frac{11}{12}$?

Your solution

Solutions on p. S6

Objective D

To solve application problems



The outside diameter of a bushing is $3\frac{3}{8}$ inches and the wall thickness is $\frac{1}{4}$ inch. Find the inside diameter of the bushing.

$$\frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

- Add $\frac{1}{4}$ to $\frac{1}{4}$ to find the total thickness of the two walls.

$$\begin{array}{rcccl} 3\frac{3}{8} & = & 3\frac{3}{8} & = & 2\frac{11}{8} \\ -\ \frac{1}{2} & = & \frac{4}{8} & = & \frac{4}{8} \\ \hline & & & & 2\frac{7}{8} \end{array}$$

- Subtract the total thickness of the two walls to find the inside diameter.

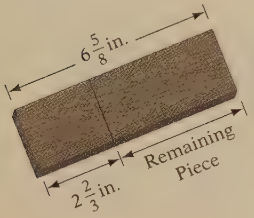
The inside diameter of the bushing is $2\frac{7}{8}$ inches.

Example 6

A $2\frac{2}{3}$-inch piece is cut from a $6\frac{5}{8}$-inch board. How much of the board is left?

Strategy

To find the length remaining, subtract the length of the piece cut from the total length of the board.



Solution

$$\begin{array}{rcrcr} & 6\frac{5}{8} & = 6\frac{15}{24} & = & 5\frac{39}{24} \\ - & 2\frac{2}{3} & = 2\frac{16}{24} & = & 2\frac{16}{24} \\ \hline & & & & 3\frac{23}{24} \end{array}$$

$3\frac{23}{24}$ inches of the board are left.

You Try It 6

A flight from New York to Los Angeles takes $5\frac{1}{2}$ hours. After the plane has been in the air for $2\frac{3}{4}$ hours, how much flight time remains?

Your strategy

Your solution

Example 7

Two painters are staining a house. In 1 day one painter stained $\frac{1}{3}$ of the house, and the other stained $\frac{1}{4}$ of the house. How much of the job remains to be done?

Strategy

To find how much of the job remains:

- Find the total amount of the house already stained $\left(\frac{1}{3} + \frac{1}{4}\right)$.
- Subtract the amount already stained from 1, which represents the complete job.

Solution

$$\begin{array}{rcr} & \frac{1}{3} & = \frac{4}{12} \\ + & \frac{1}{4} & = \frac{3}{12} \\ \hline & & \frac{7}{12} \end{array} \qquad \begin{array}{rcr} & 1 & = \frac{12}{12} \\ - & \frac{7}{12} & = \frac{7}{12} \\ \hline & & \frac{5}{12} \end{array}$$

$\frac{5}{12}$ of the house remains to be stained.

You Try It 7

A patient is put on a diet to lose 24 pounds in 3 months. The patient lost $7\frac{1}{2}$ pounds the first month and $5\frac{3}{4}$ pounds the second month. How much weight must be lost the third month to achieve the goal?

Your strategy

Your solution

Solutions on p. S6

2.5 Exercises

Objective A

Subtract.

1. $\begin{array}{r} \frac{9}{17} \\ -\frac{7}{17} \\ \hline \end{array}$

2. $\begin{array}{r} \frac{11}{15} \\ -\frac{3}{15} \\ \hline \end{array}$

3. $\begin{array}{r} \frac{11}{12} \\ -\frac{7}{12} \\ \hline \end{array}$

4. $\begin{array}{r} \frac{13}{15} \\ -\frac{4}{15} \\ \hline \end{array}$

5. $\begin{array}{r} \frac{9}{20} \\ -\frac{7}{20} \\ \hline \end{array}$

6. $\begin{array}{r} \frac{48}{55} \\ -\frac{13}{55} \\ \hline \end{array}$

7. $\begin{array}{r} \frac{42}{65} \\ -\frac{17}{65} \\ \hline \end{array}$

8. $\begin{array}{r} \frac{11}{24} \\ -\frac{5}{24} \\ \hline \end{array}$

9. $\begin{array}{r} \frac{23}{30} \\ -\frac{13}{30} \\ \hline \end{array}$

10. $\begin{array}{r} \frac{17}{42} \\ -\frac{5}{42} \\ \hline \end{array}$

11. What is $\frac{5}{14}$ less than $\frac{13}{14}$?

12. What is $\frac{7}{19}$ less than $\frac{17}{19}$?

13. Find the difference between $\frac{7}{8}$ and $\frac{5}{8}$.

14. Find the difference between $\frac{7}{12}$ and $\frac{5}{12}$.

15. What is $\frac{18}{23}$ minus $\frac{9}{23}$?

16. What is $\frac{7}{9}$ minus $\frac{3}{9}$?

17. Find $\frac{17}{24}$ decreased by $\frac{11}{24}$.

18. Find $\frac{19}{30}$ decreased by $\frac{11}{30}$.

Objective B

Subtract.

19. $\begin{array}{r} \frac{2}{3} \\ -\frac{1}{6} \\ \hline \end{array}$

20. $\begin{array}{r} \frac{7}{8} \\ -\frac{5}{16} \\ \hline \end{array}$

21. $\begin{array}{r} \frac{5}{8} \\ -\frac{2}{7} \\ \hline \end{array}$

22. $\begin{array}{r} \frac{5}{6} \\ -\frac{3}{7} \\ \hline \end{array}$

23. $\begin{array}{r} \frac{5}{7} \\ -\frac{3}{14} \\ \hline \end{array}$

24. $\begin{array}{r} \frac{5}{9} \\ -\frac{7}{15} \\ \hline \end{array}$

25. $\begin{array}{r} \frac{8}{15} \\ -\frac{7}{20} \\ \hline \end{array}$

26. $\begin{array}{r} \frac{7}{9} \\ -\frac{1}{6} \\ \hline \end{array}$

27. $\begin{array}{r} \frac{9}{14} \\ -\frac{3}{8} \\ \hline \end{array}$

28. $\begin{array}{r} \frac{5}{12} \\ -\frac{5}{16} \\ \hline \end{array}$

29. $\begin{array}{r} \frac{46}{51} \\ -\frac{3}{17} \\ \hline \end{array}$
30. $\begin{array}{r} \frac{9}{16} \\ -\frac{17}{32} \\ \hline \end{array}$
31. $\begin{array}{r} \frac{21}{35} \\ -\frac{5}{14} \\ \hline \end{array}$
32. $\begin{array}{r} \frac{19}{40} \\ -\frac{3}{16} \\ \hline \end{array}$
33. $\begin{array}{r} \frac{29}{60} \\ -\frac{3}{40} \\ \hline \end{array}$

34. What is $\frac{3}{5}$ less than $\frac{11}{12}$?

35. What is $\frac{5}{9}$ less than $\frac{11}{15}$?

36. Find the difference between $\frac{11}{24}$ and $\frac{7}{18}$.

37. Find the difference between $\frac{9}{14}$ and $\frac{5}{42}$.

38. Find $\frac{11}{12}$ decreased by $\frac{11}{15}$.

39. Find $\frac{17}{20}$ decreased by $\frac{7}{15}$.

40. What is $\frac{13}{20}$ minus $\frac{1}{6}$?

41. What is $\frac{5}{6}$ minus $\frac{7}{9}$?

Objective C

Subtract.

42. $\begin{array}{r} 5\frac{7}{12} \\ -2\frac{5}{12} \\ \hline \end{array}$
43. $\begin{array}{r} 16\frac{11}{15} \\ -11\frac{8}{15} \\ \hline \end{array}$
44. $\begin{array}{r} 72\frac{21}{23} \\ -16\frac{17}{23} \\ \hline \end{array}$
45. $\begin{array}{r} 19\frac{16}{17} \\ -9\frac{7}{17} \\ \hline \end{array}$
46. $\begin{array}{r} 6\frac{1}{3} \\ -2 \\ \hline \end{array}$

47. $\begin{array}{r} 5\frac{7}{8} \\ -1 \\ \hline \end{array}$
48. $\begin{array}{r} 10 \\ -6\frac{1}{3} \\ \hline \end{array}$
49. $\begin{array}{r} 3 \\ -2\frac{5}{21} \\ \hline \end{array}$
50. $\begin{array}{r} 6\frac{2}{5} \\ -4\frac{4}{5} \\ \hline \end{array}$
51. $\begin{array}{r} 16\frac{3}{8} \\ -10\frac{7}{8} \\ \hline \end{array}$

52. $\begin{array}{r} 25\frac{4}{9} \\ -16\frac{7}{9} \\ \hline \end{array}$
53. $\begin{array}{r} 8\frac{3}{7} \\ -2\frac{6}{7} \\ \hline \end{array}$
54. $\begin{array}{r} 16\frac{2}{5} \\ -8\frac{4}{9} \\ \hline \end{array}$
55. $\begin{array}{r} 23\frac{7}{8} \\ -16\frac{2}{3} \\ \hline \end{array}$
56. $\begin{array}{r} 6 \\ -4\frac{3}{5} \\ \hline \end{array}$

57. $\begin{array}{r} 65\frac{8}{35} \\ -16\frac{11}{14} \\ \hline \end{array}$
58. $\begin{array}{r} 82\frac{4}{33} \\ -16\frac{5}{22} \\ \hline \end{array}$
59. $\begin{array}{r} 101\frac{2}{9} \\ -16 \\ \hline \end{array}$
60. $\begin{array}{r} 77\frac{5}{18} \\ -61 \\ \hline \end{array}$
61. $\begin{array}{r} 17 \\ -7\frac{8}{13} \\ \hline \end{array}$

62. What is $5\frac{3}{8}$ less than $8\frac{1}{9}$?

63. What is $7\frac{3}{5}$ less than $23\frac{3}{20}$?

64. Find the difference between $9\frac{2}{7}$ and $3\frac{1}{4}$.

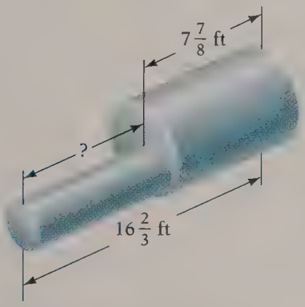
65. Find the difference between $12\frac{3}{8}$ and $7\frac{5}{12}$.

66. What is $10\frac{5}{9}$ minus $5\frac{11}{15}$?

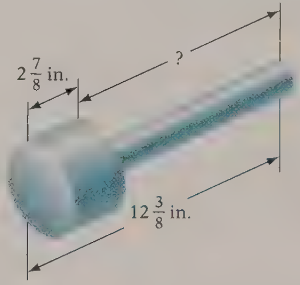
67. Find $6\frac{1}{3}$ decreased by $3\frac{3}{5}$.

Objective D *Application Problems*

68. Find the missing dimension.



69. Find the missing dimension.

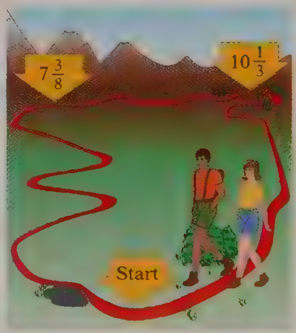


70. In the Kentucky Derby the horses run $1\frac{1}{4}$ miles. In the Belmont Stakes they run $1\frac{1}{2}$ miles, and in the Preakness Stakes they run $1\frac{3}{16}$ miles. How much farther do the horses run in the Kentucky Derby than in the Preakness Stakes? How much farther do they run in the Belmont Stakes than in the Preakness Stakes?

71. In the running high jump in the 1948 Summer Olympic Games, Alice Coachman's distance was $66\frac{1}{8}$ inches. In the same event in the 1972 Summer Olympics, Urika Meyfarth jumped $75\frac{1}{2}$ inches, and in the 1996 Olympic Games, Stefka Kostadinova jumped $80\frac{3}{4}$ inches. Find the difference between Meyfarth's distance and Coachman's distance. Find the difference between Kostadinova's distance and Meyfarth's distance.

72. Two hikers plan a 3-day $27\frac{1}{2}$-mile backpack trip carrying a total of 80 pounds. The hikers plan to travel $7\frac{3}{8}$ miles the first day and $10\frac{1}{3}$ miles the second day.

a. How many miles do the hikers plan to travel the first two days?
b. How many miles will be left to travel on the third day?



73. A 12-mile walkathon has three checkpoints. The first is $3\frac{3}{8}$ miles from the starting point. The second checkpoint is $4\frac{1}{3}$ miles from the first.

a. How many miles is it from the starting point to the second checkpoint?
b. How many miles is it from the second checkpoint to the finish line?

74. A patient with high blood pressure who weighs 225 pounds is put on a diet to lose 25 pounds in 3 months. The patient loses $8\frac{3}{4}$ pounds the first month and $11\frac{5}{8}$ pounds the second month. How much weight must be lost the third month for the goal to be achieved?

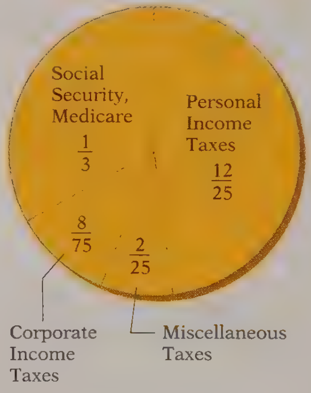
75. A wrestler is entered in the 172-pound weight class in the conference finals coming up in 3 weeks. The wrestler needs to lose $12\frac{3}{4}$ pounds. The wrestler loses $5\frac{1}{4}$ pounds the first week and $4\frac{1}{4}$ pounds the second week.

a. Without doing the calculations, determine whether the wrestler can reach his weight class by losing less in the third week than was lost in the second week.
b. How many pounds must be lost in the third week for the desired weight to be reached?

76. The figure at the right shows the sources of federal income. The category "Social Security, Medicare" includes unemployment and other retirement taxes. The category "Miscellaneous Taxes" includes excise, customs, estate, and gift taxes.

a. What is the difference between the fraction of federal income derived from personal income taxes and the fraction derived from corporate income taxes?
b. What is the difference between the fraction of federal income derived from the Social Security category and the fraction derived from miscellaneous taxes?

Sources of Federal Income



APPLYING THE CONCEPTS

77. Fill in the square to produce a true statement: $5\frac{1}{3} - \boxed{} = 2\frac{1}{2}$.

78. Fill in the square to produce a true statement: $\boxed{} - 4\frac{1}{2} = 1\frac{5}{8}$.

79. Fill in the blank squares at the right so that the sum of the numbers is the same along any row, column, or diagonal. The resulting square is called a magic square.

		$\frac{3}{4}$
	$\frac{5}{8}$	
$\frac{1}{2}$		$\frac{7}{8}$

80. If $\frac{4}{15}$ of an electrician's income is spent for housing, what fraction of the electrician's income is not spent for housing?

2.6 Multiplication of Fractions and Mixed Numbers

Objective A

To multiply fractions

The product of two fractions is the product of the numerators over the product of the denominators.

⇒ Multiply: $\frac{2}{3} \times \frac{4}{5}$

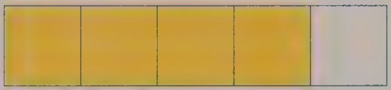
$$\frac{2}{3} \times \frac{4}{5} = \frac{2 \cdot 4}{3 \cdot 5} = \frac{8}{15}$$

- Multiply the numerators.
- Multiply the denominators.

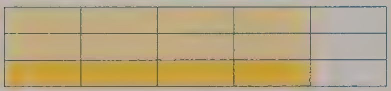
The product $\frac{2}{3} \times \frac{4}{5}$ can be read "$\frac{2}{3}$ times $\frac{4}{5}$" or "$\frac{2}{3}$ of $\frac{4}{5}$."

Reading the times sign as "of" is useful in application problems.

$\frac{4}{5}$ of the bar is shaded.



Shade $\frac{2}{3}$ of the $\frac{4}{5}$ already shaded.



$\frac{8}{15}$ of the bar is then shaded light yellow.

$$\frac{2}{3} \text{ of } \frac{4}{5} = \frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$$

After multiplying two fractions, write the product in simplest form.

⇒ Multiply: $\frac{3}{4} \times \frac{14}{15}$

$$\frac{3}{4} \times \frac{14}{15} = \frac{3 \cdot 14}{4 \cdot 15}$$

- Multiply the numerators.
- Multiply the denominators.

$$= \frac{3 \cdot 2 \cdot 7}{2 \cdot 2 \cdot 3 \cdot 5}$$

- Write the prime factorization of each number.

$$= \frac{\overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{2}} \cdot 7}{\underset{1}{\cancel{2}} \cdot 2 \cdot \underset{1}{\cancel{3}} \cdot 5} = \frac{7}{10}$$

- Eliminate the common factors. Then multiply the remaining factors in the numerator and denominator.

This example could also be worked by using the GCF.

$$\frac{3}{4} \times \frac{14}{15} = \frac{42}{60}$$

- Multiply the numerators.
- Multiply the denominators.

$$= \frac{6 \cdot 7}{6 \cdot 10}$$

- The GCF of 42 and 60 is 6. Factor 6 from 42 and 60.

$$= \frac{\overset{1}{\cancel{6}} \cdot 7}{\underset{1}{\cancel{6}} \cdot 10} = \frac{7}{10}$$

- Eliminate the GCF.

Example 1

Multiply $\frac{4}{15}$ and $\frac{5}{28}$.

Solution

$$\frac{4}{15}\times\frac{5}{28}=\frac{4\cdot 5}{15\cdot 28}=\frac{\overset{1}{\cancel{2}}\cdot\overset{1}{\cancel{2}}\cdot\overset{1}{\cancel{5}}}{3\cdot\underset{1}{\cancel{5}}\cdot\underset{1}{\cancel{2}}\cdot\underset{1}{\cancel{2}}\cdot 7}=\frac{1}{21}$$

You Try It 1

Multiply $\frac{4}{21}$ and $\frac{7}{44}$.

Your solution

Example 2

Find the product of $\frac{9}{20}$ and $\frac{33}{35}$.

Solution

$$\frac{9}{20}\times\frac{33}{35}=\frac{9\cdot 33}{20\cdot 35}=\frac{3\cdot 3\cdot 3\cdot 11}{2\cdot 2\cdot 5\cdot 5\cdot 7}=\frac{297}{700}$$

You Try It 2

Find the product of $\frac{2}{21}$ and $\frac{10}{33}$.

Your solution

Example 3

What is $\frac{14}{9}$ times $\frac{12}{7}$?

Solution

$$\frac{14}{9}\times\frac{12}{7}=\frac{14\cdot 12}{9\cdot 7}=\frac{2\cdot\overset{1}{\cancel{7}}\cdot 2\cdot 2\cdot\overset{1}{\cancel{3}}}{3\cdot\underset{1}{\cancel{3}}\cdot\underset{1}{\cancel{7}}}=\frac{8}{3}=2\frac{2}{3}$$

You Try It 3

What is $\frac{16}{5}$ times $\frac{15}{24}$?

Your solution

Solutions on p. S6

Objective B To multiply whole numbers, mixed numbers, and fractions

To multiply a whole number by a fraction or mixed number, first write the whole number as a fraction with a denominator of 1.

⇨ Multiply: $4\times\frac{3}{7}$

$$4\times\frac{3}{7}=\frac{4}{1}\times\frac{3}{7}=\frac{4\cdot 3}{1\cdot 7}=\frac{2\cdot 2\cdot 3}{7}=\frac{12}{7}=1\frac{5}{7}$$

- Write 4 with a denominator of 1; then multiply the fractions.

When one or more of the factors in a product is a mixed number, write the mixed number as an improper fraction before multiplying.

⇨ Multiply: $2\frac{1}{3}\times\frac{3}{14}$

$$2\frac{1}{3}\times\frac{3}{14}=\frac{7}{3}\times\frac{3}{14}=\frac{7\cdot 3}{3\cdot 14}=\frac{\overset{1}{\cancel{7}}\cdot\overset{1}{\cancel{3}}}{\underset{1}{\cancel{3}}\cdot 2\cdot\underset{1}{\cancel{7}}}=\frac{1}{2}$$

- Write $2\frac{1}{3}$ as an improper fraction; then multiply the fractions.

Example 4

Multiply: $4\frac{5}{6}\times\frac{12}{13}$

Solution

$$4\frac{5}{6}\times\frac{12}{13}=\frac{29}{6}\times\frac{12}{13}=\frac{29\cdot 12}{6\cdot 13}$$

$$=\frac{29\cdot\overset{1}{\cancel{2}}\cdot 2\cdot\overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}}\cdot\underset{1}{\cancel{3}}\cdot 13}=\frac{58}{13}=4\frac{6}{13}$$

You Try It 4

Multiply: $5\frac{2}{5}\times\frac{5}{9}$

Your solution

Example 5

Find $5\frac{2}{3}$ times $4\frac{1}{2}$.

Solution

$$5\frac{2}{3}\times 4\frac{1}{2}=\frac{17}{3}\times\frac{9}{2}=\frac{17\cdot 9}{3\cdot 2}$$

$$=\frac{17\cdot\overset{1}{\cancel{3}}\cdot 3}{\underset{1}{\cancel{3}}\cdot 2}=\frac{51}{2}=25\frac{1}{2}$$

You Try It 5

Multiply: $3\frac{2}{5}\times 6\frac{1}{4}$

Your solution

Example 6

Multiply: $4\frac{2}{5}\times 7$

Solution

$$4\frac{2}{5}\times 7=\frac{22}{5}\times\frac{7}{1}=\frac{22\cdot 7}{5\cdot 1}$$

$$=\frac{2\cdot 11\cdot 7}{5}=\frac{154}{5}=30\frac{4}{5}$$

You Try It 6

Multiply: $3\frac{2}{7}\times 6$

Your solution

Solutions on p. S6

Objective C **To solve application problems**

Length (ft)	*Weight (lb/ft)*
$6\frac{1}{2}$	$\frac{3}{8}$
$8\frac{5}{8}$	$1\frac{1}{4}$
$10\frac{3}{4}$	$2\frac{1}{2}$
$12\frac{7}{12}$	$4\frac{1}{3}$

The table at the left lists the length of steel rods and the weight per foot. The weight per foot is measured in pounds for each foot of rod and is abbreviated as lb/ft.

⇒ Find the weight of the steel bar that is $10\frac{3}{4}$ feet long.

Strategy

To find the weight of the steel bar, multiply its length by the weight per foot.

Solution $10\frac{3}{4}\times 2\frac{1}{2}=\frac{43}{4}\times\frac{5}{2}=\frac{43\cdot 5}{4\cdot 2}=\frac{215}{8}=26\frac{7}{8}$

The weight of the $10\frac{3}{4}$-foot rod is $26\frac{7}{8}$ pounds.

Example 7

An electrician earns \$206 for each day worked. What are the electrician's earnings for working $4\frac{1}{2}$ days?

Strategy

To find the electrician's total earnings, multiply the daily earnings (206) by the number of days worked $\left(4\frac{1}{2}\right)$.

Solution

$$206 \times 4\frac{1}{2} = \frac{206}{1} \times \frac{9}{2} = \frac{206 \cdot 9}{1 \cdot 2} = 927$$

The electrician's earnings are \$927.

You Try It 7

Over the last 10 years, a house increased in value by $2\frac{1}{2}$ times. The price of the house 10 years ago was \$70,000. What is the value of the house today?

Your strategy

Your solution

Example 8

The value of a small office building and the land on which it is built is \$190,000. The value of the land is $\frac{1}{4}$ the total value. What is the dollar value of the building?

Strategy

To find the value of the building:

- Find the value of the land $\left(\frac{1}{4} \times 190{,}000\right)$.
- Subtract the value of the land from the total value (190,000).

Solution

$$\frac{1}{4} \times 190{,}000 = \frac{190{,}000}{4} = 47{,}500$$

- **Value of the land**

$$190{,}000 - 47{,}500 = 142{,}500$$

The value of the building is \$142,500.

You Try It 8

A paint company bought a drying chamber and an air compressor for spray painting. The total cost of the two items was \$60,000. The drying chamber's cost was $\frac{4}{5}$ of the total cost. What was the cost of the air compressor?

Your strategy

Your solution

Solutions on pp. S6–S7

2.6 Exercises

Objective A

Multiply.

1. $\frac{2}{3} \times \frac{7}{8}$

2. $\frac{1}{2} \times \frac{2}{3}$

3. $\frac{5}{16} \times \frac{7}{15}$

4. $\frac{3}{8} \times \frac{6}{7}$

5. $\frac{1}{6} \times \frac{1}{8}$

6. $\frac{2}{5} \times \frac{5}{6}$

7. $\frac{11}{12} \times \frac{6}{7}$

8. $\frac{11}{12} \times \frac{3}{5}$

9. $\frac{1}{6} \times \frac{6}{7}$

10. $\frac{3}{5} \times \frac{10}{11}$

11. $\frac{1}{5} \times \frac{5}{8}$

12. $\frac{6}{7} \times \frac{14}{15}$

13. $\frac{8}{9} \times \frac{27}{4}$

14. $\frac{3}{5} \times \frac{3}{10}$

15. $\frac{5}{6} \times \frac{1}{2}$

16. $\frac{3}{8} \times \frac{5}{12}$

17. $\frac{16}{9} \times \frac{27}{8}$

18. $\frac{5}{8} \times \frac{16}{15}$

19. $\frac{3}{2} \times \frac{4}{9}$

20. $\frac{5}{3} \times \frac{3}{7}$

21. $\frac{7}{8} \times \frac{3}{14}$

22. $\frac{2}{9} \times \frac{1}{5}$

23. $\frac{1}{10} \times \frac{3}{8}$

24. $\frac{5}{12} \times \frac{6}{7}$

25. $\frac{15}{8} \times \frac{16}{3}$

26. $\frac{5}{6} \times \frac{4}{15}$

27. $\frac{1}{2} \times \frac{2}{15}$

28. $\frac{3}{8} \times \frac{5}{16}$

29. $\frac{5}{7} \times \frac{14}{15}$

30. $\frac{3}{8} \times \frac{15}{41}$

31. $\frac{5}{12} \times \frac{42}{65}$

32. $\frac{16}{33} \times \frac{55}{72}$

33. $\frac{12}{5} \times \frac{5}{3}$

34. $\frac{17}{9} \times \frac{81}{17}$

35. $\frac{16}{85} \times \frac{125}{84}$

36. $\frac{19}{64} \times \frac{48}{95}$

37. Multiply $\frac{7}{12}$ and $\frac{15}{42}$.

38. Multiply $\frac{32}{9}$ and $\frac{3}{8}$.

39. Find the product of $\frac{5}{9}$ and $\frac{3}{20}$.

40. Find the product of $\frac{7}{3}$ and $\frac{15}{14}$.

41. What is $\frac{1}{2}$ times $\frac{8}{15}$?

42. What is $\frac{3}{8}$ times $\frac{12}{17}$?

Objective B

Multiply.

43. $4 \times \frac{3}{8}$

44. $14 \times \frac{5}{7}$

45. $\frac{2}{3} \times 6$

46. $\frac{5}{12} \times 40$

47. $\frac{1}{3} \times 1\frac{1}{3}$

48. $\frac{2}{5} \times 2\frac{1}{2}$

49. $1\frac{7}{8} \times \frac{4}{15}$

50. $2\frac{1}{5} \times \frac{5}{22}$

51. $55 \times \frac{3}{10}$

52. $\frac{5}{14} \times 49$

53. $4 \times 2\frac{1}{2}$

54. $9 \times 3\frac{1}{3}$

55. $2\frac{1}{7} \times 3$

56. $5\frac{1}{4} \times 8$

57. $3\frac{2}{3} \times 5$

58. $4\frac{2}{9} \times 3$

59. $\frac{1}{2} \times 3\frac{3}{7}$

60. $\frac{3}{8} \times 4\frac{4}{5}$

61. $6\frac{1}{8} \times \frac{4}{7}$

62. $5\frac{1}{3} \times \frac{5}{16}$

63. $5\frac{1}{8} \times 5$

64. $6\frac{1}{9} \times 2$

65. $\frac{3}{8} \times 4\frac{1}{2}$

66. $\frac{5}{7} \times 2\frac{1}{3}$

67. $6 \times 2\frac{2}{3}$

68. $6\frac{1}{8} \times 0$

69. $1\frac{1}{3} \times 2\frac{1}{4}$

70. $2\frac{5}{8} \times \frac{3}{23}$

71. $2\frac{5}{8} \times 3\frac{2}{5}$

72. $5\frac{3}{16} \times 5\frac{1}{3}$

73. $3\frac{1}{7} \times 2\frac{1}{8}$

74. $16\frac{5}{8} \times 1\frac{1}{16}$

75. $2\frac{2}{5} \times 3\frac{1}{12}$

76. $2\frac{2}{3} \times \frac{3}{20}$

77. $5\frac{1}{5} \times 3\frac{1}{13}$

78. $3\frac{3}{4} \times 2\frac{3}{20}$

79. $10\frac{1}{4} \times 3\frac{1}{5}$

80. $12\frac{3}{5} \times 1\frac{3}{7}$

81. $5\frac{3}{7} \times 5\frac{1}{4}$

82. $6\frac{1}{2} \times 1\frac{3}{13}$

83. Multiply $2\frac{1}{2}$ and $3\frac{3}{5}$.

84. Multiply $4\frac{3}{8}$ and $3\frac{3}{5}$.

85. Find the product of $2\frac{1}{8}$ and $\frac{5}{17}$.

86. Find the product of $12\frac{2}{5}$ and $3\frac{7}{31}$.

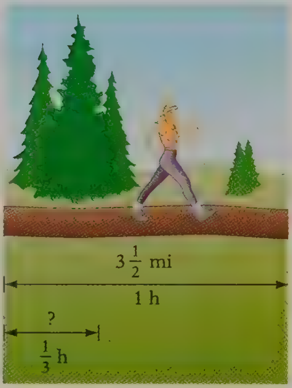
87. What is $1\frac{3}{8}$ times $2\frac{1}{5}$?

88. What is $3\frac{1}{8}$ times $2\frac{4}{7}$?

Objective C

Application Problems

89. Salmon costs $8 per pound. Find the cost of $2\frac{3}{4}$ pounds of salmon.



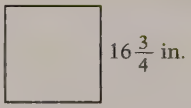
90. Maria Rivera can walk $3\frac{1}{2}$ miles in 1 hour. At this rate, how far can Maria walk in $\frac{1}{3}$ hour?

91. A board that costs $6 is $9\frac{1}{4}$ feet long. One-third of the board is cut off.

a. Without doing the calculation, is the piece being cut off at least 4 feet long?

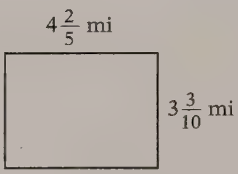
b. What is the length of the piece cut off?

92. The perimeter of a square is equal to four times the length of a side of the square. Find the perimeter of a square whose side measures $16\frac{3}{4}$ inches.



93. To find the area of a square, multiply the length of one side of the square times itself. What is the area of a square whose side measures $5\frac{1}{4}$ feet? The area of the square will be in square feet.

94. The area of a rectangle is equal to the product of the length of the rectangle times its width. Find the area of a rectangle that has a length of $4\frac{2}{5}$ miles and a width of $3\frac{3}{10}$ miles. The area will be in square miles.



95. A family budgets $\frac{2}{5}$ of its monthly income of $3200 per month for housing and utilities.

a. What amount is budgeted for housing and utilities?

b. What amount remains for purposes other than housing and utilities?

96. $\frac{5}{6}$ of a chemistry class of 36 has passing grades. $\frac{1}{5}$ of the students with passing grades received an A.

a. How many students passed the chemistry course?

b. How many students received A grades?

97. The parents of the Newton Junior High School Choir members are making robes for the choir. Each robe requires $2\frac{5}{8}$ yards of material at a cost of \$8 per yard. Find the total cost of 24 choir robes.

The table at the right shows the length of steel rods and their weight per foot. Use this table for Exercises 98 to 100.

Length (ft)	*Weight (lb/ft)*
$6\frac{1}{2}$	$\frac{3}{8}$
$8\frac{5}{8}$	$1\frac{1}{4}$
$10\frac{3}{4}$	$2\frac{1}{2}$
$12\frac{7}{12}$	$4\frac{1}{3}$

98. Find the weight of the $6\frac{1}{2}$-foot steel rod.

99. Find the weight of the $12\frac{7}{12}$-foot steel rod.

100. Find the total weight of the $8\frac{5}{8}$-foot and the $10\frac{3}{4}$-foot steel rods.

101. The manager of a mutual fund has $\frac{1}{2}$ of the portfolio invested in bonds. Of the amount invested in bonds, $\frac{3}{8}$ is invested in corporate bonds. What fraction of the total portfolio is invested in corporate bonds?

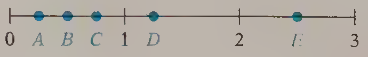
APPLYING THE CONCEPTS

102. The product of 1 and a number is $\frac{1}{2}$. Find the number.

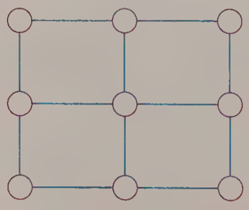
103. Our calendar is based on the solar year, which is $365\frac{1}{4}$ days. Use this fact to explain leap years.

104. Is the product of two positive fractions always greater than either one of the two numbers? If so, explain why. If not, give an example.

105. Which of the labeled points on the number line at the right could be the graph of the product of *B* and *C*?



106. Fill in the circles on the square at the right with the fractions $\frac{1}{6}, \frac{5}{18}, \frac{4}{9}, \frac{5}{9}, \frac{2}{3}, \frac{3}{4}, 1\frac{1}{9}, 1\frac{1}{2}$, and $2\frac{1}{4}$ so that the product of any row is equal to $\frac{5}{18}$. (*Note:* There is more than one possible answer.)



2.7 Division of Fractions and Mixed Numbers

Objective A To divide fractions

The **reciprocal** of a fraction is the fraction with the numerator and denominator interchanged.

The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

The process of interchanging the numerator and denominator is called **inverting** the fraction.

To find the reciprocal of a whole number, first write the whole number as a fraction with a denominator of 1; then find the reciprocal of that fraction.

The reciprocal of 5 is $\frac{1}{5}$. $\left(\text{Think } 5 = \frac{5}{1}.\right)$

Reciprocals are used to rewrite division problems as related multiplication problems. Look at the following two problems:

$8 \div 2 = 4$	$8 \times \frac{1}{2} = 4$
8 divided by 2 is 4.	8 times the reciprocal of 2 is 4.

"Divided by" means the same as "times the reciprocal of." Thus "÷ 2" can be replaced with "$\times \frac{1}{2}$," and the answer will be the same. Fractions are divided by making this replacement.

➡ Divide: $\frac{2}{3} \div \frac{3}{4}$

$$\frac{2}{3} \div \frac{3}{4} = \frac{2}{3} \times \frac{4}{3} = \frac{2 \cdot 4}{3 \cdot 3} = \frac{2 \cdot 2 \cdot 2}{3 \cdot 3} = \frac{8}{9}$$

Example 1

Divide: $\frac{5}{8} \div \frac{4}{9}$

Solution

$$\frac{5}{8} \div \frac{4}{9} = \frac{5}{8} \times \frac{9}{4} = \frac{5 \cdot 9}{8 \cdot 4}$$
$$= \frac{5 \cdot 3 \cdot 3}{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} = \frac{45}{32} = 1\frac{13}{32}$$

You Try It 1

Divide: $\frac{3}{7} \div \frac{2}{3}$

Your solution

Example 2

Divide: $\frac{3}{5} \div \frac{12}{25}$

Solution

$$\frac{3}{5} \div \frac{12}{25} = \frac{3}{5} \times \frac{25}{12} = \frac{3 \cdot 25}{5 \cdot 12}$$
$$= \frac{\overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{5}} \cdot 5}{\underset{1}{\cancel{5}} \cdot 2 \cdot 2 \cdot \underset{1}{\cancel{3}}} = \frac{5}{4} = 1\frac{1}{4}$$

You Try It 2

Divide: $\frac{3}{4} \div \frac{9}{10}$

Your solution

Solutions on p. S7

Objective B To divide whole numbers, mixed numbers, and fractions

To divide a fraction and a whole number, first write the whole number as a fraction with a denominator of 1.

Divide: $\frac{3}{7} \div 5$

$$\frac{3}{7} \div 5 = \frac{3}{7} \div \frac{5}{1} = \frac{3}{7} \times \frac{1}{5} = \frac{3 \cdot 1}{7 \cdot 5} = \frac{3}{35}$$

- Write 5 with a denominator of 1. Then divide the fractions.

When a number in a quotient is a mixed number, write the mixed number as an improper fraction before dividing.

Divide: $1\frac{13}{15} \div 4\frac{4}{5}$

Write the mixed numbers as improper fractions. Then divide the fractions.

$$1\frac{13}{15} \div 4\frac{4}{5} = \frac{28}{15} \div \frac{24}{5} = \frac{28}{15} \times \frac{5}{24} = \frac{28 \cdot 5}{15 \cdot 24} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 7 \cdot \overset{1}{\cancel{5}}}{3 \cdot \underset{1}{\cancel{5}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot 2 \cdot 3} = \frac{7}{18}$$

Example 3

Divide $\frac{4}{9}$ by 5.

Solution

$$\frac{4}{9} \div 5 = \frac{4}{9} \div \frac{5}{1} = \frac{4}{9} \times \frac{1}{5}$$

$$= \frac{4 \cdot 1}{9 \cdot 5} = \frac{2 \cdot 2}{3 \cdot 3 \cdot 5} = \frac{4}{45}$$

You Try It 3

Divide $\frac{5}{7}$ by 6.

Your solution

Example 4

Find the quotient of $\frac{3}{8}$ and $2\frac{1}{10}$.

Solution

$$\frac{3}{8} \div 2\frac{1}{10} = \frac{3}{8} \div \frac{21}{10} = \frac{3}{8} \times \frac{10}{21}$$

$$= \frac{3 \cdot 10}{8 \cdot 21} = \frac{\overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{2}} \cdot 5}{\underset{1}{\cancel{2}} \cdot 2 \cdot 2 \cdot \underset{1}{\cancel{3}} \cdot 7} = \frac{5}{28}$$

You Try It 4

Find the quotient of $12\frac{3}{5}$ and 7.

Your solution

Example 5

Divide: $2\frac{3}{4} \div 1\frac{5}{7}$

Solution

$$2\frac{3}{4} \div 1\frac{5}{7} = \frac{11}{4} \div \frac{12}{7} = \frac{11}{4} \times \frac{7}{12} = \frac{11 \cdot 7}{4 \cdot 12}$$

$$= \frac{11 \cdot 7}{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3} = \frac{77}{48} = 1\frac{29}{48}$$

You Try It 5

Divide: $3\frac{2}{3} \div 2\frac{2}{5}$

Your solution

Solutions on p. S7

Example 6

Divide: $1\frac{13}{15} \div 4\frac{1}{5}$

Solution

$$1\frac{13}{15} \div 4\frac{1}{5} = \frac{28}{15} \div \frac{21}{5} = \frac{28}{15} \times \frac{5}{21} = \frac{28 \cdot 5}{15 \cdot 21}$$

$$= \frac{2 \cdot 2 \cdot \overset{1}{\cancel{7}} \cdot \overset{1}{\cancel{5}}}{3 \cdot \underset{1}{\cancel{5}} \cdot 3 \cdot \underset{1}{\cancel{7}}} = \frac{4}{9}$$

You Try It 6

Divide: $2\frac{5}{6} \div 8\frac{1}{2}$

Your solution

Example 7

Divide: $4\frac{3}{8} \div 7$

Solution

$$4\frac{3}{8} \div 7 = \frac{35}{8} \div \frac{7}{1} = \frac{35}{8} \times \frac{1}{7}$$

$$= \frac{35 \cdot 1}{8 \cdot 7} = \frac{5 \cdot \overset{1}{\cancel{7}}}{2 \cdot 2 \cdot 2 \cdot \underset{1}{\cancel{7}}} = \frac{5}{8}$$

You Try It 7

Divide: $6\frac{2}{5} \div 4$

Your solution

Solutions on p. S7

Objective C To solve application problems

Example 8

A car used $15\frac{1}{2}$ gallons of gasoline on a 310-mile trip. How many miles can this car travel on 1 gallon of gasoline?

Strategy
To find the number of miles, divide the number of miles traveled by the number of gallons of gasoline used.

Solution

$$310 \div 15\frac{1}{2} = \frac{310}{1} \div \frac{31}{2}$$

$$= \frac{310}{1} \times \frac{2}{31} = \frac{310 \cdot 2}{1 \cdot 31}$$

$$= \frac{2 \cdot 5 \cdot \overset{1}{\cancel{31}} \cdot 2}{1 \cdot \underset{1}{\cancel{31}}} = \frac{20}{1} = 20$$

The car travels 20 miles on 1 gallon of gasoline.

You Try It 8

Leon Dern purchased a $\frac{1}{2}$-ounce gold coin for $195. What would a similar coin weighing 1 ounce cost?

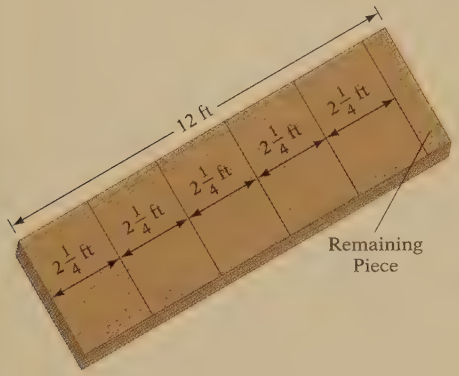
Your strategy

Your solution

Solution on p. S7

Example 9

A 12-foot board is cut into pieces $2\frac{1}{4}$ feet long for use as bookshelves. What is the length of the remaining piece after as many shelves as possible are cut?



Strategy

To find the length of the remaining piece:

- Divide the total length of the board (12) by the length of each shelf $\left(2\frac{1}{4}\right)$. This will give you the number of shelves cut, with a certain fraction of a shelf left over.
- Multiply the fractional part of the result in Step 1 by the length of one shelf to determine the length of the remaining piece.

Solution

$$12 \div 2\frac{1}{4} = \frac{12}{1} \div \frac{9}{4} = \frac{12}{1} \times \frac{4}{9}$$
$$= \frac{12 \cdot 4}{1 \cdot 9} = \frac{16}{3} = 5\frac{1}{3}$$

There are 5 pieces $2\frac{1}{4}$ feet long.

There is 1 piece that is $\frac{1}{3}$ of $2\frac{1}{4}$ feet long.

$$\frac{1}{3} \times 2\frac{1}{4} = \frac{1}{3} \times \frac{9}{4} = \frac{1 \cdot 9}{3 \cdot 4} = \frac{3}{4}$$

The length of the piece remaining is $\frac{3}{4}$ foot.

You Try It 9

A 16-foot board is cut into pieces $3\frac{1}{3}$ feet long for shelves for a bookcase. What is the length of the remaining piece after as many shelves as possible are cut?

Your strategy

Your solution

Solution on pp. S7–S8

2.7 Exercises

Objective A

Divide.

1. $\frac{1}{3} \div \frac{2}{5}$

2. $\frac{3}{7} \div \frac{3}{2}$

3. $\frac{3}{7} \div \frac{3}{7}$

4. $0 \div \frac{1}{2}$

5. $0 \div \frac{3}{4}$

6. $\frac{16}{33} \div \frac{4}{11}$

7. $\frac{5}{24} \div \frac{15}{36}$

8. $\frac{11}{15} \div \frac{1}{12}$

9. $\frac{15}{16} \div \frac{16}{39}$

10. $\frac{2}{15} \div \frac{3}{5}$

11. $\frac{8}{9} \div \frac{4}{5}$

12. $\frac{11}{15} \div \frac{5}{22}$

13. $\frac{1}{9} \div \frac{2}{3}$

14. $\frac{10}{21} \div \frac{5}{7}$

15. $\frac{2}{5} \div \frac{4}{7}$

16. $\frac{3}{8} \div \frac{5}{12}$

17. $\frac{1}{2} \div \frac{1}{4}$

18. $\frac{1}{3} \div \frac{1}{9}$

19. $\frac{1}{5} \div \frac{1}{10}$

20. $\frac{4}{15} \div \frac{2}{5}$

21. $\frac{7}{15} \div \frac{14}{5}$

22. $\frac{5}{8} \div \frac{15}{2}$

23. $\frac{14}{3} \div \frac{7}{9}$

24. $\frac{7}{4} \div \frac{9}{2}$

25. $\frac{5}{9} \div \frac{25}{3}$

26. $\frac{5}{16} \div \frac{3}{8}$

27. $\frac{2}{3} \div \frac{1}{3}$

28. $\frac{4}{9} \div \frac{1}{9}$

29. $\frac{5}{7} \div \frac{2}{7}$

30. $\frac{5}{6} \div \frac{1}{9}$

31. $\frac{2}{3} \div \frac{2}{9}$

32. $\frac{5}{12} \div \frac{5}{6}$

33. $4 \div \frac{2}{3}$

34. $\frac{2}{3} \div 4$

35. $\frac{3}{2} \div 3$

36. $3 \div \frac{3}{2}$

37. Divide $\frac{7}{8}$ by $\frac{3}{4}$.

38. Divide $\frac{7}{12}$ by $\frac{3}{4}$.

39. Find the quotient of $\frac{5}{7}$ and $\frac{3}{14}$.

40. Find the quotient of $\frac{6}{11}$ and $\frac{9}{32}$.

Objective B

Divide.

41. $\frac{5}{6} \div 25$

42. $22 \div \frac{3}{11}$

43. $6 \div 3\frac{1}{3}$

44. $5\frac{1}{2} \div 11$

45. $6\frac{1}{2} \div \frac{1}{2}$

46. $\frac{3}{8} \div 2\frac{1}{4}$

47. $\frac{5}{12} \div 4\frac{4}{5}$

48. $1\frac{1}{2} \div 1\frac{3}{8}$

49. $8\frac{1}{4} \div 2\frac{3}{4}$

50. $3\frac{5}{9} \div 32$

51. $4\frac{1}{5} \div 21$

52. $6\frac{8}{9} \div \frac{31}{36}$

53. $\frac{11}{12} \div 2\frac{1}{3}$

54. $\frac{7}{8} \div 3\frac{1}{4}$

55. $\frac{5}{16} \div 5\frac{3}{8}$

56. $\frac{9}{14} \div 3\frac{1}{7}$

57. $35 \div \frac{7}{24}$

58. $\frac{3}{8} \div 2\frac{3}{4}$

59. $\frac{11}{18} \div 2\frac{2}{9}$

60. $\frac{21}{40} \div 3\frac{3}{10}$

61. $2\frac{1}{16} \div 2\frac{1}{2}$

62. $7\frac{3}{5} \div 1\frac{7}{12}$

63. $1\frac{2}{3} \div \frac{3}{8}$

64. $16 \div \frac{2}{3}$

65. $1\frac{5}{8} \div 4$

66. $13\frac{3}{8} \div \frac{1}{4}$

67. $16 \div 1\frac{1}{2}$

68. $9 \div \frac{7}{8}$

69. $16\frac{5}{8} \div 1\frac{2}{3}$

70. $24\frac{4}{5} \div 2\frac{3}{5}$

71. $1\frac{1}{3} \div 5\frac{8}{9}$

72. $13\frac{2}{3} \div 0$

73. $82\frac{3}{5} \div 19\frac{1}{10}$

74. $45\frac{3}{5} \div 15$

75. $102 \div 1\frac{1}{2}$

76. $0 \div 3\frac{1}{2}$

77. $8\frac{2}{7} \div 1$

78. $6\frac{9}{16} \div 1\frac{3}{32}$

79. $8\frac{8}{9} \div 2\frac{13}{18}$

80. $10\frac{1}{5} \div 1\frac{7}{10}$

81. $7\frac{3}{8} \div 1\frac{27}{32}$

82. Divide $7\frac{7}{9}$ by $5\frac{5}{6}$.

83. Divide $2\frac{3}{4}$ by $1\frac{23}{32}$.

84. Find the quotient of $8\frac{1}{4}$ and $1\frac{5}{11}$.

85. Find the quotient of $\frac{14}{17}$ and $3\frac{1}{9}$.

Objective C *Application Problems*

86. Individual cereal boxes contain $\frac{3}{4}$ ounce of cereal. How many boxes can be filled with 600 ounces of cereal?

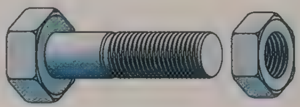
87. A box of Post's Great Grains cereal costing \$4 contains 16 ounces of cereal. How many $1\frac{1}{3}$-ounce portions can be served from this box?

88. A $\frac{5}{8}$-carat diamond was purchased for \$1200. What would a similar diamond weighing 1 carat cost?

89. The Inverness Investor Group bought $8\frac{1}{3}$ acres of land for \$200,000. What was the cost of each acre?

90. KU Energy stock is offered for $\$31\frac{5}{8}$ per share. How many shares can you buy for \$1265?

91. A nut moves $\frac{5}{32}$ inch for each turn. Find the number of turns it will take for the nut to move $1\frac{7}{8}$ inches.



92. The Hammond Company purchased $9\frac{3}{4}$ acres for a housing project. One and one-half acres were set aside for a park.

a. How many acres are available for housing?

b. How many $\frac{1}{4}$-acre parcels of land can be sold after the land for the park is set aside?

93. A chef purchased a roast that weighed $10\frac{3}{4}$ pounds. After the fat was trimmed and the bone removed, the roast weighed $9\frac{1}{3}$ pounds.

a. What was the total weight of the fat and bone?

b. How many $\frac{1}{3}$-pound servings can be cut from the trimmed roast?

94. A 15-foot board is cut into pieces $3\frac{1}{2}$ feet long for a bookcase. What is the length of the piece remaining after as many shelves as possible have been cut?

95. A scale of $\frac{1}{2}$ inch to 1 foot is used to draw the plans for a house. The scale measurements for three walls are given in the table at the right. Complete the table to determine the actual wall lengths for the three walls a, b, and c.

Wall	*Scale*	*Actual Wall Length*
a	$6\frac{1}{4}$ in.	?
b	9 in.	?
c	$7\frac{7}{8}$ in.	?

APPLYING THE CONCEPTS

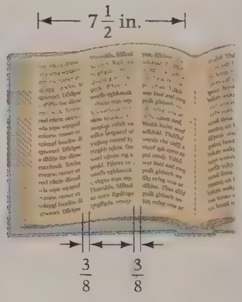
96. On a map, two cities are $4\frac{5}{8}$ inches apart. If $\frac{3}{8}$ inch on the map represents 60 miles, what is the number of miles between the two cities?

97. Is the quotient always less than the dividend in a division problem? Explain.

98. Fill in the box to make a true statement.

a. $\frac{3}{4} \cdot \square = \frac{1}{2}$ **b.** $\frac{2}{3} \cdot \square = 1\frac{3}{4}$

99. A page of type in a certain textbook is $7\frac{1}{2}$ inches wide. If the page is divided into three equal columns, with $\frac{3}{8}$ inch between columns, how wide is each column?



100. A whole number is both multiplied and divided by the same proper fraction. Which is greater, the product or the quotient?

101. Fractions are multiplied by multiplying the numerators and multiplying the denominators. Consider dividing fractions in a similar manner. Divide the numerators and divide the denominators.

$$\frac{4}{15} \div \frac{2}{5} = \frac{4 \div 2}{15 \div 5} = \frac{2}{3}$$

This gives the same answer as the traditional method.

$$\frac{4}{15} \div \frac{2}{5} = \frac{4}{15} \cdot \frac{5}{2} = \frac{20}{30} = \frac{2}{3}$$

Try this method for the following division problems.

a. $\frac{5}{6} \div \frac{1}{3}$ **b.** $\frac{4}{21} \div \frac{2}{7}$ **c.** $\frac{5}{9} \div \frac{5}{3}$ **d.** $\frac{15}{16} \div \frac{3}{8}$

102. Does the method of dividing fractions shown above always work? Give an example of when the traditional method would be a better choice.

2.8

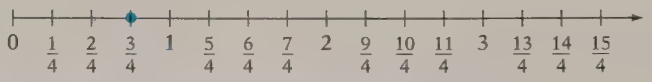
Order, Exponents, and the Order of Operations Agreement

Objective A

To identify the order relation between two fractions

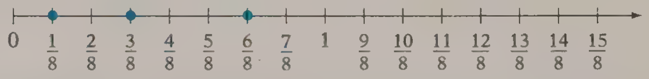
Recall that whole numbers can be graphed as points on the number line. Fractions can also be graphed as points on the number line.

The graph of $\frac{3}{4}$ on the number line



The number line can be used to determine the order relation between two fractions. A fraction that appears to the left of a given fraction is less than the given fraction. A fraction that appears to the right of a given fraction is greater than the given fraction.

$\frac{1}{8} < \frac{3}{8}$ $\qquad$ $\frac{6}{8} > \frac{3}{8}$



To find the order relation between two fractions with the same denominator, compare the numerators. The fraction that has the smaller numerator is the smaller fraction. When the denominators are different, begin by writing equivalent fractions with a common denominator; then compare numerators.

Point of Interest

Leonardo of Pisa, who was also called Fibonacci (c. 1175–1250), is credited with bringing the Hindu-Arabic number system to the Western world and promoting its use instead of the cumbersome Roman numeral system.

He was also influential in promoting the idea of the fraction bar. His notation, however, was very different from what we have today.

For instance, he wrote $\frac{3\ \ 5}{4\ \ 7}$ to mean $\frac{5}{7} + \frac{3}{7 \cdot 4}$, which equals $\frac{23}{28}$.

➡ Find the order relation between $\frac{11}{18}$ and $\frac{5}{8}$.

The LCM of 18 and 8 is 72.

$\frac{11}{18} = \frac{44}{72} \leftarrow$ Smaller numerator $\qquad$ $\frac{11}{18} < \frac{5}{8}$ or $\frac{5}{8} > \frac{11}{18}$

$\frac{5}{8} = \frac{45}{72} \leftarrow$ Larger numerator

Example 1 Place the correct symbol, $<$ or $>$, between the two numbers.

$\frac{5}{12} \qquad \frac{7}{18}$

Solution $\frac{5}{12} = \frac{15}{36} \qquad \frac{7}{18} = \frac{14}{36}$

$\frac{5}{12} > \frac{7}{18}$

You Try It 1 Place the correct symbol, $<$ or $>$, between the two numbers.

$\frac{9}{14} \qquad \frac{13}{21}$

Your solution

Solution on p. S8

Objective B

To simplify expressions containing exponents

Repeated multiplication of the same fraction can be written in two ways:

$$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \quad \text{or} \quad \left(\frac{1}{2}\right)^4 \leftarrow \text{Exponent}$$

The exponent indicates how many times the fraction occurs as a factor in the multiplication. The expression $\left(\frac{1}{2}\right)^4$ is in exponential notation.

Example 2

Simplify: $\left(\frac{5}{6}\right)^3 \cdot \left(\frac{3}{5}\right)^2$

Solution $\left(\frac{5}{6}\right)^3 \cdot \left(\frac{3}{5}\right)^2 = \left(\frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6}\right) \cdot \left(\frac{3}{5} \cdot \frac{3}{5}\right)$

$$= \frac{\overset{1}{\cancel{5}} \cdot \overset{1}{\cancel{5}} \cdot 5 \cdot \overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{3}}}{2 \cdot \underset{1}{\cancel{3}} \cdot 2 \cdot \underset{1}{\cancel{3}} \cdot 2 \cdot 3 \cdot \underset{1}{\cancel{5}} \cdot \underset{1}{\cancel{5}}} = \frac{5}{24}$$

You Try It 2

Simplify: $\left(\frac{7}{11}\right)^2 \cdot \left(\frac{2}{7}\right)$

Your solution

Solution on p. S8

Objective C

To use the Order of Operations Agreement to simplify expressions

The Order of Operations Agreement is used for fractions as well as whole numbers.

Step 1 Do all operations inside parentheses.
Step 2 Simplify any number expressions containing exponents.
Step 3 Do multiplications and divisions as they occur from left to right.
Step 4 Do additions and subtractions as they occur from left to right.

Simplify $\frac{14}{15} - \left(\frac{1}{2}\right)^2 \times \left(\frac{2}{3} + \frac{4}{5}\right)$.

$$\frac{14}{15} - \left(\frac{1}{2}\right)^2 \times \underbrace{\left(\frac{2}{3} + \frac{4}{5}\right)}$$

1. Perform operations in parentheses.

$$\frac{14}{15} - \underbrace{\left(\frac{1}{2}\right)^2} \times \frac{22}{15}$$

2. Simplify expressions with exponents.

$$\frac{14}{15} - \underbrace{\frac{1}{4} \times \frac{22}{15}}$$

3. Do multiplications and divisions as they occur from left to right.

$$\underbrace{\frac{14}{15} - \frac{11}{30}}$$

4. Do additions and subtractions as they occur from left to right.

$$\frac{17}{30}$$

One or more of the above steps may not be needed to simplify an expression. In that case, proceed to the next step in the Order of Operations Agreement.

Example 3

Simplify: $\left(\frac{3}{4}\right)^2 \div \left(\frac{3}{8} - \frac{1}{12}\right)$

Solution $\left(\frac{3}{4}\right)^2 \div \left(\frac{3}{8} - \frac{1}{12}\right)$

$$= \left(\frac{3}{4}\right)^2 \div \left(\frac{7}{24}\right) = \frac{9}{16} \div \frac{7}{24}$$

$$= \frac{9}{16} \cdot \frac{24}{7} = \frac{27}{14} = 1\frac{13}{14}$$

You Try It 3

Simplify:
$\left(\frac{1}{13}\right)^2 \cdot \left(\frac{1}{4} + \frac{1}{6}\right) \div \frac{5}{13}$

Your solution

Solution on p. S8

2.8 Exercises

Objective A

Place the correct symbol, < or >, between the two numbers.

1. $\frac{11}{40} \quad \frac{19}{40}$

2. $\frac{92}{103} \quad \frac{19}{103}$

3. $\frac{2}{3} \quad \frac{5}{7}$

4. $\frac{2}{5} \quad \frac{3}{8}$

5. $\frac{5}{8} \quad \frac{7}{12}$

6. $\frac{11}{16} \quad \frac{17}{24}$

7. $\frac{7}{9} \quad \frac{11}{12}$

8. $\frac{5}{12} \quad \frac{7}{15}$

9. $\frac{13}{14} \quad \frac{19}{21}$

10. $\frac{13}{18} \quad \frac{7}{12}$

11. $\frac{7}{24} \quad \frac{11}{30}$

12. $\frac{13}{36} \quad \frac{19}{48}$

Objective B

Simplify.

13. $\left(\frac{3}{8}\right)^2$

14. $\left(\frac{5}{12}\right)^2$

15. $\left(\frac{2}{9}\right)^3$

16. $\left(\frac{1}{2}\right) \cdot \left(\frac{2}{3}\right)^2$

17. $\left(\frac{2}{3}\right) \cdot \left(\frac{1}{2}\right)^4$

18. $\left(\frac{1}{3}\right)^2 \cdot \left(\frac{3}{5}\right)^3$

19. $\left(\frac{2}{5}\right)^3 \cdot \left(\frac{5}{7}\right)^2$

20. $\left(\frac{5}{9}\right)^3 \cdot \left(\frac{18}{25}\right)^2$

21. $\left(\frac{1}{3}\right)^4 \cdot \left(\frac{9}{11}\right)^2$

22. $\left(\frac{1}{2}\right)^6 \cdot \left(\frac{32}{35}\right)^2$

23. $\left(\frac{2}{3}\right)^4 \cdot \left(\frac{81}{100}\right)^2$

24. $\left(\frac{1}{6}\right) \cdot \left(\frac{6}{7}\right)^2 \cdot \left(\frac{2}{3}\right)$

25. $\left(\frac{2}{7}\right) \cdot \left(\frac{7}{8}\right)^2 \cdot \left(\frac{8}{9}\right)$

26. $3 \cdot \left(\frac{3}{5}\right)^3 \cdot \left(\frac{1}{3}\right)^2$

27. $4 \cdot \left(\frac{3}{4}\right)^3 \cdot \left(\frac{4}{7}\right)^2$

28. $11 \cdot \left(\frac{3}{8}\right)^3 \cdot \left(\frac{8}{11}\right)^2$

Objective C

Simplify.

29. $\frac{1}{2} - \frac{1}{3} + \frac{2}{3}$

30. $\frac{2}{5} + \frac{3}{10} - \frac{2}{3}$

31. $\frac{1}{3} \div \frac{1}{2} + \frac{3}{4}$

32. $\frac{4}{5} + \frac{3}{7} \cdot \frac{14}{15}$

33. $\left(\frac{3}{4}\right)^2 - \frac{5}{12}$

34. $\left(\frac{3}{5}\right)^3 - \frac{3}{25}$

35. $\frac{5}{6} \cdot \left(\frac{2}{3} - \frac{1}{6}\right) + \frac{7}{18}$

36. $\frac{3}{4} \cdot \left(\frac{11}{12} - \frac{7}{8}\right) + \frac{5}{16}$

37. $\frac{7}{12} - \left(\frac{2}{3}\right)^2 + \frac{5}{8}$

38. $\frac{11}{16} - \left(\frac{3}{4}\right)^2 + \frac{7}{12}$

39. $\frac{3}{4} \cdot \left(\frac{4}{9}\right)^2 + \frac{1}{2}$

40. $\frac{9}{10} \cdot \left(\frac{2}{3}\right)^3 + \frac{2}{3}$

41. $\left(\frac{1}{2} + \frac{3}{4}\right) \div \frac{5}{8}$

42. $\left(\frac{2}{3} + \frac{5}{6}\right) \div \frac{5}{9}$

43. $\frac{3}{8} \div \left(\frac{5}{12} + \frac{3}{8}\right)$

44. $\frac{7}{12} \div \left(\frac{2}{3} + \frac{5}{9}\right)$

45. $\left(\frac{3}{8}\right)^2 \div \left(\frac{3}{7} + \frac{3}{14}\right)$

46. $\left(\frac{5}{6}\right)^2 \div \left(\frac{5}{12} + \frac{2}{3}\right)$

47. $\frac{2}{5} \div \frac{3}{8} \cdot \frac{4}{5}$

APPLYING THE CONCEPTS

48. The table at the right shows the results of a survey that asked fast-food patrons their criteria for choosing where to go for fast food. For example, 3 out of every 25 people surveyed said that the speed of the service was most important.

a. According to the survey, do more people choose a fast-food restaurant on the basis of its location or the quality of the food?

b. Which criterion was cited by the most people?

Fast-Food Patrons' Top Criteria for Fast-Food Restaurants

Food Quality	$\frac{1}{4}$
Location	$\frac{13}{50}$
Menu	$\frac{4}{25}$
Price	$\frac{2}{25}$
Speed	$\frac{3}{25}$
Other	$\frac{3}{100}$

Source: Maritz Marketing Research, Inc.

49. A farmer died and left 17 horses to be divided among 3 children. The first child was to receive $\frac{1}{2}$ of the horses, the second child $\frac{1}{3}$ of the horses, and the third child $\frac{1}{9}$ of the horses. The executor for the family's estate realized that 17 horses could not be divided by halves, thirds, or ninths and so added a neighbor's horse to the farmer's. With 18 horses, the executor gave 9 horses to the first child, 6 horses to the second child, and 2 horses to the third child. This accounted for the 17 horses, so the executor returned the borrowed horse to the neighbor. Explain why this worked.

50. $\frac{2}{3} < \frac{3}{4}$. Is $\frac{2+3}{3+4}$ less than $\frac{2}{3}$, greater than $\frac{2}{3}$, or between $\frac{2}{3}$ and $\frac{3}{4}$?

Focus on Problem Solving

Common Knowledge

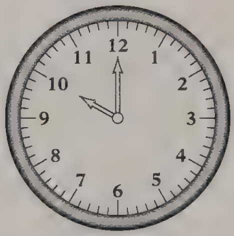
An application problem may not provide all the information that is needed to solve the problem. Sometimes, however, the necessary information is common knowledge.

➡ You are traveling by bus from Boston to New York. The trip is 4 hours long. If the bus leaves Boston at 10 A.M., what time should you arrive in New York?

What other information do you need to solve this problem?

You need to know that, using a 12-hour clock, the hours run

10 A.M.
11 A.M.
12 P.M.
1 P.M.
2 P.M.



Four hours after 10 A.M. is 2 P.M.

You should arrive in New York at 2 P.M.

➡ You purchase a 34¢ stamp at the Post Office and hand the clerk a one-dollar bill. How much change do you receive?

What information do you need to solve this problem?

You need to know that there are 100¢ in one dollar.

Your change is 100¢ − 34¢.

100 − 34 = 66

THE UNITED STATES OF AMERICA
ONE
ONE DOLLAR
1
1
1
1

You receive 66¢ in change.

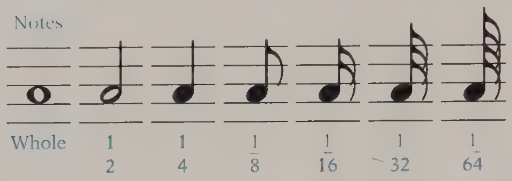
What information do you need to know to solve each of the following problems?

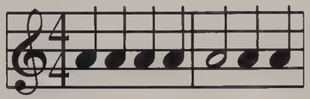
1. You sell a dozen tickets to a fundraiser. Each ticket costs $10. How much money do you collect?
2. The weekly lab period for your science course is one hour and twenty minutes long. Find the length of the science lab period in minutes.
3. An employee's monthly salary is $3750. Find the employee's annual salary.
4. A survey revealed that eighth graders spend an average of 3 hours each day watching television. Find the total time an eighth grader spends watching TV each week.
5. You want to buy a carpet for a room that is 15 feet wide and 18 feet long. Find the amount of carpet that you need.

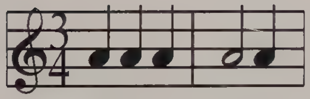
Projects and Group Activities

Music

In musical notation, notes are printed on a **staff,** which is a set of five horizontal lines and the spaces between them. The notes of a musical composition are grouped into **measures,** or **bars.** Vertical lines separate measures on a staff. The shape of a note indicates how long it should be held. The whole note has the longest time value of any note. Each time value is divided by 2 in order to find the next smallest time value.





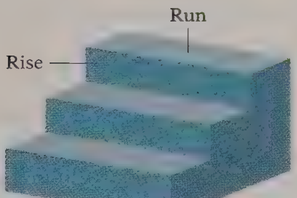


The **time signature** is a fraction that appears at the beginning of a piece of music. The numerator of the fraction indicates the number of beats in a measure. The denominator indicates what kind of note receives 1 beat. For example, music written in $\frac{2}{4}$ time has 2 beats to a measure, and a quarter note receives 1 beat. One measure in $\frac{2}{4}$ time may have 1 half note, 2 quarter notes, 4 eighth notes, or any other combination of notes totaling 2 beats. Other common time signatures include $\frac{4}{4}$, $\frac{3}{4}$, and $\frac{6}{8}$.

1. Explain the meaning of the 6 and the 8 in the time signature $\frac{6}{8}$.
2. Give some possible combinations of notes in one measure of a piece written in $\frac{4}{4}$ time.
3. What does a dot at the right of a note indicate? What is the effect of a dot at the right of a half note? At the right of a quarter note? At the right of an eighth note?
4. Symbols called rests are used to indicate periods of silence in a piece of music. What symbols are used to indicate the different time values of rests?
5. Find some examples of musical compositions written in different time signatures. Use a few measures from each to show that the sum of the time values of the notes and rests in each measure equals the numerator of the time signature.

Construction

Suppose you are involved in building your own home. Design a stairway from the first floor of the house to the second floor. Some of the questions you will need to answer follow.



What is the distance from the floor of the first story to the floor of the second story?

Typically, what is the number of steps in a stairway?

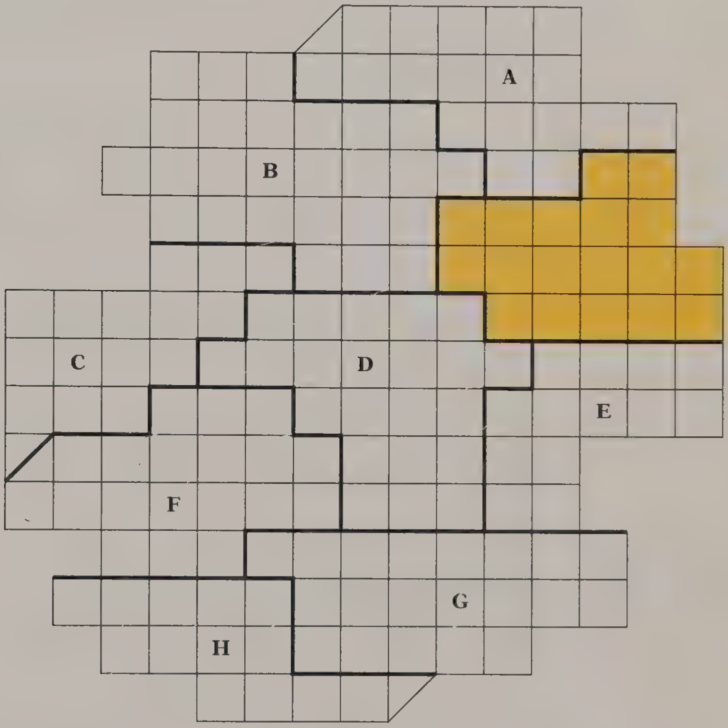
What is a reasonable length for the run of each step?

What is the width of the wood being used to build the staircase?

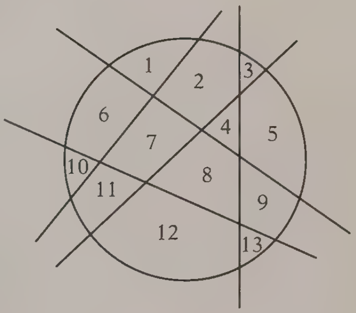
In designing the stairway, remember that each riser should be the same height, that each run should be the same length, and that the width of the wood used for the steps will have to be incorporated in the calculation.

Fractions of Diagrams

The diagram that follows has been broken up into nine areas separated by heavy lines. Eight of the areas have been labeled *A* through *H*. The ninth area is shaded. Determine which lettered areas would have to be shaded so that half of the entire diagram is shaded and half is not shaded. Write down the strategy that you or your group use to arrive at the solution. Compare your strategy with that of other individual students or groups.



Using Patterns in Experimentation



Show how to cut a pie into the greatest number of pieces with only five straight cuts of a knife. An illustration showing how five cuts can produce 13 pieces appears at the right. The correct answer, however, has more than thirteen pieces.

A reasonable question to ask is "How do I know when I have the maximum number of pieces?" To determine the answer, we suggest that you start with one cut, then two cuts, then three cuts, ... Try to discover a pattern for the greatest number of pieces that each number of cuts can produce.

Chapter Summary

Key Words

The *least common multiple* (LCM) is the smallest common multiple of two or more numbers. [p. 63]

The *greatest common factor* (GCF) is the largest common factor of two or more numbers. [p. 64]

A *fraction* can represent the number of equal parts of a whole. [p. 67]

A *proper fraction* is a fraction less than 1. [p. 67]

A *mixed number* is a number greater than 1 with a whole-number part and a fractional part. [p. 67]

An *improper fraction* is a fraction greater than or equal to 1. [p. 67]

Equal fractions with different denominators are called *equivalent fractions*. [p. 71]

A fraction is in *simplest form* when there are no common factors in the numerator and the denominator. [p. 72]

The *reciprocal* of a fraction is the fraction with the numerator and denominator interchanged. The process of interchanging the numerator and denominator is called *inverting* the fraction. [p. 99]

Essential Rules

Addition of Fractions with the Same Denominator

To add fractions with the same denominator, add the numerators and place the sum over the common denominator. [p. 75]

Addition of Fractions with Unlike Denominators

To add fractions with unlike denominators, first rewrite the fractions as equivalent fractions with the same denominator. Then add the numerators and place the sum over the common denominator. [p. 75]

Subtraction of Fractions with the Same Denominator

To subtract fractions with the same denominator, subtract the numerators and place the difference over the common denominator. [p. 83]

Subtraction of Fractions with Unlike Denominators

To subtract fractions with unlike denominators, rewrite the fractions as equivalent fractions with the same denominator. Then subtract the numerators and place the difference over the common denominator. [p. 83]

Multiplication of Fractions

To multiply two fractions, multiply the numerators and place the product over the product of the denominators. [p. 91]

Division of Fractions

To divide two fractions, multiply by the reciprocal of the divisor. [p. 99]

Order Relations Between Fractions

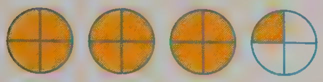
To find the order relation between two fractions with the same denominator, compare the numerators. The fraction that has the smaller numerator is the smaller fraction. [p. 107]

Chapter Review

1. Write $\frac{30}{45}$ in simplest form.

2. Simplify: $\left(\frac{3}{4}\right)^3 \cdot \frac{20}{27}$

3. Express the shaded portion of the circles as an improper fraction.



4. Find the total of $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{2}{9}$.

5. Place the correct symbol, < or >, between the two numbers.

$\frac{11}{18} \quad \frac{17}{24}$

6. Subtract:

$$\begin{array}{r} 18\frac{1}{6} \\ -\ 3\frac{5}{7} \\ \hline \end{array}$$

7. Simplify: $\frac{2}{7}\left(\frac{5}{8} - \frac{1}{3}\right) \div \frac{3}{5}$

8. Multiply: $2\frac{1}{3} \times 3\frac{7}{8}$

9. Divide: $1\frac{1}{3} \div \frac{2}{3}$

10. Find $\frac{17}{24}$ decreased by $\frac{3}{16}$.

11. Divide: $8\frac{2}{3} \div 2\frac{3}{5}$

12. Find the GCF of 20 and 48.

13. Write an equivalent fraction with the given denominator.

$\frac{2}{3} = \frac{}{36}$

14. What is $\frac{15}{28}$ divided by $\frac{5}{7}$?

15. Write an equivalent fraction with the given denominator.

$\frac{8}{11} = \frac{}{44}$

16. Multiply: $2\frac{1}{4} \times 7\frac{1}{3}$

17. Find the LCM of 18 and 12.

18. Write $\frac{16}{44}$ in simplest form.

19. Add: $\frac{3}{8} + \frac{5}{8} + \frac{1}{8}$

20. Subtract:

$$\begin{array}{r} 16 \\ -\ 5\frac{7}{8} \\ \hline \end{array}$$

21. Add: $4\frac{4}{9} + 2\frac{1}{6} + 11\frac{17}{27}$

22. Find the GCF of 15 and 25.

23. Write $\frac{17}{5}$ as a mixed number.

24. Simplify: $\left(\frac{4}{5} - \frac{2}{3}\right)^2 \div \frac{4}{15}$

25. Add: $\frac{3}{8} + 1\frac{2}{3} + 3\frac{5}{6}$

26. Find the LCM of 18 and 27.

27. Subtract: $\frac{11}{18} - \frac{5}{18}$

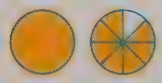
28. Write $2\frac{5}{7}$ as an improper fraction.

29. Divide: $\frac{5}{6} \div \frac{5}{12}$

30. Multiply: $\frac{5}{12} \times \frac{4}{25}$

31. What is $\frac{11}{50}$ multiplied by $\frac{25}{44}$?

32. Express the shaded portion of the circles as a mixed number.



33. During three months of the rainy season, $5\frac{7}{8}$, $6\frac{2}{3}$, and $8\frac{3}{4}$ inches of rain fell. Find the total rainfall for the three months.

34. A home building contractor bought $4\frac{2}{3}$ acres for $168,000. What was the cost of each acre?

35. A 15-mile race has three checkpoints. The first checkpoint is $4\frac{1}{2}$ miles from the starting point. The second checkpoint is $5\frac{3}{4}$ miles from the first checkpoint. How many miles is the second checkpoint from the finish line?

36. A compact car gets 36 miles on each gallon of gasoline. How many miles can the car travel on $6\frac{3}{4}$ gallons of gasoline?

Chapter Test

1. Multiply: $\frac{9}{11} \times \frac{44}{81}$

2. Find the GCF of 24 and 80.

3. Divide: $\frac{5}{9} \div \frac{7}{18}$

4. Simplify: $\left(\frac{3}{4}\right)^2 \div \left(\frac{2}{3} + \frac{5}{6}\right) - \frac{1}{12}$

5. Write $9\frac{4}{5}$ as an improper fraction.

6. What is $5\frac{2}{3}$ multiplied by $1\frac{7}{17}$?

7. Write $\frac{40}{64}$ in simplest form.

8. Place the correct symbol, < or >, between the two numbers.

$\frac{3}{8} \quad \frac{5}{12}$

9. Simplify: $\left(\frac{1}{4}\right)^3 \div \left(\frac{1}{8}\right)^2 - \frac{1}{6}$

10. Find the LCM of 24 and 40.

11. Subtract: $\frac{17}{24} - \frac{11}{24}$

12. Write $\frac{18}{5}$ as a mixed number.

13. Find the quotient of $6\frac{2}{3}$ and $3\frac{1}{6}$.

14. Write an equivalent fraction with the given denominator.

$\frac{5}{8} = \frac{}{72}$

15. Add:
$$\begin{array}{r} \frac{5}{6} \\ \frac{7}{9} \\ +\ \frac{1}{15} \\ \hline \end{array}$$

16. Subtract:
$$\begin{array}{r} 23\frac{1}{8} \\ -\ 9\frac{9}{44} \\ \hline \end{array}$$

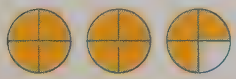
17. What is $\frac{9}{16}$ minus $\frac{5}{12}$?

18. Simplify: $\left(\frac{2}{3}\right)^4 \cdot \frac{27}{32}$

19. Add: $\frac{7}{12} + \frac{11}{12} + \frac{5}{12}$

20. What is $12\frac{5}{12}$ more than $9\frac{17}{20}$?

21. Express the shaded portion of the circles as an improper fraction.



22. An electrician earns $240 for each day worked. What is the total of the electrician's earnings for working $3\frac{1}{2}$ days?

23. Grant Miura bought $7\frac{1}{4}$ acres of land for a housing project. One and three-fourths acres were set aside for a park, and the remaining land was developed into $\frac{1}{2}$-acre lots. How many lots were available for sale?

24. Chris Aguilar bought 100 shares of a utility stock at \$$24\frac{1}{2}$ per share. The stock gained \$$5\frac{5}{8}$ during the first month of ownership and lost \$$2\frac{1}{4}$ during the second month. Find the value of one share of the utility stock at the end of the second month.

25. In 3 successive months, the rainfall measured $11\frac{1}{2}$ inches, $7\frac{5}{8}$ inches, and $2\frac{1}{3}$ inches. Find the total rainfall for the 3 months.

Cumulative Review

1. Round 290,496 to the nearest thousand.

2. Subtract: $\begin{array}{r} 390{,}047 \\ -\ 98{,}769 \\ \hline \end{array}$

3. Find the product of 926 and 79.

4. Divide: $57\overline{)30{,}792}$

5. Simplify: $4 \cdot (6 - 3) \div 6 - 1$

6. Find the prime factorization of 44.

7. Find the LCM of 30 and 42.

8. Find the GCF of 60 and 80.

9. Write $7\frac{2}{3}$ as an improper fraction.

10. Write $\frac{25}{4}$ as a mixed number.

11. Write an equivalent fraction with the given denominator.

$$\frac{5}{16} = \frac{}{48}$$

12. Write $\frac{24}{60}$ in simplest form.

13. What is $\frac{9}{16}$ more than $\frac{7}{12}$?

14. Add: $\begin{array}{r} 3\frac{7}{8} \\ 7\frac{5}{12} \\ +\ 2\frac{15}{16} \\ \hline \end{array}$

15. Find $\frac{3}{8}$ less than $\frac{11}{12}$.

16. Subtract: $\begin{array}{r} 5\frac{1}{6} \\ -\ 3\frac{7}{18} \\ \hline \end{array}$

17. Multiply: $\frac{3}{8} \times \frac{14}{15}$

18. Multiply: $3\frac{1}{8} \times 2\frac{2}{5}$

19. Divide: $\frac{7}{16} \div \frac{5}{12}$

20. Find the quotient of $6\frac{1}{8}$ and $2\frac{1}{3}$.

21. Simplify: $\left(\frac{1}{2}\right)^3 \cdot \frac{8}{9}$

22. Simplify: $\left(\frac{1}{2} + \frac{1}{3}\right) \div \left(\frac{2}{5}\right)^2$

23. Molly O'Brien had $1359 in a checking account. During the week, Molly wrote checks for $128, $54, and $315. Find the amount in the checking account at the end of the week.

24. The tickets for a movie were $5 for an adult and $2 for a student. Find the total income from the sale of 87 adult tickets and 135 student tickets.

25. Find the total weight of three packages that weigh $1\frac{1}{2}$ pounds, $7\frac{7}{8}$ pounds, and $2\frac{2}{3}$ pounds.

26. A board $2\frac{5}{8}$ feet long is cut from a board $7\frac{1}{3}$ feet long. What is the length of the remaining piece?

27. A car travels 27 miles on each gallon of gasoline. How many miles can the car travel on $8\frac{1}{3}$ gallons of gasoline?

28. Jimmy Santos purchased $10\frac{1}{3}$ acres of land to build a housing development. Jimmy donated 2 acres for a park. How many $\frac{1}{3}$-acre parcels can be sold from the remaining land?

Chapter 3

Decimals

RECYCLING CENTRE

Recycling empty soda cans is easy and enables the tin to be reused, which saves our natural resources. Recycling centers offer money as an incentive for dropping off empty cans. The cans are weighed and, for each pound donated, a certain amount of money is offered. Both the weight and the payment are numbers calculated using decimals. **Exercises 122 and 123 on page 142** are examples of how a recycling center rewards the recycling of products.

Objective

Section 3.1

- **A** To write decimals in standard form and in words
- **B** To round a decimal to a given place value

Section 3.2

- **A** To add decimals
- **B** To solve application problems

Section 3.3

- **A** To subtract decimals
- **B** To solve application problems

Section 3.4

- **A** To multiply decimals
- **B** To solve application problems

Section 3.5

- **A** To divide decimals
- **B** To solve application problems

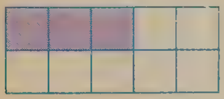
Section 3.6

- **A** To convert fractions to decimals
- **B** To convert decimals to fractions
- **C** To identify the order relation between two decimals or between a decimal and a fraction

Need help? For on-line student resources, such as section quizzes, visit this textbook's web site at **math.college.hmco.com/students.**

Prep Test

1. Express the shaded portion of the rectangle as a fraction.



2. Round 36,852 to the nearest hundred.

3. Write 4791 in words.

4. Write six thousand eight hundred forty-two in standard form.

For Exercises 5 to 8, add, subtract, multiply or divide.

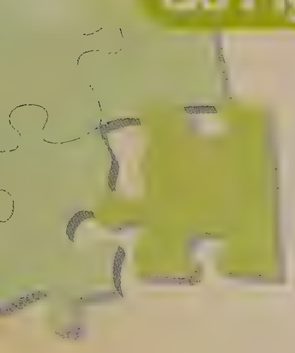
5. 37 + 8892 + 465

6. 2403 − 765

7. 844 × 91

8. $23\overline{)6412}$

Go Figure



Maria and Pedro are siblings. Pedro has as many brothers as sisters. Maria has twice as many brothers as sisters. How many children are in the family?

3.1

Introduction to Decimals

Objective A To write decimals in standard form and in words

Point of Interest

Leonardo of Pisa (Fibonacci) anticipated the decimal system. He wrote $\frac{2}{10}\frac{7}{10}5$ to mean $5 + \frac{7}{10} + \frac{2}{10 \cdot 10}$.

The smallest human bone is found in the middle ear and measures 0.135 inch in length. The number 0.135 is in **decimal notation.**

Note the relationship between fractions and numbers written in decimal notation.

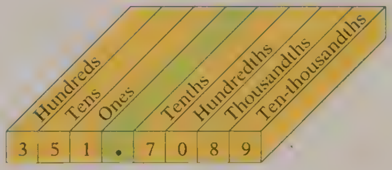
Three tenths	Three hundredths	Three thousandths
$\frac{3}{1\underline{0}} = 0.\underline{3}$	$\frac{3}{1\underline{00}} = 0.\underline{03}$	$\frac{3}{1\underline{000}} = 0.\underline{003}$
1 zero 1 decimal place	2 zeros 2 decimal places	3 zeros 3 decimal places

A number written in decimal notation has three parts.

351	.	7089
Whole-number part	**Decimal point**	**Decimal part**

A number written in decimal notation is often called simply a **decimal.** The position of a digit in a decimal determines the digit's place value.

In the decimal 351.7089, the position of the digit 9 determines that its place value is ten-thousandths.


Hundreds
Tens
Ones
Tenths
Hundredths
Thousandths
Ten-thousandths
3 5 1 . 7 0 8 9

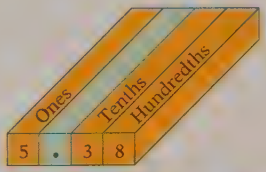
When writing a decimal in words, write the decimal part as though it were a whole number. Then name the place value of the last digit.

0.6481	Six thousand four hundred eighty-one ten-thousandths
549.238	Five hundred forty-nine and two hundred thirty-eight thousandths (The decimal point is read as "and.")

To write a decimal in standard form, zeros may have to be inserted after the decimal point so that the last digit is in the given place-value position.

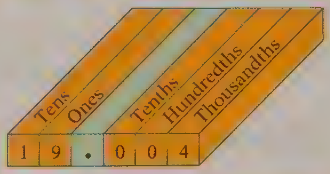
Five and thirty-eight hundredths

8 is in the hundredths place. $5.3\underline{8}$


Ones
Tenths
Hundredths
5 . 3 8

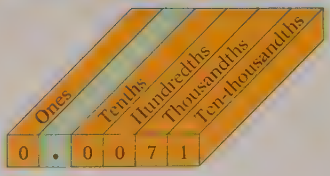
Nineteen and four thousandths

4 is in the thousandths place. Insert two zeros so that 4 is in the thousandths place. $19.00\underline{4}$


Tens
Ones
Tenths
Hundredths
Thousandths
1 9 . 0 0 4

Seventy-one ten-thousandths

1 is in the ten-thousandths place. Insert two zeros so that 1 is in the ten-thousandths place. $0.007\underline{1}$


Ones
Tenths
Hundredths
Thousandths
Ten-thousandths
0 . 0 0 7 1

Example 1 Write 307.4027 in words.

Solution Three hundred seven and four thousand twenty-seven ten-thousandths

You Try It 1 Write 209.05838 in words.

Your solution

Example 2 Write six hundred seven and seven hundred eight hundred-thousandths in standard form.

Solution 607.00708

You Try It 2 Write forty-two thousand and two hundred seven millionths in standard form.

Your solution

Solutions on p. S8

Objective B To round a decimal to a given place value

Rounding decimals is similar to rounding whole numbers, except that the digits to the right of the given place value are dropped instead of being replaced by zeros.

If the digit to the right of the given place value is less than 5, drop that digit and all digits to the right. If the digit to the right of the given place value is greater than or equal to 5, increase the number in the given place value by 1 and drop all digits to its right.

⇨ Round 26.3799 to the nearest hundredth.

Given place value
26.3799
$9 > 5$ Increase 7 by 1 and drop all digits to the right of 7.

26.3799 rounded to the nearest hundredth is 26.38.

Example 3 Round 0.39275 to the nearest ten-thousandth.

Solution

Given place value
0.39275
$5 = 5$

0.3928

You Try It 3 Round 4.349254 to the nearest hundredth.

Your solution

Example 4 Round 42.0237412 to the nearest hundred-thousandth.

Solution

Given place value
42.0237412
$1 < 5$

42.02374

You Try It 4 Round 3.290532 to the nearest hundred-thousandth.

Your solution

Solutions on p. S8

3.1 Exercises

Objective A

Name the place value of the digit 4.

1. 36.8497
2. 0.473129
3. 107.35846
4. 2.03415

Write each decimal in words.

5. 0.27
6. 0.92
7. 1.005
8. 3.067
9. 36.4
10. 59.7
11. 0.00035
12. 0.00092
13. 10.007
14. 20.009
15. 52.00095
16. 64.00037
17. 0.0293
18. 0.0717
19. 6.324
20. 8.916
21. 276.3297
22. 418.3115
23. 216.0729
24. 976.0317
25. 4625.0379
26. 2986.0925
27. 1.00001
28. 3.00003

Write each decimal in standard form.

29. Seven hundred sixty-two thousandths
30. Two hundred ninety-five thousandths
31. Sixty-two millionths
32. Forty-one millionths
33. Eight and three hundred four ten-thousandths
34. Four and nine hundred seven ten-thousandths
35. Three hundred four and seven hundredths
36. Eight hundred ninety-six and four hundred seven thousandths

37. Three hundred sixty-two and forty-eight thousandths

38. Seven hundred eighty-four and eighty-four thousandths

39. Three thousand forty-eight and two thousand two ten-thousandths

40. Seven thousand sixty-one and nine thousand one ten-thousandths

Objective B

Round each decimal to the given place value.

41. 7.359 Tenths

42. 6.405 Tenths

43. 23.009 Tenths

44. 89.19204 Tenths

45. 22.68259 Hundredths

46. 16.30963 Hundredths

47. 7.072854 Thousandths

48. 1946.3745 Thousandths

49. 62.009435 Thousandths

50. 0.029876 Ten-thousandths

51. 0.012346 Ten-thousandths

52. 1.702596 Nearest whole number

53. 2.079239 Hundred-thousandths

54. 0.0102903 Millionths

55. 0.1009754 Millionths

APPLYING THE CONCEPTS

56. To what decimal place value are timed events in the Olympics recorded? Provide some specific examples of events and the winning times in each.

0
5
10
15
20
25
30
35
40
45
50
55

57. Provide an example of a situation in which a decimal is always rounded up, even if the digit to the right is less than 5. Provide an example of a situation in which a decimal is always rounded down, even if the digit to the right is 5 or greater than 5. (*Hint*: Think about situations in which money changes hands.)

58. Indicate which zeros of the number, if any, need not be entered on a calculator.

a. 23.500 **b.** 0.000235 **c.** 300.0005 **d.** 0.004050

59. **a.** A decimal number was rounded to 6. Between what two numbers, to the nearest tenth, was the number?

b. A decimal number was rounded to 10.2. Between what two numbers, to the nearest hundredth, was the number?

3.2 Addition of Decimals

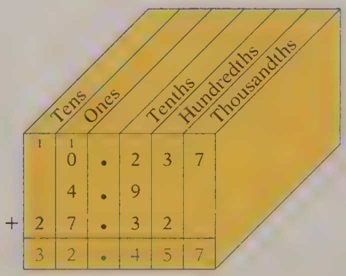
Objective A To add decimals

Point of Interest

Simon Stevin (1548–1620) was the first to name decimal numbers. He wrote the number 2.345 as 2 0 3 1 4 2 5 3. He called the whole-number part the *commencement*, the tens digit was *prime*, the hundredths digit was *seconds*, the thousandths digit was *thirds*, and so on.

To add decimals, write the numbers so that the decimal points are on a vertical line. Add as for whole numbers, and write the decimal point in the sum directly below the decimal points in the addends.

➡ Add: 0.237 + 4.9 + 27.32



Note that by placing the decimal points on a vertical line, we make sure that digits of the same place value are added.

Example 1 Find the sum of 42.3, 162.903, and 65.0729.

Solution

$$\begin{array}{r} {\scriptstyle 1\,1\,1}\\ 42.3\\ 162.903\\ +\ \ 65.0729\\ \hline 270.2759 \end{array}$$

You Try It 1 Find the sum of 4.62, 27.9, and 0.62054.

Your solution

Example 2 Add: 0.83 + 7.942 + 15

Solution

$$\begin{array}{r} {\scriptstyle 1\,1}\\ 0.83\\ 7.942\\ +\ 15.\\ \hline 23.772 \end{array}$$

You Try It 2 Add: 6.05 + 12 + 0.374

Your solution

Solutions on pp. S8–S9

ESTIMATION

Estimating the Sum of Two or More Decimals

Estimate and then use your calculator to find 23.037 + 16.7892.

To estimate the sum of two or more numbers, round each number to the same place value. In this case, we will round to the nearest whole number. Then add. The estimated answer is 40.

$$\begin{array}{rcr} 23.037 & \approx & 23\\ +\ 16.7892 & \approx & +\ 17\\ \hline & & 40 \end{array}$$

Now use your calculator to find the exact sum. The exact answer is 39.8262.

$$23.037\ \boxed{+}\ 16.7892\ \boxed{=}\ 39.8262$$

Objective B To solve application problems 3

A study determined the average amounts that boys and girls aged 12 to 16 are paid for the chores shown in the table at the right. Use the figures in the table for Example 3 and You Try It 3.

Chore	*Girls*	*Boys*
Clean room	\$1.93	\$2.61
Make bed	\$1.30	\$1.64
Do homework	\$1.41	\$2.86
Pick up or put away clothes	\$1.35	\$1.29
Clean bathroom	\$1.73	\$9.02
Do dishes	\$1.65	\$2.56
Dry or fold clothes	\$1.52	\$1.63
Load or empty dishwasher	\$1.07	\$2.06
Vacuum	\$2.21	\$2.76

Source: Copyright © 2000 *Time, Inc.* Reprinted by permission.

Example 3

On average, what would a girl be paid during a week in which she did each one of the chores listed in the table?

Strategy

To find the amount she would be paid, add the amounts in the column headed "Girls."

Solution

$$1.93 + 1.30 + 1.41 + 1.35 + 1.73 + 1.65 + 1.52 + 1.07 + 2.21 = 14.17$$

For performing each one of the chores listed, a girl would be paid, on average, \$14.17.

You Try It 3

On average, what would a boy be paid during a week in which he did each one of the chores listed in the table?

Your strategy

Your solution

Example 4

Dan Burhoe earned a salary of \$138.50 for working 3 days this week as a food server. He also received \$42.75, \$35.80, and \$59.25 in tips during the 3 days. Find his total income for the 3 days of work.

Strategy

To find the total income, add the tips (42.75, 35.80, and 59.25) to the salary (138.50).

Solution

$$138.50 + 42.75 + 35.80 + 59.25 = 276.30$$

Dan's total income for the 3 days of work was \$276.30.

You Try It 4

Anita Khavari, an insurance executive, earns a salary of \$875 every 4 weeks. During the past 4-week period, she received commissions of \$985.80, \$791.46, \$829.75, and \$635.42. Find her total income for the past 4-week period.

Your strategy

Your solution

Solutions on p. S9

3.2 Exercises

Objective A

Add.

1. 16.008 + 2.0385 + 132.06

2. 17.32 + 1.0579 + 16.5

3. 1.792 + 67 + 27.0526

4. 8.772 + 1.09 + 26.5027

5. 3.02 + 62.7 + 3.924

6. 9.06 + 4.976 + 59.6

7. 82.006 + 9.95 + 0.927

8. 0.826 + 8.76 + 79.005

9. 4.307 + 99.82 + 9.078

10.
0.3
+ 0.07

11.
0.29
+ 0.4

12.
1.007
+ 2.1

13.
7.3
+ 9.005

14.
4.9257
27.05
+ 9.0063

15.
8.72
99.073
+ 2.9736

16.
62.4
9.827
+ 692.44

17.
8
89.43
+ 7.0659

Estimate by rounding to the nearest whole number. Then use your calculator to add.

18.
342.42
89.625
+ 176.2

19.
219.9
0.872
+ 13.42

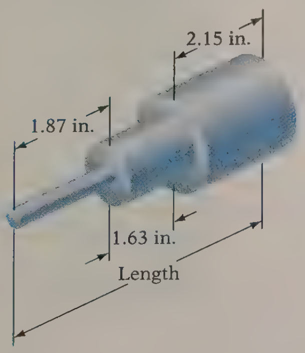
20.
823.9
82.65
+ 46.923

21.
678.92
97.6
+ 5.423

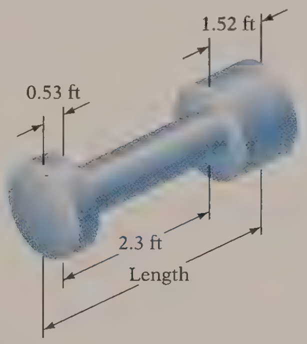
Objective B *Application Problems*

22. A family has a mortgage of $814.72, a Visa bill of $216.40, and an electric bill of $87.32. Estimate by rounding the numbers to the nearest hundred dollars, and then calculate the exact amount of the three payments.

23. Find the length of the shaft.

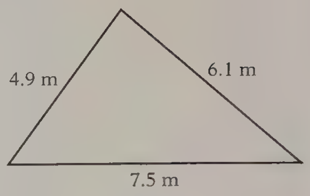


24. Find the length of the shaft.



25. You have $2143.57 in your checking account. You make deposits of $210.98, $45.32, $1236.34, and $27.99. Find the amount in your checking account after you have made the deposits if no money has been withdrawn.

26. The perimeter of a triangle is the sum of the lengths of the three sides of the triangle. Find the perimeter of a triangle that has sides that measure 4.9 meters, 6.1 meters, and 7.5 meters.



27. The world's population in 2050 is expected to be 8.9 billion people. It is projected that in that year, Asia's population will be 5.3 billion and Africa's population will be 1.8 billion. What are the combined populations of Asia and Africa expected to be in 2050? (Source: United Nations Population Division, World Population Prospects)

The table at the right shows the number of self-employed people, in millions, in the United States. They are listed by annual earnings. Use this table for Exercises 28–30.

Annual Earnings	*Number of Self-Employed (in millions)*
Less than $5000	5.1
$5000 - $24,999	3.9
$25,000 – $49,999	1.6
$50,000 or more	1.0

Source: U.S. Small Business Administration

28. How many self-employed people earn less than $50,000?

29. How many self-employed people earn $5000 or more?

30. How many people in the United States are self-employed?

APPLYING THE CONCEPTS

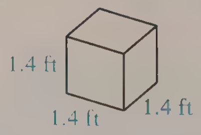
The table at the right gives the prices for selected products in a grocery store. Use this table for Exercises 31 and 32.

Product	*Cost*
Raisin bran	$3.29
Butter	$2.79
Bread	$1.49
Popcorn	$1.19
Potatoes	$2.49
Cola (6-pack)	$1.99
Mayonnaise	$2.99
Lunch meat	$3.39
Milk	$2.59
Toothpaste	$2.69

31. Does a customer with $10 have enough money to purchase raisin bran, bread, milk, and butter?

32. Name three items that would cost more than $7 but less than $8. (There is more than one answer.)

33. Can a piece of rope 4 feet long be wrapped around the box shown at the right?

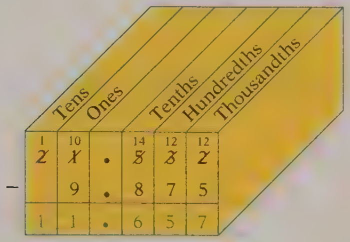


3.3 Subtraction of Decimals

Objective A **To subtract decimals**

To subtract decimals, write the numbers so that the decimal points are on a vertical line. Subtract as for whole numbers, and write the decimal point in the difference directly below the decimal point in the subtrahend.

Subtract 21.532 − 9.875 and check.



Placing the decimal points on a vertical line ensures that digits of the same place value are subtracted.

Check:

$$\begin{array}{lr} & {\scriptstyle 1\,1\;\,1\,1} \\ \text{Subtrahend} & 9.875 \\ +\ \text{Difference} & +\ 11.657 \\ \hline =\ \text{Minuend} & 21.532 \end{array}$$

Subtract 4.3 − 1.7942 and check.

$$\begin{array}{r} \overset{3}{\cancel{4}}\,.\,\overset{12}{\cancel{3}}\,\overset{9}{\cancel{0}}\,\overset{9}{\cancel{0}}\,\overset{10}{\cancel{0}} \\ -\,1\,.\,7\,9\,4\,2 \\ \hline 2\,.\,5\,0\,5\,8 \end{array}$$

If necessary, insert zeros in the minuend before subtracting.

Check:

$$\begin{array}{r} {\scriptstyle 1\;\;1\,1\,1} \\ 1.7942 \\ +\ 2.5058 \\ \hline 4.3000 \end{array}$$

Example 1 Subtract 39.047 − 7.96 and check.

Solution

$$\begin{array}{r} 3\,\overset{8}{\cancel{9}}\,.\,\overset{9}{\cancel{0}}\,\overset{14}{\cancel{4}}\,7 \\ -\ \ 7\,.\,9\,6 \\ \hline 3\,1\,.\,0\,8\,7 \end{array} \qquad \textit{Check:}\ \begin{array}{r} {\scriptstyle 1\;1} \\ 7.96 \\ +\ 31.087 \\ \hline 39.047 \end{array}$$

You Try It 1 Subtract 72.039 − 8.47 and check.

Your solution

Example 2 Find 9.23 less than 29 and check.

Solution

$$\begin{array}{r} \overset{1}{\cancel{2}}\,\overset{18}{\cancel{9}}\,.\,\overset{9}{\cancel{0}}\,\overset{10}{\cancel{0}} \\ -\ \ 9\,.\,2\,3 \\ \hline 1\,9\,.\,7\,7 \end{array} \qquad \textit{Check:}\ \begin{array}{r} {\scriptstyle 1\;1\;1} \\ 9.23 \\ +\ 19.77 \\ \hline 29.00 \end{array}$$

You Try It 2 Subtract 35 − 9.67 and check.

Your solution

Example 3 Subtract 1.2 − 0.8235 and check.

Solution

$$\begin{array}{r} \overset{0}{\cancel{1}}\,.\,\overset{11}{\cancel{2}}\,\overset{9}{\cancel{0}}\,\overset{9}{\cancel{0}}\,\overset{10}{\cancel{0}} \\ -\,0\,.\,8\,2\,3\,5 \\ \hline 0\,.\,3\,7\,6\,5 \end{array} \qquad \textit{Check:}\ \begin{array}{r} {\scriptstyle 1\;1\,1\,1} \\ 0.8235 \\ +\ 0.3765 \\ \hline 1.2000 \end{array}$$

You Try It 3 Subtract 3.7 − 1.9715 and check.

Your solution

Solutions on p. S9

ESTIMATION

Estimating the Difference Between Two Decimals

Estimate and then use your calculator to find 820.2306 − 475.74815.

To estimate the difference between two numbers, round each number to the same place value. In this case we will round to the nearest ten. Then subtract. The estimated answer is 340.

$$\begin{array}{rcr} 820.2306 & \approx & 820 \\ \underline{-\ 475.74815} & \approx & \underline{-\ 480} \\ & & 340 \end{array}$$

Now use your calculator to find the exact difference. The exact answer is 344.48245.

820.2306 $\boxed{-}$ 475.74815 $\boxed{=}$ 344.48245

Objective B To solve application problems

Example 4

You bought a book for $15.87. How much change did you receive from a $20.00 bill?

Strategy

To find the amount of change, subtract the cost of the book (15.87) from 20.00.

Solution

$$\begin{array}{r} 20.00 \\ \underline{-\ 15.87} \\ 4.13 \end{array}$$

You received $4.13 in change.

You Try It 4

Your breakfast cost $6.85. How much change did you receive from a $10.00 bill?

Your strategy

Your solution

Example 5

You had a balance of $87.93 on your student debit card. You then used the card, deducting $15.99 for a CD, $6.85 for lunch, and $8.50 for a ticket to the football game. What is your new student debit card balance?

Strategy

To find your new debit card balance:

- Add to find the total of the three deductions (15.99 + 6.85 + 8.50).
- Subtract the total of the three deductions from the old balance (87.93).

Solution

$$\begin{array}{rl} 15.99 & \\ 6.85 & \\ \underline{+\ \ 8.50} & \\ 31.34 & \text{total of deductions} \end{array} \qquad \begin{array}{r} 87.93 \\ \underline{-\ 31.34} \\ 56.59 \end{array}$$

Your new debit card balance is $56.59.

You Try It 5

You had a balance of $2472.69 in your checking account. You then wrote checks for $1025.60, $79.85, and $162.47. Find the new balance in your checking account.

Your strategy

Your solution

Solutions on p. S9

3.3 Exercises

Objective A

Subtract and check.

1. 24.037 − 18.41

2. 26.029 − 19.31

3. 123.07 − 9.4273

4. 214 − 7.143

5. 16.5 − 9.7902

6. 13.2 − 8.6205

7. 235.79 − 20.093

8. 463.27 − 40.095

9. 63.005 − 9.1274

10. 23.004 − 7.2175

11. 92 − 19.2909

12. 41.2405 − 25.2709

13. $\begin{array}{r} 0.32 \\ -\ 0.0058 \\ \hline \end{array}$

14. $\begin{array}{r} 0.78 \\ -\ 0.0073 \\ \hline \end{array}$

15. $\begin{array}{r} 3.005 \\ -\ 1.982 \\ \hline \end{array}$

16. $\begin{array}{r} 6.007 \\ -\ 2.734 \\ \hline \end{array}$

17. $\begin{array}{r} 352.16 \\ -\ 90.994 \\ \hline \end{array}$

18. $\begin{array}{r} 872 \\ -\ 80.753 \\ \hline \end{array}$

19. $\begin{array}{r} 724.32 \\ -\ 69 \\ \hline \end{array}$

20. $\begin{array}{r} 625.46 \\ -\ 77.509 \\ \hline \end{array}$

21. $\begin{array}{r} 362.394 \\ -\ 19.4672 \\ \hline \end{array}$

22. $\begin{array}{r} 421.385 \\ -\ 17.5293 \\ \hline \end{array}$

23. $\begin{array}{r} 19 \\ -\ 10.372 \\ \hline \end{array}$

24. $\begin{array}{r} 23.4 \\ -\ 0.921 \\ \hline \end{array}$

Use the relationship between addition and subtraction to complete the statement.

25. ________ + 2.325 = 7.01

26. 5.392 + ________ = 8.07

27. ________ + 8.967 = 19.35

Estimate by rounding to the nearest whole number (ones place). Then use your calculator to subtract.

28. $\begin{array}{r} 93.079256 \\ -\ 66.09249 \\ \hline \end{array}$

29. $\begin{array}{r} 3.7529 \\ -\ 1.00784 \\ \hline \end{array}$

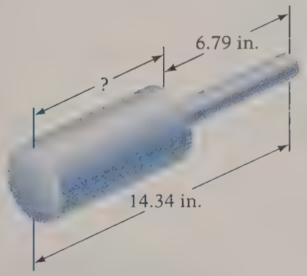
30. $\begin{array}{r} 76.53902 \\ -\ 45.73005 \\ \hline \end{array}$

31. $\begin{array}{r} 9.07325 \\ -\ 1.924 \\ \hline \end{array}$

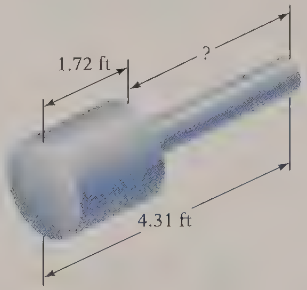
Objective B Application Problems

32. The manager of the Edgewater Cafe takes a reading of the cash register tape each hour. At 1:00 P.M. the tape read $967.54. At 2:00 P.M. the tape read $1437.15. Find the amount of sales between 1:00 P.M. and 2:00 P.M.

33. Find the missing dimension.



34. Find the missing dimension.



35. You had a balance of $1029.74 in your checking account. You then wrote checks for $67.92, $43.10, and $496.34.
a. Find the total amount of the checks written.
b. Find the new balance in your checking account.

36. The price of gasoline is $1.42 per gallon after the price rose $.07 one month and $.12 the next month. Find the price of gasoline before these increases in price.

37. The average annual rainfall in Oakland, California, is 21.79 inches. Oakland's normal rainfall for July 1 to February 4 is 13.25 inches. From July 1 to February 4, 1998, Oakland, California, had 30.02 inches of rain. How many inches above normal was Oakland's rainfall from July 1 to February 4, 1998? (Source: National Weather Service)

38. Grace Herrera owned 357.448 shares of a mutual fund on January 1. On December 31 of the same year, she had 439.917 shares. What was the increase in the number of shares Grace owned during that year?

The table at the right shows the actual and projected number of households shopping online, in millions. Use the table for Exercises 39 and 40.

Year	*Number of Households Shopping Online (in millions)*
1998	8.7
1999	13.1
2000	17.7
2001	23.1
2002	30.3
2003	40.3

Source: Forrester Research Inc.

39. Calculate the growth in online shopping from 2000 to 2003.

40. Calculate the growth in online shopping from 1998 to 2003.

APPLYING THE CONCEPTS

41. Find the largest amount by which the estimate of the sum of two decimals rounded to **a.** tenths, **b.** hundredths, and **c.** thousandths places could differ from the exact sum.

3.4 Multiplication of Decimals

Objective A To multiply decimals

Point of Interest

Benjamin Banneker (1731–1806) was the first African American to earn distinction as a mathematician and scientist. He was on the survey team that determined the boundaries of Washington, DC. The mathematics of surveying requires extensive use of decimals.

Decimals are multiplied as though they were whole numbers. Then the decimal point is placed in the product. Writing the decimals as fractions shows where to write the decimal point in the product.

$$0.\underline{3} \times 5 = \frac{3}{10} \times \frac{5}{1} = \frac{15}{10} = 1.\underline{5}$$

1 decimal place 1 decimal place

$$0.\underline{3} \times 0.\underline{5} = \frac{3}{10} \times \frac{5}{10} = \frac{15}{100} = 0.\underline{15}$$

1 decimal place 1 decimal place 2 decimal places

$$0.\underline{3} \times 0.\underline{05} = \frac{3}{10} \times \frac{5}{100} = \frac{15}{1000} = 0.\underline{015}$$

1 decimal place 2 decimal places 3 decimal places

To multiply decimals, multiply the numbers as with whole numbers. Write the decimal point in the product so that the number of decimal places in the product is the sum of the decimal places in the factors.

➡ Multiply: 21.4 × 0.36

$$\begin{array}{r l}
21.4 & \text{1 decimal place} \\
\times\ 0.36 & \text{2 decimal places} \\
\hline
1284 & \\
642 & \\
\hline
7.704 & \text{3 decimal places}
\end{array}$$

➡ Multiply: 0.037 × 0.08

$$\begin{array}{r l}
0.037 & \text{3 decimal places} \\
\times\ \ 0.08 & \text{2 decimal places} \\
\hline
0.00296 & \text{5 decimal places}
\end{array}$$

- Two zeros must be inserted between the 2 and the decimal point so that there are 5 decimal places in the product.

To multiply a decimal by a power of 10 (10, 100, 1000, . . .), move the decimal point to the right the same number of places as there are zeros in the power of 10.

$3.8925 \times 1\underline{0} = 38.925$

1 zero 1 decimal place

$3.8925 \times 1\underline{00} = 389.25$

2 zeros 2 decimal places

$3.8925 \times 1\underline{000} = 3892.5$

3 zeros 3 decimal places

$3.8925 \times 1\underline{0{,}000} = 38{,}925.$

4 zeros 4 decimal places

$3.8925 \times 1\underline{00{,}000} = 389{,}250.$

5 zeros 5 decimal places

- Note that a zero must be inserted before the decimal point.

Note that if the power of 10 is written in exponential notation, the exponent indicates how many places to move the decimal point.

$$3.8925 \times 10^1 = 3\underbrace{8}_{\text{1 decimal place}}.925$$

$$3.8925 \times 10^2 = 3\underbrace{89}_{\text{2 decimal places}}.25$$

$$3.8925 \times 10^3 = 3\underbrace{892}_{\text{3 decimal places}}.5$$

$$3.8925 \times 10^4 = 3\underbrace{8,925}_{\text{4 decimal places}}.$$

$$3.8925 \times 10^5 = 3\underbrace{89,250}_{\text{5 decimal places}}.$$

Example 1 Multiply: 920 × 3.7

Solution

$$\begin{array}{r} 920 \\ \times \quad 3.7 \\ \hline 644\ 0 \\ 2760 \\ \hline 3404.0 \end{array}$$

You Try It 1 Multiply: 870 × 4.6

Your solution

Example 2 Find 0.00079 multiplied by 0.025.

Solution

$$\begin{array}{r} 0.00079 \\ \times \quad 0.025 \\ \hline 395 \\ 158 \\ \hline 0.00001975 \end{array}$$

You Try It 2 Find 0.000086 multiplied by 0.057.

Your solution

Example 3 Find the product of 3.69 and 2.07.

Solution

$$\begin{array}{r} 3.69 \\ \times\ 2.07 \\ \hline 2583 \\ 7380 \\ \hline 7.6383 \end{array}$$

You Try It 3 Find the product of 4.68 and 6.03.

Your solution

Example 4 Multiply: 42.07 × 10,000

Solution 42.07 × 10,000 = 420,700

You Try It 4 Multiply: 6.9 × 1000

Your solution

Example 5 Find 3.01 times 10^3.

Solution $3.01 \times 10^3 = 3010$

You Try It 5 Find 4.0273 times 10^2.

Your solution

Solutions on p. S9

ESTIMATION

Estimating the Product of Two Decimals

Estimate and then use your calculator to find 28.259 × 0.029.

To estimate a product, round each number so that there is one nonzero digit. Then multiply.

$$\begin{array}{rcr} 28.259 & \approx & 30 \\ \times\ 0.029 & \approx & \times\ 0.03 \\ \hline & & 0.90 \end{array}$$

The estimated answer is 0.90.

Now use your calculator to find the exact product. The exact answer is 0.819511.

28.259 $\boxed{\times}$ 0.029 $\boxed{=}$ 0.819511

Objective B **To solve application problems**

The tables below list water rates and meter fees for a city. These tables are used for Example 6 and You Try It 6.

Water Charges	
Commercial	$1.39/1000 gal
Comm Restaurant	$1.39/1000 gal
Industrial	$1.39/1000 gal
Institutional	$1.39/1000 gal
Res—No Sewer	
Residential—SF	
>0 <200 gal per day	$1.15/1000 gal
>200 <1500 gal per day	$1.39/1000 gal
>1500 gal per day	$1.54/1000 gal

Meter Charges	
Meter	*Meter Fee*
5/8" & 3/4"	$13.50
1"	$21.80
1-1/2"	$42.50
2"	$67.20
3"	$133.70
4"	$208.20
6"	$415.10
8"	$663.70

Example 6

Find the total bill for an industrial water user with a 6-inch meter that used 152,000 gallons of water for July and August.

Strategy

To find the total cost of water:

- Find the cost of water by multiplying the cost per 1000 gallons (1.39) by the number of 1000-gallon units used.
- Add the cost of the water to the meter fee (415.10).

Solution

$$\text{Cost of water} = \frac{152{,}000}{1000} \cdot 1.39 = 211.28$$

$$\text{Total cost} = 211.28 + 415.10 = 626.38$$

The total cost is $626.38.

You Try It 6

Find the total bill for a commercial user that used 5000 gallons of water per day for July and August. The user has a 3-inch meter.

Your strategy

Your solution

Solution on pp. S9–S10

Example 7

It costs \$.036 an hour to operate an electric motor. How much does it cost to operate the motor for 120 hours?

Strategy

To find the cost of running the motor for 120 hours, multiply the hourly cost (0.036) by the number of hours the motor is run (120).

Solution

$$\begin{array}{r} 0.036 \\ \times \quad 120 \\ \hline 720 \\ 36 \\ \hline 4.320 \end{array}$$

The cost of running the motor for 120 hours is \$4.32.

You Try It 7

The cost of electricity to run a freezer for 1 hour is \$.035. This month the freezer has run for 210 hours. Find the total cost of running the freezer this month.

Your strategy

Your solution

Example 8

Jason Ng earns a salary of \$360 for a 40-hour work week. This week he worked 12 hours of overtime at a rate of \$13.50 for each hour of overtime worked. Find his total income for the week.

Strategy

To find Jason's total income for the week:

- Find the overtime pay by multiplying the hourly overtime rate (13.50) by the number of hours of overtime worked (12).
- Add the overtime pay to the weekly salary (360).

Solution

$$\begin{array}{r} 13.50 \\ \times \quad 12 \\ \hline 27\ 00 \\ 135\ 0 \\ \hline 162.00 \end{array} \ \text{Overtime pay} \qquad \begin{array}{r} 360.00 \\ +\ 162.00 \\ \hline 522.00 \end{array}$$

Jason's total income for this week is \$522.00.

You Try It 8

You make a down payment of \$175 on a stereo and agree to make payments of \$37.18 a month for the next 18 months to repay the remaining balance. Find the total cost of the stereo.

Your strategy

Your solution

Solutions on p. S10

3.4 Exercises

Objective A

Multiply.

1. $\begin{array}{r} 0.9 \\ \times\ 0.4 \\ \hline \end{array}$ **2.** $\begin{array}{r} 0.7 \\ \times\ 0.9 \\ \hline \end{array}$ **3.** $\begin{array}{r} 0.5 \\ \times\ 0.6 \\ \hline \end{array}$ **4.** $\begin{array}{r} 0.3 \\ \times\ 0.7 \\ \hline \end{array}$ **5.** $\begin{array}{r} 0.5 \\ \times\ 0.5 \\ \hline \end{array}$

6. $\begin{array}{r} 0.7 \\ \times\ 0.7 \\ \hline \end{array}$ **7.** $\begin{array}{r} 0.9 \\ \times\ 0.5 \\ \hline \end{array}$ **8.** $\begin{array}{r} 0.2 \\ \times\ 0.6 \\ \hline \end{array}$ **9.** $\begin{array}{r} 7.7 \\ \times\ 0.9 \\ \hline \end{array}$ **10.** $\begin{array}{r} 3.4 \\ \times\ 0.4 \\ \hline \end{array}$

11. $\begin{array}{r} 9.2 \\ \times\ 0.2 \\ \hline \end{array}$ **12.** $\begin{array}{r} 2.6 \\ \times\ 0.7 \\ \hline \end{array}$ **13.** $\begin{array}{r} 7.2 \\ \times\ 0.6 \\ \hline \end{array}$ **14.** $\begin{array}{r} 6.8 \\ \times\ 0.4 \\ \hline \end{array}$ **15.** $\begin{array}{r} 7.4 \\ \times\ 0.1 \\ \hline \end{array}$

16. $\begin{array}{r} 3.8 \\ \times\ 0.1 \\ \hline \end{array}$ **17.** $\begin{array}{r} 7.9 \\ \times\ \ \ 5 \\ \hline \end{array}$ **18.** $\begin{array}{r} 9.3 \\ \times\ \ \ 7 \\ \hline \end{array}$ **19.** $\begin{array}{r} 0.68 \\ \times\ \ \ \ 4 \\ \hline \end{array}$ **20.** $\begin{array}{r} 0.83 \\ \times\ \ \ \ 9 \\ \hline \end{array}$

21. $\begin{array}{r} 0.67 \\ \times\ \ 0.9 \\ \hline \end{array}$ **22.** $\begin{array}{r} 0.84 \\ \times\ \ 0.3 \\ \hline \end{array}$ **23.** $\begin{array}{r} 0.16 \\ \times\ \ 0.6 \\ \hline \end{array}$ **24.** $\begin{array}{r} 0.47 \\ \times\ \ 0.8 \\ \hline \end{array}$ **25.** $\begin{array}{r} 2.5 \\ \times\ 5.4 \\ \hline \end{array}$

26. $\begin{array}{r} 3.9 \\ \times\ 1.9 \\ \hline \end{array}$ **27.** $\begin{array}{r} 8.4 \\ \times\ 9.5 \\ \hline \end{array}$ **28.** $\begin{array}{r} 7.6 \\ \times\ 5.8 \\ \hline \end{array}$ **29.** $\begin{array}{r} 0.83 \\ \times\ \ 5.2 \\ \hline \end{array}$ **30.** $\begin{array}{r} 0.24 \\ \times\ \ 2.7 \\ \hline \end{array}$

31. $\begin{array}{r} 0.46 \\ \times\ \ 3.9 \\ \hline \end{array}$ **32.** $\begin{array}{r} 0.78 \\ \times\ \ 6.8 \\ \hline \end{array}$ **33.** $\begin{array}{r} 0.2 \\ \times\ 0.3 \\ \hline \end{array}$ **34.** $\begin{array}{r} 0.3 \\ \times\ 0.3 \\ \hline \end{array}$ **35.** $\begin{array}{r} 0.24 \\ \times\ \ 0.3 \\ \hline \end{array}$

36. $\begin{array}{r} 0.17 \\ \times\ \ 0.5 \\ \hline \end{array}$ **37.** $\begin{array}{r} 1.47 \\ \times\ 0.09 \\ \hline \end{array}$ **38.** $\begin{array}{r} 6.37 \\ \times\ 0.05 \\ \hline \end{array}$ **39.** $\begin{array}{r} 8.92 \\ \times\ 0.004 \\ \hline \end{array}$ **40.** $\begin{array}{r} 6.75 \\ \times\ 0.007 \\ \hline \end{array}$

41. $\begin{array}{r} 0.49 \\ \times\ 0.16 \\ \hline \end{array}$

42. $\begin{array}{r} 0.38 \\ \times\ 0.21 \\ \hline \end{array}$

43. $\begin{array}{r} 7.6 \\ \times\ 0.01 \\ \hline \end{array}$

44. $\begin{array}{r} 5.1 \\ \times\ 0.01 \\ \hline \end{array}$

45. $\begin{array}{r} 8.62 \\ \times\quad 4 \\ \hline \end{array}$

46. $\begin{array}{r} 5.83 \\ \times\quad 7 \\ \hline \end{array}$

47. $\begin{array}{r} 64.5 \\ \times\quad 9 \\ \hline \end{array}$

48. $\begin{array}{r} 37.8 \\ \times\quad 8 \\ \hline \end{array}$

49. $\begin{array}{r} 2.19 \\ \times\ 9.2 \\ \hline \end{array}$

50. $\begin{array}{r} 1.25 \\ \times\ 5.6 \\ \hline \end{array}$

51. $\begin{array}{r} 1.85 \\ \times\ 0.023 \\ \hline \end{array}$

52. $\begin{array}{r} 37.8 \\ \times\ 0.052 \\ \hline \end{array}$

53. $\begin{array}{r} 0.478 \\ \times\ 0.37 \\ \hline \end{array}$

54. $\begin{array}{r} 0.526 \\ \times\ 0.22 \\ \hline \end{array}$

55. $\begin{array}{r} 48.3 \\ \times\ 0.0041 \\ \hline \end{array}$

56. $\begin{array}{r} 67.2 \\ \times\ 0.0086 \\ \hline \end{array}$

57. $\begin{array}{r} 2.437 \\ \times\ 6.1 \\ \hline \end{array}$

58. $\begin{array}{r} 4.237 \\ \times\ 0.54 \\ \hline \end{array}$

59. $\begin{array}{r} 0.413 \\ \times\ 0.0016 \\ \hline \end{array}$

60. $\begin{array}{r} 0.517 \\ \times\ 0.0029 \\ \hline \end{array}$

61. $\begin{array}{r} 94.73 \\ \times\ 0.57 \\ \hline \end{array}$

62. $\begin{array}{r} 89.23 \\ \times\ 0.62 \\ \hline \end{array}$

63. $\begin{array}{r} 8.005 \\ \times\ 0.067 \\ \hline \end{array}$

64. $\begin{array}{r} 9.032 \\ \times\ 0.019 \\ \hline \end{array}$

65. 4.29×0.1

66. 6.78×0.1

67. 5.29×0.4

68. 6.78×0.5

69. 0.68×0.7

70. 0.56×0.9

71. 1.4×0.73

72. 6.3×0.37

73. 5.2×7.3

74. 7.4×2.9

75. 3.8×0.61

76. 7.2×0.72

77. 0.32×10

78. 6.93×10

79. 0.065×100

80. 0.039×100

81. 6.2856×1000

82. 3.2954×1000

83. 3.2×1000

84. $0.006 \times 10{,}000$

85. $3.57 \times 10{,}000$

86. 8.52×10^1

87. 0.63×10^1

88. 82.9×10^2

89. 0.039×10^2

90. 6.8×10^3

91. 4.9×10^4

92. 6.83×10^4

93. 0.067×10^2

94. 0.052×10^2

95. Find the product of 0.0035 and 3.45.

96. Find the product of 237 and 0.34.

97. Multiply 3.005 by 0.00392.

98. Multiply 20.34 by 1.008.

99. Multiply 1.348 by 0.23.

100. Multiply 0.000358 by 3.56.

101. Find the product of 23.67 and 0.0035.

102. Find the product of 0.00346 and 23.1.

103. Find the product of 5, 0.45, and 2.3.

104. Find the product of 0.03, 23, and 9.45.

Estimate and then use your calculator to multiply.

105. $\begin{array}{r} 28.5 \\ \times\ \ 3.2 \\ \hline \end{array}$

106. $\begin{array}{r} 86.3 \\ \times\ \ 4.4 \\ \hline \end{array}$

107. $\begin{array}{r} 2.38 \\ \times 0.44 \\ \hline \end{array}$

108. $\begin{array}{r} 9.82 \\ \times 0.77 \\ \hline \end{array}$

109. $\begin{array}{r} 0.866 \\ \times\ \ \ 4.5 \\ \hline \end{array}$

110. $\begin{array}{r} 0.239 \\ \times\ \ \ 8.2 \\ \hline \end{array}$

111. $\begin{array}{r} 4.34 \\ \times 2.59 \\ \hline \end{array}$

112. $\begin{array}{r} 6.87 \\ \times 9.98 \\ \hline \end{array}$

113. $\begin{array}{r} 8.434 \\ \times 0.044 \\ \hline \end{array}$

114. $\begin{array}{r} 7.037 \\ \times 0.094 \\ \hline \end{array}$

115. $\begin{array}{r} 28.44 \\ \times\ \ 1.12 \\ \hline \end{array}$

116. $\begin{array}{r} 86.57 \\ \times\ \ 7.33 \\ \hline \end{array}$

117. $\begin{array}{r} 49.6854 \\ \times 39.0672 \\ \hline \end{array}$

118. $\begin{array}{r} 2.00547 \\ \times\ \ \ \ 9.672 \\ \hline \end{array}$

119. $\begin{array}{r} 0.00456 \\ \times 0.009542 \\ \hline \end{array}$

120. $\begin{array}{r} 7.00637 \\ \times\ \ 0.0128 \\ \hline \end{array}$

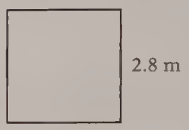
Objective E Application Problems

121. An electric motor costing \$315.45 has an operating cost of \$.027 for 1 hour of operation. Find the cost to run the motor for 56 hours. Round to the nearest cent.

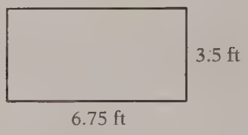
122. Four hundred empty soft drink cans weigh 18.75 pounds. A recycling center pays \$.75 per pound for the cans. Find the amount received for the 400 cans. Round to the nearest cent.

123. A recycling center pays \$.045 per pound for newspapers. Estimate the payment for recycling 520 pounds of newspapers. Find the actual amount received from recycling the newspapers.

124. The perimeter of a square is equal to four times the length of a side of the square. Find the perimeter of a square whose side measures 2.8 meters.



125. The area of a rectangle is equal to the product of the length of the rectangle times its width. Find the area of a rectangle that has a length of 6.75 feet and a width of 3.5 feet. The area will be in square feet.

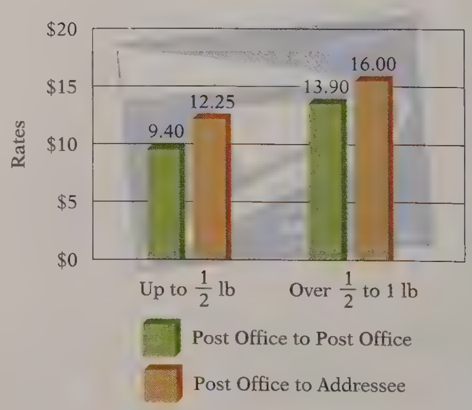


126. You bought a car for \$5000 down and made payments of \$399.50 each month for 36 months.
a. Find the amount of the payments over the 36 months.
b. Find the total cost of the car.

127. A nurse earns a salary of \$756 for a 40-hour week. This week the nurse worked 15 hours of overtime at a rate of \$28.35 for each hour of overtime worked.
a. Find the nurse's overtime pay.
b. Find the nurse's total income for the week.

128. Bay Area Rental Cars charges \$12 a day and \$.12 per mile for renting a car. You rented a car for 3 days and drove 235 miles. Find the total cost of renting the car.

129. The graph at the right shows United States Postal Service rates for express mail. How much would it cost a company to mail 25 express mail packages, each weighing $\frac{1}{4}$ pound, post office to addressee?



U.S. Postal Service Rates for Express Mail

130. A taxi costs \$1.50 and \$.20 for each $\frac{1}{8}$ mile driven. Find the cost of hiring a taxi to get from the airport to the hotel—a distance of 5.5 miles.

The table at the right lists three pieces of steel required for a repair project. Use this table for Exercise 131.

Grade of Steel	Weight (pounds per foot)	Required Number of Feet	Cost per Pound
1	2.2	8	$1.20
2	3.4	6.5	$1.35
3	6.75	15.4	$1.94

131. **a.** Find the total cost of each of the grades of steel.
b. Find the total cost of the three pieces of steel.

A confectioner ships holiday packs of candy and nuts anywhere in the United States. At the right is a price list for nuts and candy, and below is a table of shipping charges to zones in the United States. For any fraction of a pound, use the next higher weight. Use these tables for Exercise 132. (16 oz = 1 lb)

Code	Description	Price
112	Almonds 16 oz	$4.75
116	Cashews 8 oz	$2.90
117	Cashews 16 oz	$5.50
130	Macadamias 7 oz	$5.25
131	Macadamias 16 oz	$9.95
149	Pecan halves 8 oz	$6.25
155	Mixed nuts 8 oz	$4.80
160	Cashew brittle 8 oz	$1.95
182	Pecan roll 8 oz	$3.70
199	Chocolate peanuts 8 oz	$1.90

Pounds	Zone 1	Zone 2	Zone 3	Zone 4
1–3	$6.55	$6.85	$7.25	$7.75
4–6	$7.10	$7.40	$7.80	$8.30
7–9	$7.50	$7.80	$8.20	$8.70
10–12	$7.90	$8.20	$8.60	$9.10

132. Find the cost of sending the following orders to the given mail zone.

a. Code	Quantity	b. Code	Quantity	c. Code	Quantity
116	2	112	1	117	3
130	1	117	4	131	1
149	3	131	2	155	2
182	4	160	3	160	4
Mail to zone 4.		182	5	182	1
		Mail to zone 3.		199	3
				Mail to zone 2.	

133. An emissions test for cars requires that of the total engine exhaust, less than 1 part per thousand $\left(\frac{1}{1000} = 0.001\right)$ be hydrocarbon emissions. Using this figure, determine which of the cars in the table below would fail the emissions test.

Car	Total Engine Exhaust	Hydrocarbon Emission
1	367,921	36
2	401,346	42
3	298,773	21
4	330,045	32
5	432,989	45

APPLYING THE CONCEPTS

Chris works at B & W Garage as an auto mechanic and has just completed an engine overhaul for a customer. To determine the cost of the repair job, Chris keeps a list of times worked and parts used. A parts list and a list of the times worked are shown below. Use these tables for Exercise 134.

Parts Used		*Time Spent*	
Item	*Quantity*	*Day*	*Hours*
Gasket set	1	Monday	7.0
Ring set	1	Tuesday	7.5
Valves	8	Wednesday	6.5
Wrist pins	8	Thursday	8.5
Valve springs	16	Friday	9.0
Rod bearings	8		
Main bearings	5		
Valve seals	16		
Timing chain	1		

Price List		
Item Number	*Description*	*Unit Price*
27345	Valve spring	$1.85
41257	Main bearing	$3.40
54678	Valve	$4.79
29753	Ring set	$33.98
45837	Gasket set	$48.99
23751	Timing chain	$42.95
23765	Fuel pump	$77.59
28632	Wrist pin	$2.71
34922	Rod bearing	$2.67
2871	Valve seal	$0.42

134. **a.** Organize a table of data showing the parts used, the unit price for each, and the price of the quantity used. *Hint:* Use the following headings for the table.

Quantity	*Item Number*	*Description*	*Unit Price*	*Total*

b. Add up the numbers in the "Total" column to find the total cost of the parts.
c. If the charge for labor is $36.75 per hour, compute the cost of labor.
d. What is the total cost for parts and labor?

135. Explain how the decimal point is placed when a number is multiplied by 10, 100, 1000, 10,000, etc.

136. Explain how the decimal point is placed in the product of two decimals.

137. Show how the decimal is placed in the product of 1.3×2.31 by first writing each number as a fraction and then multiplying. Now change back to decimal notation.

3.5

Division of Decimals

Objective A

To divide decimals

To divide decimals, move the decimal point in the divisor to the right to make the divisor a whole number. Move the decimal point in the dividend the same number of places to the right. Place the decimal point in the quotient directly over the decimal point in the dividend, and then divide as with whole numbers.

➡ Divide: $3.25\overline{)15.275}$

$$\begin{array}{r} \,. \\ 3\underset{\curvearrowright}{.25}.\overline{)15.2\underset{\curvearrowright}{7.}5} \end{array}$$

- **Move the decimal point 2 places to the right in the divisor and then in the dividend. Place the decimal point in the quotient.**

$$\begin{array}{r} 4.7 \\ 325.\overline{)\ 1527.5} \\ -1300 \\ \hline 227\ 5 \\ -227\ 5 \\ \hline 0 \end{array}$$

- **Divide as with whole numbers.**

Moving the decimal point the same number of decimal places in the divisor and dividend does not change the value of the quotient, because this process is the same as multiplying the numerator and denominator of a fraction by the same number. In the example above,

$$3.25\overline{)15.275} = \frac{15.275}{3.25} = \frac{15.275 \times 100}{3.25 \times 100} = \frac{1527.5}{325} = 325\overline{)1527.5}$$

When dividing decimals, we usually round the quotient off to a specified place value, rather than writing the quotient with a remainder.

➡ Divide: $0.3\overline{)0.56}$
Round to the nearest hundredth.

$$\begin{array}{r} 1.866 \approx 1.87 \\ 0\underset{\curvearrowright}{.3}.\overline{)\ 0.5\underset{\curvearrowright}{.}600} \\ -\ 3 \\ \hline 2\ 6 \\ -2\ 4 \\ \hline 20 \\ -18 \\ \hline 20 \\ -18 \\ \hline \end{array}$$

We must carry the division to the thousandths place to round the quotient to the nearest hundredth. Therefore, zeros must be inserted in the dividend so that the quotient has a digit in the thousandths place.

➡ Divide 57.93 by 3.24. Round to the nearest thousandth.

$$\begin{array}{r l}
 & 17.8796 \approx 17.880 \\
3\underset{\curvearrowright}{.24}.\overline{)\;57\underset{\curvearrowright}{.93}.0000} & \\
-32\ 4 & \\
\hline
25\ 53 & \\
-22\ 68 & \\
\hline
2\ 85\ 0 & \\
-2\ 59\ 2 & \\
\hline
25\ 80 & \\
-22\ 68 & \\
\hline
3\ 120 & \\
-2\ 916 & \\
\hline
2040 & \\
-1944 & \\
\hline
\end{array}$$

Zeros must be inserted in the dividend so that the quotient has a digit in the ten-thousandths place.

To divide a decimal by a power of 10 (10, 100, 1000, . . .), move the decimal point to the left the same number of places as there are zeros in the power of 10.

$34.65 \div \underline{10} = 3.465$ 1 zero 1 decimal place

$34.65 \div \underline{100} = 0.3465$ 2 zeros 2 decimal places

$34.65 \div \underline{1000} = 0.03465$ 3 zeros 3 decimal places

- Note that a zero must be inserted between the 3 and the decimal point.

$34.65 \div \underline{10{,}000} = 0.003465$ 4 zeros 4 decimal places

- Note that two zeros must be inserted between the 3 and the decimal point.

If the power of 10 is written in exponential notation, the exponent indicates how many places to move the decimal point.

$34.65 \div 10^1 = 3.465$ 1 decimal place

$34.65 \div 10^2 = 0.3465$ 2 decimal places

$34.65 \div 10^3 = 0.03465$ 3 decimal places

$34.65 \div 10^4 = 0.003465$ 4 decimal places

Example 1 Divide: $0.1344 \div 0.032$

Solution

$$\begin{array}{r}
4.2 \\
0\underset{\curvearrowright}{.032}.\overline{)0\underset{\curvearrowright}{.134}.4} \\
-128 \\
\hline
6\ 4 \\
-6\ 4 \\
\hline
0
\end{array}$$

You Try It 1 Divide: $0.1404 \div 0.052$

Your solution

Solution on p. S10

Example 2 Divide: 58.092 ÷ 82
Round to the nearest thousandth.

Solution

$$\begin{array}{r} 0.7084 \approx 0.708 \\ 82\overline{)\ 58.0920} \\ -57\ 4 \\ \hline 69 \\ -\ \ 0 \\ \hline 692 \\ -656 \\ \hline 360 \\ -328 \\ \hline \end{array}$$

You Try It 2 Divide: 37.042 ÷ 76
Round to the nearest thousandth.

Your solution

Example 3 Divide: 420.9 ÷ 7.06
Round to the nearest tenth.

Solution

$$\begin{array}{r} 59.61 \approx 59.6 \\ 7.06.\overline{)\ 420.90.00} \\ -353\ 0 \\ \hline 67\ 90 \\ -63\ 54 \\ \hline 4\ 36\ 0 \\ -4\ 23\ 6 \\ \hline 12\ 40 \\ -\ \ 7\ 06 \\ \hline \end{array}$$

You Try It 3 Divide: 370.2 ÷ 5.09
Round to the nearest tenth.

Your solution

Example 4 Divide: 402.75 ÷ 1000

Solution 402.75 ÷ 1000 = 0.40275

You Try It 4 Divide: 309.21 ÷ 10,000

Your solution

Example 5 What is 0.625 divided by 10^2?

Solution $0.625 \div 10^2 = 0.00625$

You Try It 5 What is 42.93 divided by 10^4?

Your solution

Solutions on p. S10

ESTIMATION

Estimating the Quotient of Two Decimals

Estimate and then use your calculator to find 282.18 ÷ 0.48.

To estimate a quotient, round each number so that there is one nonzero digit. Then divide.

$282.18 \div 0.48 \approx$
$300 \div 0.5 = 600$

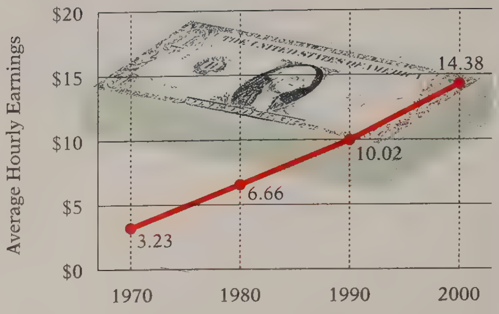
The estimated answer is 600.

Now use your calculator to find the exact quotient. The exact answer is 587.875.

$282.18\ \boxed{\div}\ 0.48\ \boxed{=}\ 587.875$

Objective B To solve application problems

The graph at the right shows average hourly earnings in the United States. Use this table for Example 6 and You Try It 6.



Average Hourly Earnings in the United States

Source: Statistical Abstract of the United States; Bureau of Labor Statistics

Example 6
How many times greater were the average hourly earnings in 2000 than in 1970? Round to the nearest whole number.

Strategy
To find how many times greater the average hourly earnings were, divide the 2000 average hourly earnings (14.38) by the 1970 average hourly earnings (3.23).

Solution
$14.38 \div 3.23 \approx 4$

The average hourly earnings in 2000 were about 4 times greater than in 1970.

You Try It 6
How many times greater were the average hourly earnings in 1990 than in 1970? Round to the nearest tenth.

Your strategy

Your solution

Example 7
A one-year subscription to a monthly magazine costs $45. The price of each issue at the newsstand is $4.90. How much would you save per issue by buying a year's subscription rather than buying each issue at the newsstand?

Strategy
To find the amount saved:

- Find the subscription price per issue by dividing the cost of the subscription (45) by the number of issues (12).
- Subtract the subscription price per issue from the newsstand price (4.90).

Solution
$45 \div 12 = 3.75$

$4.90 - 3.75 = 1.15$

The savings would be $1.15 per issue.

You Try It 7
A Nielsen survey of the number of people (in millions) who watch television during the week is given in the table below.

Mon.	*Tues.*	*Wed.*	*Thu.*	*Fri.*	*Sat.*	*Sun.*
91.9	89.8	90.6	93.9	78.0	77.1	87.7

Find the average number of people who watch television per day.

Your strategy

Your solution

Solutions on p. S10

3.5 Exercises

Objective A

Divide.

1. $3\overline{)2.46}$

2. $7\overline{)3.71}$

3. $0.8\overline{)3.84}$

4. $0.9\overline{)6.93}$

5. $0.7\overline{)62.3}$

6. $0.4\overline{)52.8}$

7. $0.4\overline{)24}$

8. $0.5\overline{)65}$

9. $0.7\overline{)59.01}$

10. $0.9\overline{)8.721}$

11. $0.5\overline{)16.15}$

12. $0.8\overline{)77.6}$

13. $0.7\overline{)3.542}$

14. $0.6\overline{)2.436}$

15. $6.3\overline{)8.19}$

16. $3.2\overline{)7.04}$

17. $3.6\overline{)0.396}$

18. $2.7\overline{)0.648}$

19. $6.9\overline{)26.22}$

20. $1.7\overline{)84.66}$

Divide. Round to the nearest tenth.

21. $55.62 \div 8.8$

22. $25.43 \div 5.4$

23. $5.427 \div 9.5$

24. $1.837 \div 1.4$

25. $18.4 \div 7.3$

26. $52.9 \div 8.1$

27. $0.183 \div 0.17$

28. $0.381 \div 0.47$

29. $6.924 \div 0.053$

Divide. Round to the nearest hundredth.

30. $4.817 \div 16$

31. $6.467 \div 8$

32. $0.0418 \div 0.53$

33. $0.0647 \div 0.72$

34. $7 \div 0.55$

35. $38.665 \div 0.95$

36. $13.97 \div 25.4$

37. $27.738 \div 60.8$

38. $3.171 \div 45.6$

Divide. Round to the nearest thousandth.

39. 1.028 ÷ 54

40. 6.729 ÷ 27

41. 0.0437 ÷ 0.5

42. 75.469 ÷ 77.8

43. 34.31 ÷ 95.3

44. 0.2695 ÷ 2.67

45. 0.4871 ÷ 4.72

46. 0.1142 ÷ 17.2

47. 0.2307 ÷ 26.7

Divide. Round to the nearest whole number.

48. 16.5 ÷ 4

49. 89.76 ÷ 90

50. 1.94 ÷ 0.3

51. 1.0478 ÷ 0.413

52. 2.148 ÷ 0.519

53. 0.79 ÷ 0.778

54. 3.092 ÷ 0.075

55. 392 ÷ 6.9

56. 8.729 ÷ 0.075

Divide.

57. 4.07 ÷ 10

58. 0.039 ÷ 10

59. 42.67 ÷ 10

60. 389.7 ÷ 100

61. 1.037 ÷ 100

62. 237.835 ÷ 100

63. 8.295 ÷ 1000

64. 82,547 ÷ 1000

65. 825.37 ÷ 1000

66. $8.35 \div 10^1$

67. $0.32 \div 10^1$

68. $87.65 \div 10^1$

69. $23.627 \div 10^2$

70. $2.954 \div 10^2$

71. $0.0053 \div 10^2$

72. $289.32 \div 10^3$

73. $1.8932 \div 10^3$

74. $0.139 \div 10^3$

75. Divide 44.208 by 2.4.

76. Divide 0.04664 by 0.44.

77. Find the quotient of 723.15 and 45.

78. Find the quotient of 3.3463 and 3.07.

79. Divide 13.5 by 10^3.

80. Divide 0.045 by 10^5.

81. Find the quotient of 23.678 and 1000.

82. Find the quotient of 7.005 and 10,000.

83. What is 0.0056 divided by 0.05?

84. What is 123.8 divided by 0.02?

Estimate and then use your calculator to divide. Round your calculated answer to the nearest ten-thousandth.

85. 42.42 ÷ 3.8

86. 69.8 ÷ 7.2

87. 389 ÷ 0.44

88. 642 ÷ 0.83

89. 6.394 ÷ 3.5

90. 8.429 ÷ 4.2

91. 1.235 ÷ 0.021

92. 7.456 ÷ 0.072

93. 95.443 ÷ 1.32

94. 423.0925 ÷ 4.0927

95. 1.000523 ÷ 429.07

96. 0.03629 ÷ 0.00054

Objective B *Application Problems*

97. Ramon, a high school football player, gained 162 yards on 26 carries in a high school football game. Find the average number of yards gained per carry. Round to the nearest hundredth.

98. A car with an odometer reading of 17,814.2 is filled with 9.4 gallons of gas. At an odometer reading of 18,130.4, the tank is empty and the car is filled with 12.4 gallons of gas. How many miles does the car travel on 1 gallon of gasoline?

99. A case of diet cola costs $6.79. If there are 24 cans in a case, find the cost per can. Round to the nearest cent.

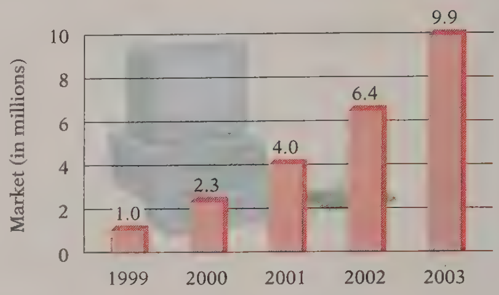
100. Anne is building bookcases that are 3.4 feet long. How many complete shelves can be cut from a 12-foot board?

101. Earl is 52 years old and is buying $70,000 of life insurance for an annual premium of $703.80. If he pays each annual premium in 12 equal installments, how much is each monthly payment?

102. An oil company has issued 3,541,221,500 shares of stock. The company paid $6,090,990,120 in dividends. Find the dividend for each share of stock. Round to the nearest cent.

103. When the Massachusetts Turnpike opened, the toll for a passenger car that traveled the entire 136 miles of it was $5.60. Calculate the cost per mile. Round to the nearest cent.

104. The graph at the right shows the expected growth of digital subscriber lines (DSL). Figures given are in millions. How many times greater was the DSL market expected to be in 2003 than in 2001? Round to the nearest tenth.



Digital Subscriber Lines (DSL) Market

Source: Copyright © 1999 *USA Today.* Reprinted with permission.

APPLYING THE CONCEPTS

105. Explain how the decimal point is moved when a number is divided by 10, 100, 1000, 10,000, etc.

106. A ballpoint pen priced at 50¢ was not selling. When the price was reduced to a different whole number of cents, the entire stock sold for $31.95. How many cents were charged per pen when the price was reduced? (*Hint*: There is more than one possible answer.)

107. Explain how baseball batting averages are determined. Then find Boston Red Sox shortstop Nomar Garciaparra's batting average with 190 hits out of 532 at bats. Round to the nearest thousandth.

108. Explain how the decimal point is placed in the quotient when a number is divided by a decimal.

For each of the problems below, insert +, −, ×, or ÷ into the square so that the statement is true.

109. 3.45 ☐ 0.5 = 6.9

110. 3.46 ☐ 0.24 = 0.8304

111. 6.009 ☐ 4.68 = 1.329

112. 0.064 ☐ 1.6 = 0.1024

113. 9.876 ☐ 23.12 = 32.996

114. 3.0381 ☐ 1.23 = 2.47

Fill in the square to make a true statement.

115. 6.47 − ☐ = 1.253

116. 6.47 + ☐ = 9

117. 0.009 ÷ ☐ = 0.36

3.6 Comparing and Converting Fractions and Decimals

Objective A To convert fractions to decimals

Every fraction can be written as a decimal. To write a fraction as a decimal, divide the numerator of the fraction by the denominator. The quotient can be rounded to the desired place value.

➡ Convert $\frac{3}{7}$ to a decimal.

$$\begin{array}{r} 0.42857 \\ 7\overline{)3.00000} \end{array}$$

$\frac{3}{7}$ rounded to the nearest hundredth is 0.43.

$\frac{3}{7}$ rounded to the nearest thousandth is 0.429.

$\frac{3}{7}$ rounded to the nearest ten-thousandth is 0.4286.

➡ Convert $3\frac{2}{9}$ to a decimal. Round to the nearest thousandth.

$$3\frac{2}{9} = \frac{29}{9} \qquad \begin{array}{r} 3.2222 \\ 9\overline{)29.0000} \end{array}$$

$3\frac{2}{9}$ rounded to the nearest thousandth is 3.222.

Example 1

Convert $\frac{3}{8}$ to a decimal.
Round to the nearest hundredth.

Solution

$$\begin{array}{r} 0.375 \\ 8\overline{)3.000} \end{array} \approx 0.38$$

You Try It 1

Convert $\frac{9}{16}$ to a decimal.
Round to the nearest tenth.

Your solution

Example 2

Convert $2\frac{3}{4}$ to a decimal.
Round to the nearest tenth.

Solution

$$2\frac{3}{4} = \frac{11}{4} \qquad \begin{array}{r} 2.75 \\ 4\overline{)11.00} \end{array} \approx 2.8$$

You Try It 2

Convert $4\frac{1}{6}$ to a decimal.
Round to the nearest hundredth.

Your solution

Solutions on p. S10

Objective B To convert decimals to fractions

To convert a decimal to a fraction, remove the decimal point and place the decimal part over a denominator equal to the place value of the last digit in the decimal.

$$\overset{\text{hundredths}}{0.47} = \frac{47}{100}$$

$$\overset{\text{hundredths}}{7.45} = 7\frac{45}{100} = 7\frac{9}{20}$$

$$\overset{\text{thousandths}}{0.275} = \frac{275}{1000} = \frac{11}{40}$$

$$\overset{\text{hundredths}}{0.16\frac{2}{3}} = \frac{16\frac{2}{3}}{100} = 16\frac{2}{3} \div 100 = \frac{50}{3} \times \frac{1}{100} = \frac{1}{6}$$

Example 3 Convert 0.82 and 4.75 to fractions.

Solution

$$0.82 = \frac{82}{100} = \frac{41}{50}$$

$$4.75 = 4\frac{75}{100} = 4\frac{3}{4}$$

You Try It 3 Convert 0.56 and 5.35 to fractions.

Your solution

Example 4 Convert $0.15\frac{2}{3}$ to a fraction.

Solution

$$0.15\frac{2}{3} = \frac{15\frac{2}{3}}{100} = 15\frac{2}{3} \div 100$$

$$= \frac{47}{3} \times \frac{1}{100} = \frac{47}{300}$$

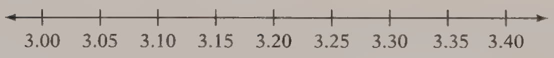
You Try It 4 Convert $0.12\frac{7}{8}$ to a fraction.

Your solution

Solutions on pp. S10–S11

Objective C To identify the order relation between two decimals or between a decimal and a fraction

Decimals, like whole numbers and fractions, can be graphed as points on the number line. The number line can be used to show the order of decimals. A decimal that appears to the right of a given number is greater than the given number. A decimal that appears to the left of a given number is less than the given number.



Note that 3, 3.0, and 3.00 represent the same number.

➡ Find the order relation between $\frac{3}{8}$ and 0.38.

$$\frac{3}{8} = 0.375 \qquad 0.38 = 0.380$$

$$0.375 < 0.380$$

$$\frac{3}{8} < 0.38$$

Example 5 Place the correct symbol, < or >, between the numbers.

$\frac{5}{16}$ 0.32

Solution

$$\frac{5}{16} \approx 0.313$$

$$0.313 < 0.32$$

$$\frac{5}{16} < 0.32$$

You Try It 5 Place the correct symbol, < or >, between the numbers.

0.63 $\frac{5}{8}$

Your solution

Solution on p. S11

3.6 Exercises

Objective A

Convert the fraction to a decimal. Round to the nearest thousandth.

1. $\frac{5}{8}$
2. $\frac{7}{12}$
3. $\frac{2}{3}$
4. $\frac{5}{6}$
5. $\frac{1}{6}$
6. $\frac{7}{8}$

7. $\frac{5}{12}$
8. $\frac{9}{16}$
9. $\frac{7}{4}$
10. $\frac{5}{3}$
11. $1\frac{1}{2}$
12. $2\frac{1}{3}$

13. $\frac{16}{4}$
14. $\frac{36}{9}$
15. $\frac{3}{1000}$
16. $\frac{5}{10}$
17. $7\frac{2}{25}$
18. $16\frac{7}{9}$

19. $37\frac{1}{2}$
20. $\frac{5}{24}$
21. $\frac{4}{25}$
22. $3\frac{1}{3}$
23. $8\frac{2}{5}$
24. $5\frac{4}{9}$

Objective B

Convert the decimal to a fraction.

25. 0.8
26. 0.4
27. 0.32
28. 0.48
29. 0.125

30. 0.485
31. 1.25
32. 3.75
33. 16.9
34. 17.5

35. 8.4
36. 10.7
37. 8.437
38. 9.279
39. 2.25

40. 7.75
41. $0.15\frac{1}{3}$
42. $0.17\frac{2}{3}$
43. $0.87\frac{7}{8}$
44. $0.12\frac{5}{9}$

45. 7.38
46. 0.33
47. 0.57
48. $0.33\frac{1}{3}$
49. $0.66\frac{2}{3}$

Objective C

Place the correct symbol, < or >, between the numbers.

50. 0.15 0.5

51. 0.6 0.45

52. 6.65 > 6.56

53. 3.89 < 3.98

54. 2.504 2.054

55. 0.025 0.105

56. $\frac{3}{8}$ 0.365

57. $\frac{4}{5}$ 0.802

58. $\frac{2}{3}$ 0.65

59. 0.85 $\frac{7}{8}$

60. $\frac{5}{9}$ 0.55

61. $\frac{7}{12}$ 0.58

62. 0.62 $\frac{7}{15}$

63. $\frac{11}{12}$ 0.92

64. 0.161 $\frac{1}{7}$

65. 0.623 0.6023

66. 0.86 0.855

67. 0.87 0.087

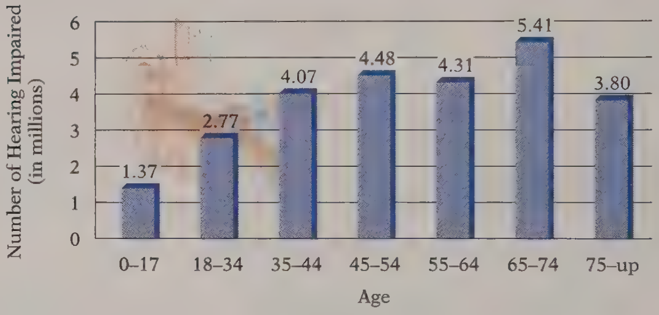
68. 1.005 0.5

69. 0.033 0.3

APPLYING THE CONCEPTS

The graph at the right shows the breakdown by age group of Americans who are hearing impaired. Use this graph for Exercises 70 and 71.

70. Determine whether the number of hearing-impaired individuals under the age of 45 is more than or less than the number of hearing impaired individuals over the age of 64.



Breakdown by Age Group of Hearing-Impaired Americans

Source: American Speech-Language-Hearing Association

71. Is the number of hearing-impaired individuals who are aged 65–74 more than or less than 4 times the number of hearing-impaired individuals who are aged 0–17?

72. Is $\frac{7}{13}$ in decimal form a repeating decimal? Why or why not?

73. If a number is rounded to the nearest thousandth, is it always greater than if it were rounded to the nearest hundredth? Give examples to support your answer.

74. Explain how terminating, repeating, and nonrepeating decimals differ. Give an example of each kind of decimal.

Focus on Problem Solving

Relevant Information

Problems in mathematics or real life involve a question or a need and information or circumstances related to that need. Solving problems in the sciences usually involves a question, an observation, and measurements of some kind.

One of the challenges of problem solving in the sciences is to separate the information that is relevant to the problem from other information. Following is an example from the physical sciences in which some relevant information was omitted.

Hooke's Law states that the distance that a weight will stretch a spring is directly proportional to the weight on the spring. That is, $d = kF$, where d is the distance the spring is stretched and F is the force. In an experiment to verify this law, some physics students were continually getting inconsistent results. Finally, the instructor discovered that the heat produced when the lights were turned on was affecting the experiment. In this case, relevant information was omitted—namely, that the temperature of the spring can affect the distance it will stretch.

A lawyer drove 8 miles to the train station. After a 35-minute ride of 18 miles, the lawyer walked 10 minutes to the office. Find the total time it took the lawyer to get to work.

From this situation, answer the following before reading on.

a. What is asked for?

b. Is there enough information to answer the question?

c. Is information given that is not needed?

Here are the answers.

a. We want the total time for the lawyer to get to work.

b. No. We do not know the time it takes the lawyer to get to the train station.

c. Yes. Neither the distance to the train station nor the distance of the train ride is necessary to answer the question.

For each of the following problems, answer these questions:

a. What is asked for?

b. Is there enough information to answer the question?

c. Is information given that is not needed?

1. A customer bought 6 boxes of strawberries and paid with a $20 bill. What was the change?

2. A board is cut into two pieces. One piece is 3 feet longer than the other piece. What is the length of the original board?

3. A family rented a car for their vacation and drove 680 miles. The cost of the rental car was $21 per day with 150 free miles per day and $.15 for each mile driven above the number of free miles allowed. How many miles did the family drive per day?

4. An investor bought 8 acres of land for $80,000. One and one-half acres were set aside for a park, and the remainder was developed into one-half-acre lots. How many lots were available for sale?

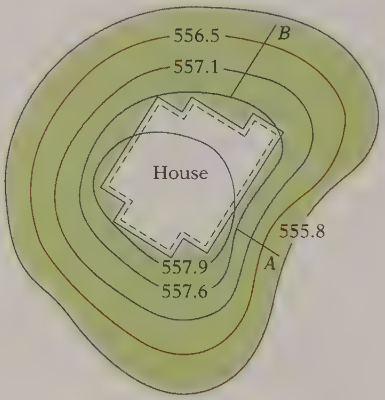
5. You wrote checks of \$43.67, \$122.88, and \$432.22 after making a deposit of \$768.55. How much do you have left in your checking account?

Projects and Group Activities

Topographical Maps

Carpenters use fractions to measure the wood that is used to frame a house. For instance, a door entry may measure $42\frac{1}{2}$ inches. However, the grading contractor (the person who levels the lot on which the house is built) measures the height of the lot by using decimals. For instance, a certain place may be at a height of 554.2 feet. (The height is measured above sea level, so 554.2 means 554.2 feet above sea level.)

A surveyor provides the grading contractor with a grading plan that shows the elevation of each point of the lot. These plans are drawn so that the house can be sited in such a way that water will drain away from it. The diagram below is a **topographical map** for a lot. Along each closed curve, called a **contour curve,** the lot is at the same height above sea level. For instance, every point on the curve in red is 556.5 feet above sea level.



Using the map above, answer the following questions.

1. What is the elevation of the highest point on the lot?

2. What is the elevation of the lowest point on the lot?

3. What is the difference in elevation between the highest and lowest points on the lot?

4. Assuming that this map is drawn to scale, describe the significance of how steep the slope is along line *A* compared to that along line *B*.

5. Acquire a forest service contour map of a portion of a national forest and plan a cross-country hike, choosing a trail that avoids the steepest ascent.

International Math and Science Test Scores

The Third International Mathematics and Science Study (TIMSS) is responsible for comparing the performance of half a million students from 41 countries at five different grade levels on tests of their mathematics and science knowledge. The table below shows the performance of the fourth-graders from the 26 nations that participated at this level.

1. Calculate the average score in mathematics and the average score in science. (Find the sum of all the scores and then divide by the number of scores.) Round to the nearest tenth.
2. How far above or below each average are the U.S. scores?
3. Which countries scored above the United States in math? In science? Are they the same countries?
4. Discuss the following: The study reported that U.S. fourth-graders receive, on average, 4.2 hours of mathematics instruction per week; the international average is 3.9 hours. However, four of the nations that scored higher than the United States in mathematics spend less time per week on mathematics instruction.
5. One area of the mathematics testing in which the U.S. average was below the international average was measurement. On the international test, units cited in the questions were metric units. Discuss how these two facts might be related.
6. Where do you think the United States should rank internationally, given its resources and population?
7. What would you do to improve math and science education in the United States?

Nation	*Average Score in Mathematics*	*Average Score in Science*
Australia	546	562
Austria	559	565
Canada	532	549
Cyprus	502	475
Czech Republic	567	557
England	513	551
Greece	492	497
Hong Kong	587	533
Hungary	548	532
Iceland	474	505
Iran, Islamic Republic	429	416
Ireland	550	539
Israel	531	505
Japan	597	574
Korea	611	597
Kuwait	400	401
Latvia (LSS)	525	512
Netherlands	577	557
New Zealand	499	531
Norway	502	530
Portugal	475	480
Scotland	520	536
Singapore	625	547
Slovenia	552	546
Thailand	490	473
United States	545	565

Chapter Summary

Key Words

A number written in *decimal notation* has three parts: a whole-number part, a decimal point, and a decimal part. The *decimal part* of a number represents a number less than 1. [p. 123]

Essential Rules

To Write a Decimal in Words
Write the decimal part as though it were a whole number. Then name the place value of the last digit. [p. 123]

To Write a Decimal in Standard Form When It Is Written in Words
Write the whole-number part, replace the word "and" with a decimal point, and write the decimal part so that the last digit is in the given place-value position. [p. 123]

To Round a Decimal to a Given Place Value
If the digit to the right of the given place value is less than 5, drop that digit and all digits to the right. If the digit to the right of the given place value is greater than or equal to 5, increase the number in the given place value by 1, and drop all digits to its right. [p. 124]

To Add Decimals
Write the numbers so that the decimal points are on a vertical line. Add as you would with whole numbers. Then write the decimal point in the sum directly below the decimal points in the addends. [p. 127]

To Subtract Decimals
Write the numbers so that the decimal points are on a vertical line. Subtract as you would with whole numbers. Then write the decimal point in the difference directly below the decimal point in the subtrahend. [p. 131]

To Multiply Decimals
Multiply the numbers as you would whole numbers. Then write the decimal point in the product so that the number of decimal places in the product is the sum of the decimal places in the factors. [p. 135]

To Multiply a Decimal by a Power of 10
Move the decimal point to the right the same number of places as there are zeros in the power of 10. If the power of 10 is written in exponential notation, the exponent indicates how many places to move the decimal point. [pp. 135–136]

To Divide Decimals
Move the decimal point in the divisor to the right so that it is a whole number. Move the decimal point in the dividend the same number of places to the right. Place the decimal point in the quotient directly above the decimal point in the dividend. Then divide as you would with whole numbers. [p. 145]

To Divide a Decimal by a Power of 10
Move the decimal point to the left the same number of places as there are zeros in the power of 10. If the power of 10 is written in exponential notation, the exponent indicates how many places to move the decimal point. [p. 146]

To Convert a Fraction to a Decimal
Divide the numerator of the fraction by the denominator. Round the quotient to the desired place value. [p. 153]

To Convert a Decimal to a Fraction
Remove the decimal point and place the decimal part over a denominator equal to the place value of the last digit in the decimal. [p. 153]

To Find the Order Relation Between a Decimal and a Fraction
First rewrite the fraction as a decimal. Then compare the two decimals. [p. 154]

Chapter Review

1. Find the quotient of 3.6515 and 0.067.

2. Find the sum of 369.41, 88.3, 9.774, and 366.474.

3. Place the correct symbol, < or >, between the two numbers.
0.055 0.1

4. Write 22.0092 in words.

5. Round 0.05678235 to the nearest hundred-thousandth.

6. Convert $2\frac{1}{3}$ to a decimal. Round to the nearest hundredth.

7. Convert 0.375 to a fraction.

8. Add: $3.42 + 0.794 + 32.5$

9. Write thirty-four and twenty-five thousandths in standard form.

10. Place the correct symbol, < or >, between the two numbers.
$\frac{5}{8}$ 0.62

11. Convert $\frac{7}{9}$ to a decimal. Round to the nearest thousandth.

12. Convert $0.66\frac{2}{3}$ to a fraction.

13. Subtract: $27.31 - 4.4465$

14. Round 7.93704 to the nearest hundredth.

15. Find the product of 3.08 and 2.9.

16. Write 342.37 in words.

17. Write three and six thousand seven hundred fifty-three hundred-thousandths in standard form.

18. Multiply:

$$\begin{array}{r} 34.79 \\ \times\ \ 0.74 \\ \hline \end{array}$$

19. Divide: $0.053\overline{)0.349482}$

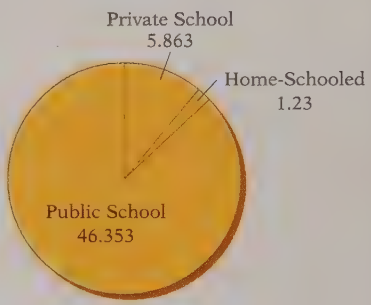
20. What is 7.796 decreased by 2.9175?

21. You had a balance of $895.68 in your checking account. You then wrote checks of $145.72 and $88.45. Find the new balance in your checking account.

The graph at the right shows where American children in grades K–12 are being educated. Figures are in millions of children. Use this graph for Exercises 22 and 23.

22. Find the total number of American children in grades K–12.

23. How many more children are being educated in public school than in private school?

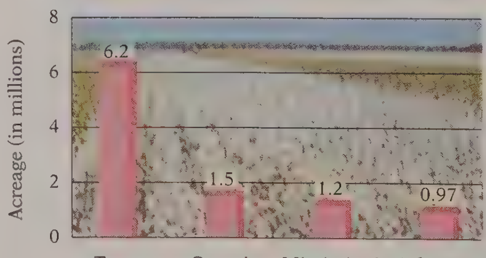


Where Children in Grades K–12 Are Being Educated in America (in millions)

Source: U.S. Department of Education; Home School Legal Defense Association

24. According to the American School Food Service Association, 1.9 million gallons of milk are served in school cafeterias every day. How many gallons of milk are served in school cafeterias during a 5-day school week?

25. Almost 15 million acres are planted in cotton in the United States. The top four cotton states are shown in the graph at the right. How many times greater is the number of acres planted in Texas than the number planted in Mississippi? Round to the nearest tenth.



Acres Planted in Cotton

Source: Agriculture Department, *Cotton and Wool Outlook*, April 2000

Chapter Test

1. Place the correct symbol, < or >, between the two numbers.
0.66 0.666

2. Subtract:
$$\begin{array}{r} 13.027 \\ -\ \ 8.94 \\ \hline \end{array}$$

3. Write 45.0302 in words.

4. Convert $\frac{9}{13}$ to a decimal. Round to the nearest thousandth.

5. Convert 0.825 to a fraction.

6. Round 0.07395 to the nearest ten-thousandth.

7. Find 0.0569 divided by 0.037. Round to the nearest thousandth.

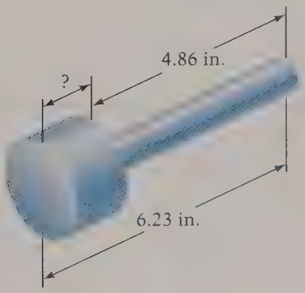
8. Find 9.23674 less than 37.003.

9. Round 7.0954625 to the nearest thousandth.

10. Divide: $0.006\overline{)1.392}$

11. Add:
$$\begin{array}{r} 270.93 \\ 97. \\ 1.976 \\ +\ \ 88.675 \\ \hline \end{array}$$

12. Find the missing dimension.



13. Multiply: $\begin{array}{r} 1.37 \\ \times\, 0.004 \\ \hline \end{array}$

14. What is the total of 62.3, 4.007, and 189.65?

15. Write two hundred nine and seven thousand eighty-six hundred-thousandths in standard form.

16. A car was bought for $16,734.40, with a down payment of $2500. The balance was paid in 36 monthly payments. Find the amount of each monthly payment.

17. You received a salary of $727.50, a commission of $1909.64, and a bonus of $450. Find your total income.

18. A long-distance telephone call costs $.85 for the first 3 minutes and $.42 for each additional minute. Find the cost of a 12-minute long-distance telephone call.

The table at the right shows the average number of hours per week that students use a computer. Use this table for Exercises 19 and 20.

Grade Level	*Average Number of Hours of Computer Use per Week*
Prekindergarten – kindergarten	3.9
1st – 3rd	4.9
4th – 6th	4.2
7th – 8th	6.9
9th – 12th	6.7

Source: Find/SVP American Learning Household Survey

19. On average, how many hours per year does a 10th-grade student use a computer? Use a 52-week year.

20. On average, how many more hours per year does a 2nd-grade student use a computer than a 5th-grade student? Use a 52-week year.

Cumulative Review

1. Divide: $89\overline{)20{,}932}$

2. Simplify: $2^3 \cdot 4^2$

3. Simplify: $2^2 - (7 - 3) \div 2 + 1$

4. Find the LCM of 9, 12, and 24.

5. Write $\frac{22}{5}$ as a mixed number.

6. Write $4\frac{5}{8}$ as an improper fraction.

7. Write an equivalent fraction with the given denominator.

$$\frac{5}{12} = \frac{}{60}$$

8. Add: $\frac{3}{8} + \frac{5}{12} + \frac{9}{16}$

9. What is $5\frac{7}{12}$ increased by $3\frac{7}{18}$?

10. Subtract: $9\frac{5}{9} - 3\frac{11}{12}$

11. Multiply: $\frac{9}{16} \times \frac{4}{27}$

12. Find the product of $2\frac{1}{8}$ and $4\frac{5}{17}$.

13. Divide: $\frac{11}{12} \div \frac{3}{4}$

14. What is $2\frac{3}{8}$ divided by $2\frac{1}{2}$?

15. Simplify: $\left(\frac{2}{3}\right)^2 \cdot \left(\frac{3}{4}\right)^3$

16. Simplify: $\left(\frac{2}{3}\right)^2 - \left(\frac{2}{3} - \frac{1}{2}\right) + 2$

17. Write 65.0309 in words.

18. Add:

$$\begin{array}{r} 379.006 \\ 27.523 \\ 9.8707 \\ +\ \ 88.2994 \\ \hline \end{array}$$

19. What is 29.005 decreased by 7.9286?

20. Multiply: $\begin{array}{r} 9.074 \\ \times\ 6.09 \\ \hline \end{array}$

21. Divide: $8.09\overline{)17.42963}$.
Round to the nearest thousandth.

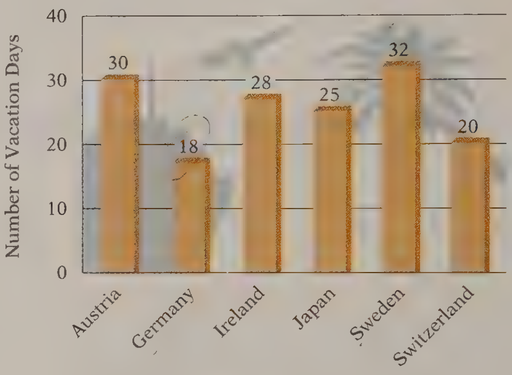
22. Convert $\frac{11}{15}$ to a decimal. Round to the nearest thousandth.

23. Convert $0.16\frac{2}{3}$ to a fraction.

24. Place the correct symbol, < or >, between the two numbers.

$\frac{8}{9} \quad 0.98$

25. The graph at the right shows the number of vacation days per year that are legally mandated in several countries. How many more vacation days does Sweden mandate than Germany?



Number of Legally Mandated Vacation Days

Source: Economic Policy Institute; *World Almanac*

26. An investor purchased stock at $\$32\frac{1}{8}$ per share. During the first 2 months of ownership, the stock lost $\$\frac{3}{4}$ and then gained $\$1\frac{1}{2}$. Find the value of each share of stock at the end of the 2 months.

27. You have a checking account balance of \$814.35. You then write checks for \$42.98, \$16.43, and \$137.56. Find your checking account balance after you write the checks.

28. A machine lathe takes 0.017 inch from a brass bushing that is 1.412 inches thick. Find the resulting thickness of the bushing.

29. The state income tax on your business is \$820 plus 0.08 times your profit. You made a profit of \$64,860 last year. Find the amount of income tax you paid last year.

30. You bought a camera costing \$210.96. The down payment was \$20, and the balance is to be paid in 8 equal monthly payments. Find the monthly payment.

Chapter

4

Ratio and Proportion

Objectives

Section 4.1

A To write the ratio of two quantities in simplest form

B To solve application problems

Section 4.2

A To write rates

B To write unit rates

C To solve application problems

Section 4.3

A To determine whether a proportion is true

B To solve proportions

C To solve application problems

What cargo might this freighter be carrying? Auto parts, clothing, watches, toys, agricultural goods? These are only a few of the products imported into the United States each year. When an American company imports goods from another country, U.S. dollars are exchanged for the currency of the other country in order to pay for the products. This exchange of currencies is discussed in **Exercise 33 on page 176.**

Need help? For on-line student resources, such as section quizzes, visit this textbook's web site at **math.college.hmco.com/students.**

Prep Test

1. Simplify: $\frac{8}{10}$

2. Simplify: $\frac{450}{650 + 250}$

3. Write as a decimal: $\frac{372}{15}$

4. Which is greater, 4×33 or 62×2?

5. Complete: $? \times 5 = 20$

Go Figure

Luis, Kim, Reggie, and Dave are standing in line. Dave is not first. Kim is between Luis and Reggie. Luis is between Dave and Kim. Give the order in which the men are standing.

4.1 Ratio

Objective A To write the ratio of two quantities in simplest form

Quantities such as 3 feet, 12 cents, and 9 cars are number quantities written with **units**.

3 feet
12 cents
9 cars
↑
units

These are only some examples of units. Shirts, dollars, trees, miles, and gallons are further examples.

A **ratio** is a comparison of two quantities that have the *same* units. This comparison can be written three different ways:

1. As a fraction
2. As two numbers separated by a colon (:)
3. As two numbers separated by the word *to*

The ratio of the lengths of two boards, one 8 feet long and the other 10 feet long, can be written as

1. $\frac{8\text{ feet}}{10\text{ feet}} = \frac{8}{10} = \frac{4}{5}$
2. 8 feet : 10 feet = 8 : 10 = 4 : 5
3. 8 feet to 10 feet = 8 to 10 = 4 to 5

A ratio is in **simplest form** when the two numbers do not have a common factor. Note that in a ratio, the units are not written.

This ratio means that the smaller board is $\frac{4}{5}$ the length of the longer board.

Point of Interest

In the 1990s, the major-league pitchers with the best strikeout-to-walk ratios (having pitched a minimum of 100 innings) were

Dennis Eckersley	6.46:1
Shane Reynolds	4.13:1
Greg Maddux	4:1
Bret Saberhagen	3.92:1
Rod Beck	3.81:1

The best single-season strikeout-to-walk ratio for starting pitchers in the same period was that of Bret Saberhagen, 11:1. (*Source:* Elias Sports Bureau)

Example 1

Write the comparison \$6 to \$8 as a ratio in simplest form using a fraction, a colon, and the word *to*.

Solution $\frac{\$6}{\$8} = \frac{6}{8} = \frac{3}{4}$

\$6 : \$8 = 6 : 8 = 3 : 4

\$6 to \$8 = 6 to 8 = 3 to 4

You Try It 1

Write the comparison 20 pounds to 24 pounds as a ratio in simplest form using a fraction, a colon, and the word *to*.

Your solution

Example 2

Write the comparison 18 quarts to 6 quarts as a ratio in simplest form using a fraction, a colon, and the word *to*.

Solution $\frac{18\text{ quarts}}{6\text{ quarts}} = \frac{18}{6} = \frac{3}{1}$

18 quarts : 6 quarts = 18 : 6 = 3 : 1

18 quarts to 6 quarts = 18 to 6 = 3 to 1

You Try It 2

Write the comparison 64 miles to 8 miles as a ratio in simplest form using a fraction, a colon, and the word *to*.

Your solution

Solutions on p. S11

Objective B To solve application problems

Use the table below for Example 3 and You Try It 3.

Board Feet of Wood at a Lumber Store

Pine	*Ash*	*Oak*	*Cedar*
20,000	18,000	10,000	12,000

Example 3
Find, as a fraction in simplest form, the ratio of the number of board feet of pine to the number of board feet of oak.

Strategy
To find the ratio, write the ratio of board feet of pine (20,000) to board feet of oak (10,000) in simplest form.

Solution

$$\frac{20{,}000}{10{,}000} = \frac{2}{1}$$

The ratio is $\frac{2}{1}$.

You Try It 3
Find, as a fraction in simplest form, the ratio of the number of board feet of cedar to the number of board feet of ash.

Your strategy

Your solution

Example 4
The cost of building a patio cover was \$250 for labor and \$350 for materials. What, as a fraction in simplest form, is the ratio of the cost of materials to the total cost for labor and materials?

Strategy
To find the ratio, write the ratio of the cost of materials (\$350) to the total cost (\$250 + \$350) in simplest form.

Solution

$$\frac{\$350}{\$250 + \$350} = \frac{350}{600} = \frac{7}{12}$$

The ratio is $\frac{7}{12}$.

You Try It 4
A company spends \$20,000 a month for television advertising and \$15,000 a month for radio advertising. What, as a fraction in simplest form, is the ratio of the cost of radio advertising to the total cost of radio and television advertising?

Your strategy

Your solution

Solutions on p. S11

4.1 Exercises

Objective A

Write the comparison as a ratio in simplest form using a fraction, a colon (:), and the word *to*.

1. 3 pints to 15 pints

2. 6 pounds to 8 pounds

3. \$40 to \$20

4. 10 feet to 2 feet

5. 3 miles to 8 miles

6. 2 hours to 3 hours

7. 37 hours to 24 hours

8. 29 inches to 12 inches

9. 6 minutes to 6 minutes

10. 8 days to 12 days

11. 35 cents to 50 cents

12. 28 inches to 36 inches

13. 30 minutes to 60 minutes

14. 25 cents to 100 cents

15. 32 ounces to 16 ounces

16. 12 quarts to 4 quarts

17. 3 cups to 4 cups

18. 6 years to 7 years

19. \$5 to \$3

20. 30 yards to 12 yards

21. 12 quarts to 18 quarts

22. 20 gallons to 28 gallons

23. 14 days to 7 days

24. 9 feet to 3 feet

Objective B *Application Problems*

Write ratios in simplest form using a fraction.

Family Budget

Housing	*Food*	*Transportation*	*Taxes*	*Utilities*	*Miscellaneous*	*Total*
\$800	\$400	\$300	\$350	\$150	\$400	\$2400

25. Use the table to find the ratio of housing cost to total expenses.

26. Use the table to find the ratio of food cost to total expenses.

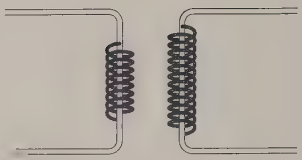
27. Use the table to find the ratio of utilities cost to food cost.

28. Use the table to find the ratio of transportation cost to housing cost.

29. National Collegiate Athletic Association (NCAA) statistics show that for every 154,000 high school seniors playing basketball, only 4000 will play college basketball as first-year students. Write the ratio of the number of first-year students playing college basketball to the number of high school seniors playing basketball.

30. NCAA statistics show that for every 2800 college seniors playing college basketball, only 50 will play as rookies in the National Basketball Association. Write the ratio of the number of National Basketball Association rookies to the number of college seniors playing basketball.

31. A transformer has 40 turns in the primary coil and 480 turns in the secondary coil. State the ratio of the number of turns in the primary coil to the number of turns in the secondary coil.



32. Rita Sterling bought a computer system for \$2400. Five years later she sold the computer for \$900. Find the ratio of the amount she received for the computer to the cost of the computer.

33. A house with an original value of \$90,000 increased in value to \$110,000 in 5 years.
   a. Find the increase in the value of the house.
   b. What is the ratio of the increase in value to the original value of the house?

34. The price of gasoline jumped from \$1.35 to \$1.62 in 1 year.
   a. What was the increase in the price per gallon?
   b. What is the ratio of the increase in price to the original price?

APPLYING THE CONCEPTS

A bank uses the ratio of a borrower's total monthly debts to the borrower's total monthly income to determine the maximum monthly payment for a potential homeowner. This ratio is called the debt–income ratio. Use the homeowner's income–debt table at the right for Exercises 35 and 36.

Income	*Debts*
\$3500	\$900
\$250	\$170
\$140	\$160
	\$95

35. Compute the debt–income ratio for the potential homeowner.

36. If Central Trust Bank will make a loan to a customer whose debt–income ratio is less than $\frac{1}{3}$, will the potential homeowner qualify? Explain your answer.

37. To make a home loan, First National Bank requires a debt–income ratio that is less than $\frac{2}{5}$. Would the homeowner whose income–debt table is given at the right qualify for a loan using these standards? Explain.

Income		*Debts*	
Salary	\$3400	Mortgage	\$1800
Interest	\$83	Property tax	\$104
Rent	\$650	Insurance	\$35
Dividends	\$34	Liabilities	\$120
		Credit card	\$234
		Car loan	\$197

38. Is the value of a ratio always less than 1? Explain.

4.2 Rates

Objective A To write rates

Point of Interest

Listed below are rates at which some crimes are committed in our nation.

Crime	*Every*
Larceny	4 seconds
Burglary	14 seconds
Robbery	60 seconds
Rape	6 minutes
Murder	31 minutes

A **rate** is a comparison of two quantities that have *different* units. A rate is written as a fraction.

A distance runner ran 26 miles in 4 hours. The distance-to-time rate is written

$$\frac{26\text{ miles}}{4\text{ hours}} = \frac{13\text{ miles}}{2\text{ hours}}$$

A rate is in **simplest form** when the numbers that form the rate have no common factors. Note that the units are written as part of the rate.

Example 1 Write "6 roof supports for every 9 feet" as a rate in simplest form.

Solution $\frac{6\text{ supports}}{9\text{ feet}} = \frac{2\text{ supports}}{3\text{ feet}}$

You Try It 1 Write "15 pounds of fertilizer for 12 trees" as a rate in simplest form.

Your solution

Solution on p. S11

Objective B To write unit rates

Point of Interest

According to a Gallup Poll, women see doctors more often than men do. On average, men visit the doctor 3.8 times per year, whereas women go to the doctor 5.8 times per year.

A **unit rate** is a rate in which the number in the denominator is 1.

$\frac{\$3.25}{1\text{ pound}}$ or \$3.25/pound is read "\$3.25 per pound."

To find unit rates, divide the number in the numerator of the rate by the number in the denominator of the rate.

A car traveled 344 miles on 16 gallons of gasoline. To find the miles per gallon (unit rate), divide the numerator of the rate by the denominator of the rate.

$\frac{344\text{ miles}}{16\text{ gallons}}$ is the rate.

$$16\overline{)344.0} = 21.5$$

21.5 miles/gallon is the unit rate.

Example 2 Write "300 feet in 8 seconds" as a unit rate.

Solution $\frac{300\text{ feet}}{8\text{ seconds}}$ $\quad 8\overline{)300.0} = 37.5$

37.5 feet/second

You Try It 2 Write "260 miles in 8 hours" as a unit rate.

Your solution

Solution on p. S11

Objective C To solve application problems

The table at the right shows typical air fares for long routes. Find the cost per mile for the four routes in order to determine the most expensive route and the least expensive route on the basis of mileage flown.

Long Routes	*Miles*	*Fare*
New York–Los Angeles	2475	$683
San Francisco–Dallas	1464	$536
Denver–Pittsburgh	1302	$525
Minneapolis–Hartford	1050	$483

Strategy
To find the cost per mile, divide the fare by the miles flown for each route. Compare the costs per mile to determine the most expensive and least expensive routes per mile.

Solution New York–Los Angeles $\frac{683}{2475} \approx 0.28$

San Francisco–Dallas $\frac{536}{1464} \approx 0.37$

Denver–Pittsburgh $\frac{525}{1302} \approx 0.40$

Minneapolis–Hartford $\frac{483}{1050} = 0.46$

$0.28 < 0.37 < 0.40 < 0.46$

The Minneapolis–Hartford route is the most expensive per mile, and the New York–Los Angeles route is the least expensive per mile.

Example 3
As an investor, Jung Ho purchased 100 shares of stock for $1500. One year later, Jung sold the 100 shares for $1800. What was his profit per share?

Strategy
To find Jung's profit per share:

- Find the total profit by subtracting the original cost (1500) from the selling price (1800).
- Find the profit per share (unit rate) by dividing the total profit by the number of shares of stock (100).

Solution
$1800 - 1500 = 300$

$300 \div 100 = 3$

Jung Ho's profit was $3/share.

You Try It 3
Erik Peltier, a jeweler, purchased 5 ounces of gold for $1625. Later, he sold the 5 ounces for $1720. What was Erik's profit per ounce?

Your strategy

Your solution

Solution on p. S11

4.2 Exercises

Objective A

Write each phrase as a rate in simplest form.

1. 3 pounds of meat for 4 people
2. 30 ounces in 24 glasses
3. $80 for 12 boards
4. 84 cents for 6 bars of soap
5. 300 miles on 15 gallons
6. 88 feet in 8 seconds
7. 20 children in 8 families
8. 48 leaves on 9 plants
9. 16 gallons in 2 hours
10. 25 ounces in 5 minutes

Objective B

Write each phrase as a unit rate.

11. 10 feet in 4 seconds
12. 816 miles in 6 days
13. $1300 earned in 4 weeks
14. $27,000 earned in 12 months
15. 1100 trees planted on 10 acres
16. 3750 words on 15 pages
17. $32.97 earned in 7 hours
18. $315.70 earned in 22 hours
19. 628.8 miles in 12 hours
20. 388.8 miles in 8 hours
21. 344.4 miles on 12.3 gallons of gasoline
22. 409.4 miles on 11.5 gallons of gasoline
23. $349.80 for 212 pounds
24. $11.05 for 3.4 pounds

Objective C *Application Problems*

25. An automobile was driven 326.6 miles on 11.5 gallons of gas. Find the number of miles driven per gallon of gas.
26. You drive 246.6 miles in 4.5 hours. Find the number of miles you drove per hour.
27. The Saturn-5 rocket uses 534,000 gallons of fuel in 2.5 minutes. How much fuel does the rocket use per minute?

28. Assume that Regency Computer produced 5000 zip disks for $26,536.32. Of the disks made, 122 did not meet company standards.
  a. How many disks did meet company standards?
  b. What was the cost per disk for those disks that met company standards?

29. The Pierre family purchased a 250-pound side of beef for $365.75 and had it packaged. During the packaging, 75 pounds of beef were discarded as waste.
  a. How many pounds of beef were packaged?
  b. What was the cost per pound for the packaged beef?

30. The Bear Valley Fruit Stand purchased 250 boxes of strawberries for $162.50. All the strawberries were sold for $312.50. What was the profit per box of strawberries?

31. During filming, an IMAX camera uses 65-mm film at a rate of 5.6 feet per second.
  a. At what rate per minute does the camera go through film?
  b. At what rate does it use a 500-foot roll of 65-mm film? Round to the nearest second.

32. The table at the right shows the population and area of three countries. The population density of a country is the number of people per square mile.
  a. Which country has the least population density?
  b. How many more people per square mile are there in India than in the United States? Round to the nearest whole number.

Country	*Population*	*Area (in square miles)*
Australia	18,322,000	2,968,000
India	936,546,000	1,269,000
United States	267,000,000	3,619,000

Another application of rates is in the area of international trade. Suppose a company in Canada purchases a shipment of sneakers from an American company. The Canadian company must exchange the Canadian dollar for U.S. dollars in order to pay for the order. The number of Canadian dollars that are equivalent to one U.S. dollar is called the **exchange rate.**

33. The table at the right shows the exchange rates per U.S. dollar for three foreign countries and for the Euro at the time of this writing.
  a. How many euros would be paid for an order of American computer hardware costing $38,000?
  b. Calculate the cost, in Japanese yen, of an American car costing $24,000.

Exchange Rates per U.S. Dollar	
Australian Dollar	1.741
Canadian Dollar	1.477
Japanese Yen	106.500
The Euro	1.118

APPLYING THE CONCEPTS

34. You have a choice of receiving a wage of $34,000 per year, $2840 per month, $650 per week, or $18 per hour. Which pay choice would you take? Assume a 40-hour week with 52 weeks per year.

35. The price–earnings ratio of a company's stock is one measure used by stock market analysts to assess the financial well-being of the company. Explain the meaning of the price–earnings ratio.

4.3 Proportions

Objective A To determine whether a proportion is true

Point of Interest

Proportions were studied by the earliest mathematicians. Clay tablets uncovered by archaeologists show evidence of proportions in Egyptian and Babylonian cultures dating from 1800 B.C.

A **proportion** is an expression of the equality of two ratios or rates.

$$\frac{50 \text{ miles}}{4 \text{ gallons}} = \frac{25 \text{ miles}}{2 \text{ gallons}}$$

Note that the units of the numerators are the same and the units of the denominators are the same.

$$\frac{3}{6} = \frac{1}{2}$$

This is the equality of two ratios.

A proportion is **true** if the fractions are equal when written in lowest terms.

In any true proportion, the "cross products" are equal.

➡ Is $\frac{2}{3} = \frac{8}{12}$ a true proportion?

$$\frac{2}{3} \times \frac{8}{12} \quad \begin{array}{l} \rightarrow 3 \times 8 = 24 \\ \rightarrow 2 \times 12 = 24 \end{array}$$

The cross products *are* equal.
$\frac{2}{3} = \frac{8}{12}$ is a true proportion.

A proportion is **not true** if the fractions are not equal when reduced to lowest terms.

If the cross products are not equal, then the proportion is not true.

➡ Is $\frac{4}{5} = \frac{8}{9}$ a true proportion?

$$\frac{4}{5} \times \frac{8}{9} \quad \begin{array}{l} \rightarrow 5 \times 8 = 40 \\ \rightarrow 4 \times 9 = 36 \end{array}$$

The cross products *are not* equal.
$\frac{4}{5} = \frac{8}{9}$ is not a true proportion.

Example 1

Is $\frac{5}{8} = \frac{10}{16}$ a true proportion?

Solution

$$\frac{5}{8} \times \frac{10}{16} \quad \begin{array}{l} \rightarrow 8 \times 10 = 80 \\ \rightarrow 5 \times 16 = 80 \end{array}$$

The cross products are equal.
The proportion is true.

You Try It 1

Is $\frac{6}{10} = \frac{9}{15}$ a true proportion?

Your solution

Example 2

Is $\frac{62 \text{ miles}}{4 \text{ gallons}} = \frac{33 \text{ miles}}{2 \text{ gallons}}$ a true proportion?

Solution

$$\frac{62}{4} \times \frac{33}{2} \quad \begin{array}{l} \rightarrow 4 \times 33 = 132 \\ \rightarrow 62 \times 2 = 124 \end{array}$$

The cross products are not equal.
The proportion is not true.

You Try It 2

Is $\frac{\$32}{6 \text{ hours}} = \frac{\$90}{8 \text{ hours}}$ a true proportion?

Your solution

Solutions on p. S11

Objective B

To solve proportions

4

Sometimes one of the numbers in a proportion is unknown. In this case, it is necessary to *solve* the proportion.

To **solve** a proportion, find a number to replace the unknown so that the proportion is true.

Solve: $\frac{9}{6} = \frac{3}{n}$

$$\frac{9}{6} = \frac{3}{n}$$

$9 \times n = 6 \times 3$ • Find the cross products.

$9 \times n = 18$

$n = 18 \div 9$ • Think of $9 \times n = 18$ as $9\overset{\times\, n}{\overline{)\,18}}$.

$n = 2$

Check:

$$\frac{9}{6} \quad \frac{3}{2} \begin{array}{l} \rightarrow 6 \times 3 = 18 \\ \rightarrow 9 \times 2 = 18 \end{array}$$

Example 3

Solve $\frac{n}{12} = \frac{25}{60}$ and check.

Solution

$n \times 60 = 12 \times 25$

$n \times 60 = 300$

$n = 300 \div 60$

$n = 5$

Check:

$$\frac{5}{12} \quad \frac{25}{60} \begin{array}{l} \rightarrow 12 \times 25 = 300 \\ \rightarrow 5 \times 60 = 300 \end{array}$$

You Try It 3

Solve $\frac{n}{14} = \frac{3}{7}$ and check.

Your solution

Example 4

Solve $\frac{4}{9} = \frac{n}{16}$. Round to the nearest tenth.

Solution

$4 \times 16 = 9 \times n$

$64 = 9 \times n$

$64 \div 9 = n$

$7.1 \approx n$

Note: A rounded answer is an approximation. Therefore, the answer to a check will not be exact.

You Try It 4

Solve $\frac{5}{7} = \frac{n}{20}$. Round to the nearest tenth.

Your solution

Solutions on pp. S11–S12

Example 5

Solve $\frac{28}{52} = \frac{7}{n}$ and check.

Solution

$$\begin{aligned} 28 \times n &= 52 \times 7 \\ 28 \times n &= 364 \\ n &= 364 \div 28 \\ n &= 13 \end{aligned}$$

Check:

$$\frac{28}{52} \quad \frac{7}{13} \quad \begin{aligned} &\rightarrow 52 \times 7 = 364 \\ &\rightarrow 28 \times 13 = 364 \end{aligned}$$

Example 6

Solve $\frac{15}{n} = \frac{8}{3}$. Round to the nearest hundredth.

Solution

$$\begin{aligned} 15 \times 3 &= n \times 8 \\ 45 &= n \times 8 \\ 45 \div 8 &= n \\ 5.63 &\approx n \end{aligned}$$

Example 7

Solve $\frac{n}{9} = \frac{3}{1}$ and check.

Solution

$$\begin{aligned} n \times 1 &= 9 \times 3 \\ n \times 1 &= 27 \\ n &= 27 \div 1 \\ n &= 27 \end{aligned}$$

Check:

$$\frac{27}{9} \quad \frac{3}{1} \quad \begin{aligned} &\rightarrow 9 \times 3 = 27 \\ &\rightarrow 27 \times 1 = 27 \end{aligned}$$

Example 8

Solve $\frac{5}{9} = \frac{15}{n}$ and check.

Solution

$$\begin{aligned} 5 \times n &= 9 \times 15 \\ 5 \times n &= 135 \\ n &= 135 \div 5 \\ n &= 27 \end{aligned}$$

Check:

$$\frac{5}{9} \quad \frac{15}{27} \quad \begin{aligned} &\rightarrow 9 \times 15 = 135 \\ &\rightarrow 5 \times 27 = 135 \end{aligned}$$

You Try It 5

Solve $\frac{15}{20} = \frac{12}{n}$ and check.

Your solution

You Try It 6

Solve $\frac{12}{n} = \frac{7}{4}$. Round to the nearest hundredth.

Your solution

You Try It 7

Solve $\frac{n}{12} = \frac{4}{1}$ and check.

Your solution

You Try It 8

Solve $\frac{3}{8} = \frac{12}{n}$ and check.

Your solution

Solutions on p. S12

Objective C To solve application problems

Example 9

A mason determines that 9 cement blocks are required for a retaining wall 2 feet long. At this rate, how many cement blocks are required for a retaining wall that is 24 feet long?

Strategy

To find the number of cement blocks for a retaining wall 24 feet long, write and solve a proportion using n to represent the number of blocks required.

Solution

$$\frac{9 \text{ cement blocks}}{2 \text{ feet}} = \frac{n \text{ cement blocks}}{24 \text{ feet}}$$

$$\begin{aligned} 9 \times 24 &= 2 \times n \\ 216 &= 2 \times n \\ 216 \div 2 &= n \\ 108 &= n \end{aligned}$$

A 24-foot retaining wall requires 108 cement blocks.

You Try It 9

Twenty-four jars can be packed in 6 identical boxes. At this rate, how many jars can be packed in 15 boxes?

Your strategy

Your solution

Example 10

The dosage of a certain medication is 2 ounces for every 50 pounds of body weight. How many ounces of this medication are required for a person who weighs 175 pounds?

Strategy

To find the number of ounces of medication for a person weighing 175 pounds, write and solve a proportion using n to represent the number of ounces of medication for a 175-pound person.

Solution

$$\frac{2 \text{ ounces}}{50 \text{ pounds}} = \frac{n \text{ ounces}}{175 \text{ pounds}}$$

$$\begin{aligned} 2 \times 175 &= 50 \times n \\ 350 &= 50 \times n \\ 350 \div 50 &= n \\ 7 &= n \end{aligned}$$

A 175-pound person requires 7 ounces of medication.

You Try It 10

Three tablespoons of a liquid plant fertilizer are to be added to every 4 gallons of water. How many tablespoons of fertilizer are required for 10 gallons of water?

Your strategy

Your solution

Solutions on p. S12

4.3 Exercises

Objective A

Determine whether the proportion is true or not true.

1. $\frac{4}{8} = \frac{10}{20}$

2. $\frac{39}{48} = \frac{13}{16}$

3. $\frac{7}{8} = \frac{11}{12}$

4. $\frac{15}{7} = \frac{17}{8}$

5. $\frac{27}{8} = \frac{9}{4}$

6. $\frac{3}{18} = \frac{4}{19}$

7. $\frac{45}{135} = \frac{3}{9}$

8. $\frac{3}{4} = \frac{54}{72}$

9. $\frac{16}{3} = \frac{48}{9}$

10. $\frac{15}{5} = \frac{3}{1}$

11. $\frac{7}{40} = \frac{7}{8}$

12. $\frac{9}{7} = \frac{6}{5}$

13. $\frac{\text{50 miles}}{\text{2 gallons}} = \frac{\text{25 miles}}{\text{1 gallon}}$

14. $\frac{\text{16 feet}}{\text{10 seconds}} = \frac{\text{24 feet}}{\text{15 seconds}}$

15. $\frac{\text{6 minutes}}{\text{5 cents}} = \frac{\text{30 minutes}}{\text{25 cents}}$

16. $\frac{\text{16 pounds}}{\text{12 days}} = \frac{\text{20 pounds}}{\text{14 days}}$

17. $\frac{\$15}{\text{4 pounds}} = \frac{\$45}{\text{12 pounds}}$

18. $\frac{\text{270 trees}}{\text{6 acres}} = \frac{\text{90 trees}}{\text{2 acres}}$

19. $\frac{\text{300 feet}}{\text{4 rolls}} = \frac{\text{450 feet}}{\text{7 rolls}}$

20. $\frac{\text{1 gallon}}{\text{4 quarts}} = \frac{\text{7 gallons}}{\text{28 quarts}}$

21. $\frac{\$65}{\text{5 days}} = \frac{\$26}{\text{2 days}}$

22. $\frac{\text{80 miles}}{\text{2 hours}} = \frac{\text{110 miles}}{\text{3 hours}}$

23. $\frac{\text{7 tiles}}{\text{4 feet}} = \frac{\text{42 tiles}}{\text{20 feet}}$

24. $\frac{\text{15 feet}}{\text{3 yards}} = \frac{\text{90 feet}}{\text{18 yards}}$

Objective B

Solve. Round to the nearest hundredth.

25. $\frac{n}{4} = \frac{6}{8}$

26. $\frac{n}{7} = \frac{9}{21}$

27. $\frac{12}{18} = \frac{n}{9}$

28. $\frac{7}{21} = \frac{35}{n}$

29. $\frac{6}{n} = \frac{24}{36}$

30. $\frac{3}{n} = \frac{15}{10}$

31. $\frac{n}{45} = \frac{17}{135}$

32. $\frac{9}{4} = \frac{18}{n}$

33. $\frac{n}{6} = \frac{2}{3}$

34. $\frac{5}{12} = \frac{n}{144}$

35. $\frac{n}{5} = \frac{7}{8}$

36. $\frac{4}{n} = \frac{9}{5}$

37. $\frac{n}{11} = \frac{32}{4}$

38. $\frac{3}{4} = \frac{8}{n}$

39. $\frac{5}{12} = \frac{n}{8}$

40. $\frac{36}{20} = \frac{12}{n}$

41. $\frac{n}{15} = \frac{21}{12}$

42. $\frac{40}{n} = \frac{15}{8}$

43. $\frac{32}{n} = \frac{1}{3}$

44. $\frac{5}{8} = \frac{42}{n}$

45. $\frac{18}{11} = \frac{16}{n}$

46. $\frac{25}{4} = \frac{n}{12}$

47. $\frac{28}{8} = \frac{12}{n}$

48. $\frac{n}{30} = \frac{65}{120}$

49. $\frac{0.3}{5.6} = \frac{n}{25}$

50. $\frac{1.3}{16} = \frac{n}{30}$

51. $\frac{0.7}{9.8} = \frac{3.6}{n}$

52. $\frac{1.9}{7} = \frac{13}{n}$

Objective C Application Problems

Solve. Round to the nearest hundredth.

53. A 6-ounce package of Puffed Wheat contains 600 calories. How many calories are in a 0.5-ounce serving of the cereal?

54. A car travels 70.5 miles on 3 gallons of gas. Find the distance that the car can travel on 14 gallons of gas.

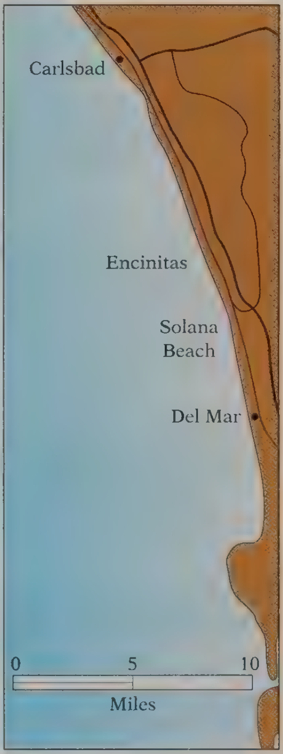
55. Ron Stokes uses 2 pounds of fertilizer for every 100 square feet of lawn for landscape maintenance. At this rate, how many pounds of fertilizer did he use on a lawn that measures 3500 square feet?

56. A nursery prepares a liquid plant food by adding 1 gallon of water for each 2 ounces of plant food. At this rate, how many gallons of water are required for 25 ounces of plant food?

57. A manufacturer of baseball equipment makes 4 aluminum bats for every 15 bats made from wood. On a day when 100 aluminum bats are made, how many wooden bats are produced?

58. A brick wall 20 feet in length contains 1040 bricks. At the same rate, how many bricks would it take to build a wall 48 feet in length?

59. The scale on the map at the right is "1.25 inches equals 10 miles." Find the distance between Carlsbad and Del Mar, which are 2 inches apart on the map.



60. The scale on the plans for a new house is "1 inch equals 3 feet." Find the length and width of a room that measures 5 inches by 8 inches on the drawing.

61. The dosage for a medication is $\frac{1}{3}$ ounce for every 40 pounds of body weight. At this rate, how many ounces of medication should a physician prescribe for a patient who weighs 150 pounds?

62. A bank requires a monthly payment of $33.45 on a $2500 loan. At the same rate, find the monthly payment on a $10,000 loan.

63. A pre-election survey showed that 2 out of every 3 eligible voters would cast ballots in the county election. At this rate, how many people in a county of 240,000 eligible voters would vote in the election?

64. A paint manufacturer suggests using 1 gallon of paint for every 400 square feet of a wall. At this rate, how many gallons of paint would be required for a room that has 1400 square feet of wall?

65. A 60-year-old male can obtain $10,000 of life insurance for $35.35 per month. At this rate, what is the monthly cost of $50,000 of life insurance?

66. Suppose a computer chip manufacturer knows from experience that in an average production run of 2000 circuit boards, 60 will be defective. How many defective circuit boards can be expected from a run of 25,000 circuit boards?

67. You own 240 shares of a computer stock. The company declares a stock split of 5 shares for every 3 owned. How many shares of stock will you own after the stock split?

68. The director of data processing at a college estimates that the ratio of student time to administrative time on a certain computer is 3:2. During a month in which the computer was used 200 hours for administration, how many hours was it used by students?

69. The ratio of weight on the moon to weight on Earth is 1:6. If a bowling ball weighs 16 pounds on Earth, what would it weigh on the moon?

70. When engineers designed a new car, they first built a model of the car. The ratio of the size of a part on the model to the actual size of the part is 2:5. If a door is 1.3 feet long on the model, what is the length of the door on the car?

71. Carlos Capasso owns 50 shares of Texas Utilities that pay dividends of $153. At this rate, what dividend would Carlos receive after buying 300 additional shares of Texas Utilities?

APPLYING THE CONCEPTS

The table at the right shows how each dollar of projected spending by the federal government for a recent year was distributed. The costs for Social Security, health, income security, net interest, and the category education, veterans, and transportation are fixed expenditures. Defense and "Other" represent the only discretionary spending by the government. The projected budget for the year was $1977.6 billion.

How Your Federal Tax Dollar Is Spent	
Social Security	23 cents
Health	21 cents
Income security	14 cents
Net interest	11 cents
Edu., Vets, Transp.	10 cents
Defense	16 cents
Other	5 cents

Source: Tax Foundation, Office of Management and Budget

72. **a.** Is at least one-half of federal spending discretionary spending?
  b. Using a colon, write the ratio of the expenditures for "health" to the discretionary spending.
  c. Find the amount of the budget to be spent on fixed expenditures.
  d. Find the amount of the budget to be spent on Social Security.

73. A survey of voters in a city claimed that 2 people of every 5 who voted cast a ballot in favor of city amendment A and that 3 people of every 4 who voted cast a ballot against amendment A. Is this possible? Explain your answer.

74. Write a word problem that requires solving a proportion to find the answer.

Focus on Problem Solving

Looking for a Pattern

A very useful problem-solving strategy is looking for a pattern.

Problem A legend says that a peasant invented the game of chess and gave it to a very rich king as a present. The king so enjoyed the game that he gave the peasant the choice of anything in the kingdom. The peasant's request was simple: "Place one grain of wheat on the first square, 2 grains on the second square, 4 grains on the third square, 8 on the fourth square, and continue doubling the number of grains until the last square of the chessboard is reached." How many grains of wheat must the king give the peasant?

Solution A chessboard consists of 64 squares. To find the total number of grains of wheat on the 64 squares, we begin by looking at the amount of wheat on the first few squares.

Square 1	Square 2	Square 3	Square 4	Square 5	Square 6	Square 7	Square 8
1	2	4	8	16	32	64	128
1	3	7	15	31	63	127	255

The bottom row of numbers represents the sum of the number of grains of wheat up to and including that square. For instance, the number of grains of wheat on the first 7 squares is $1 + 2 + 4 + 8 + 16 + 32 + 64 = 127$.

One pattern to observe is that the number of grains of wheat on a square can be expressed as a power of 2.

The number of grains on square $n = 2^{n-1}$.

For example, the number of grains on square $7 = 2^{7-1} = 2^6 = 64$.

A second pattern of interest is that **the number *below* a square** (the total number of grains up to and including that square) **is one less than the number of grains of wheat *on the next* square.** For example, the number *below* square 7 is one less than the number *on* square 8 ($128 - 1 = 127$). From this observation, the number of grains of wheat on the first 8 squares is the number on square 8 (128) plus one less than the number on square 8 (127): The total number of grains of wheat on the first 8 squares is $128 + 127 = 255$.

From this observation,

$$\begin{aligned}\text{Number of grains of wheat on the chessboard} &= \text{number of grains on square 64} + \text{one less than the number of grains on square 64}\\ &= 2^{64-1} + (2^{64-1} - 1)\\ &= 2^{63} + 2^{63} - 1 \approx 18{,}000{,}000{,}000{,}000{,}000{,}000\end{aligned}$$

To give you an idea of the magnitude of this number, this is more wheat than has been produced in the world since chess was invented.

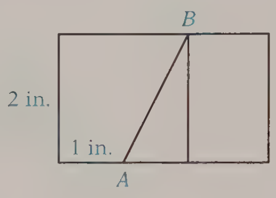
The same king decided to have a banquet in the long banquet room of the palace to celebrate the invention of chess. The king had 50 square tables, and each table could seat only one person on each side. The king pushed the tables together to form one long banquet table. How many people could sit at this table? *Hint:* Try constructing a pattern by using 2 tables, 3 tables, and 4 tables.

Projects and Group Activities

The Golden Ratio

There are certain designs that have been repeated over and over in both art and architecture. One of these involves the **golden rectangle.**

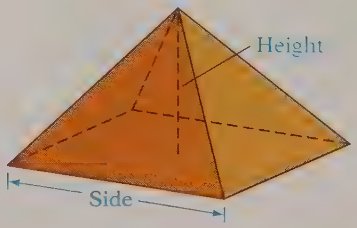
A golden rectangle is drawn at the right. Begin with a square that measures, say, 2 inches on a side. Now measure the distance from *A* to *B*. Place this length along the bottom of the square, starting at *A*. The resulting rectangle is a golden rectangle.



Golden Rectangle

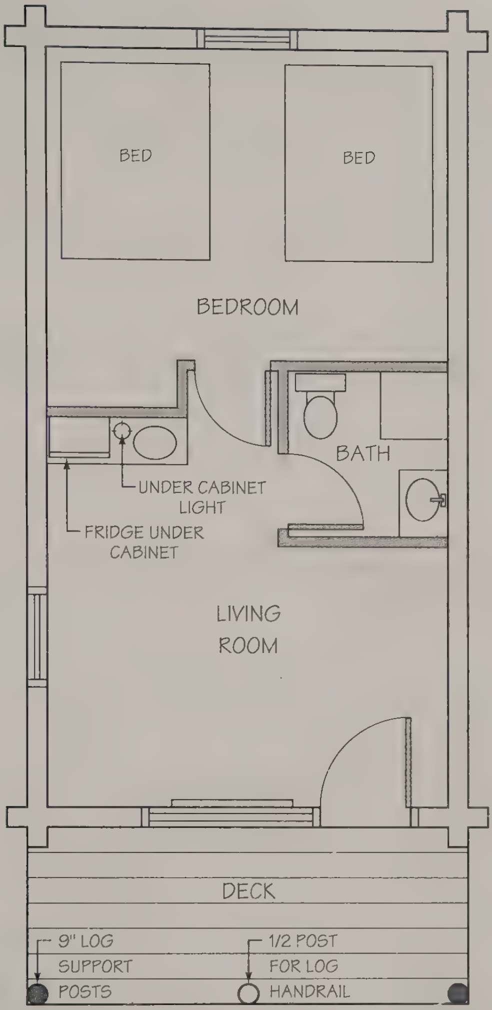
The **golden ratio** is the ratio of the length of the golden rectangle to its width. If you have drawn the rectangle following the procedure above, you will find that the golden ratio is approximately 1.6.

The golden ratio appears in many different situations. Some historians claim that some of the great pyramids of Egypt are based on the golden ratio. The drawing at the right shows the Pyramid of Gizeh, which dates from approximately 2600 B.C. The ratio of the height to a side of the base is approximately 1.6.



1. The canvas of the Mona Lisa painted by Leonardo da Vinci is a golden rectangle. However, there are other instances of the golden rectangle in the painting itself. Do some research on this painting and write a few paragraphs summarizing your findings.
2. What do 3 × 5 and 5 × 8 index cards have to do with the golden rectangle?
3. What does the United Nations Building in New York City have to do with the golden rectangle?
4. When was the Parthenon in Athens, Greece, built? What does the front of that building have to do with the golden rectangle?

Drawing the Floor Plans for a Building


BED
BED
BEDROOM
BATH
UNDER CABINET LIGHT
FRIDGE UNDER CABINET
LIVING ROOM
DECK
9" LOG SUPPORT POSTS
1/2 POST FOR LOG HANDRAIL

The drawing at the left is a sketch of the floor plan for a cabin at a resort in the mountains of Utah. The measurements are missing. Assume that you are the architect and will finish the drawing. You will have to decide the size of the rooms and put in the measurements to scale.

Design a cabin that you would like to own. Select a scale and draw all the rooms to scale.

If you are interested in architecture, visit an architect who is using CAD (computer-aided design) to create a floor plan. Computer technology has revolutionized the field of architectural design.

Small Business and Gross Income

The Ourtown Times
The Town Council Met Monday Night
The new Shopping Mall opened yesterday
"My Mother the Cat" to come back to TV

Do some research on your local newspaper. Answer each of the following questions.

1. What is the newspaper's circulation?
2. How often is an edition of the newspaper published?
3. What is the cost per issue when it is purchased at a newsstand?
4. Is there a special rate for subscribers to the newspaper?
5. What is the cost of a subscription?
6. What is the length of time for which a subscription is paid?
7. How many of the newspaper's readers are subscribers?
8. Use the answers to the above questions to determine the number of copies sold at newsstands.
9. Use the figures that you have gathered to determine the total annual income, or **gross income,** derived from sales of the newspaper.

The U.S. House of Representatives

The framers of the Constitution decided to use a ratio to determine the number of representatives from each state. It was determined that each state would have one representative for every 30,000 citizens, with a minimum of one representative. Congress has changed this ratio over the years, and we now have 435 representatives.

Find the number of representatives from your state. Determine the ratio of citizens to representatives. Also do this for the most populous state and for the least populous state.

You might consider getting information on the number of representatives for each state and the populations of different states via the Internet.

Chapter Summary

Key Words

Quantities such as 8 feet and 60 miles are number quantities written with *units*. [p. 169]

A *ratio* is a comparison of two quantities that have the same units. [p. 169]

A ratio is in *simplest form* when the two numbers that make up the ratio have no common factors. [p. 169]

A *rate* is a comparison of two quantities that have different units. [p. 173]

A rate is in *simplest form* when the numbers that make up the rate have no common factors. [p. 173]

A *unit rate* is a rate in which the number in the denominator is 1. [p. 173]

A *proportion* is an expression of the equality of two ratios or rates. [p. 177]

A proportion is true if the fractions are equal when written in lowest terms. In any true proportion, the "cross products" are equal. [p. 177]

Essential Rules

Ways to Express a Ratio

A ratio can be written in three different ways:

1. As a fraction
2. As two numbers separated by a colon (:)
3. As two numbers separated by the word *to* [p. 169]

To Find Unit Rates

To find unit rates, divide the number in the numerator of the rate by the number in the denominator of the rate. [p. 173]

To Solve a Proportion

One of the numbers in a proportion may be unknown. To solve a proportion, find a number to replace the unknown so that the proportion is true. [p. 178]

Chapter Review

1. Determine whether the proportion is true or not true.
$\frac{2}{9} = \frac{10}{45}$

2. Write the comparison 32 dollars to 80 dollars as a ratio in simplest form using a fraction, a colon (:), and the word *to*.

3. Write "250 miles in 4 hours" as a unit rate.

4. Determine whether the proportion is true or not true.
$\frac{8}{15} = \frac{32}{60}$

5. Solve the proportion.
$\frac{16}{n} = \frac{4}{17}$

6. Write "$300 earned in 40 hours" as a unit rate.

7. Write "$8.75 for 5 pounds" as a unit rate.

8. Write the comparison 8 feet to 28 feet as a ratio in simplest form using a fraction, a colon (:), and the word *to*.

9. Solve the proportion.
$\frac{n}{8} = \frac{9}{2}$

10. Solve the proportion. Round to the nearest hundredth.
$\frac{18}{35} = \frac{10}{n}$

11. Write the comparison 6 inches to 15 inches as a ratio in simplest form using a fraction, a colon (:), and the word *to*.

12. Determine whether the proportion is true or not true.
$\frac{3}{8} = \frac{10}{24}$

13. Write "$15 in 4 hours" as a rate in simplest form.

14. Write "326.4 miles on 12 gallons" as a unit rate.

15. Write the comparison 12 days to 12 days as a ratio in simplest form using a fraction, a colon (:), and the word *to*.

16. Determine whether the proportion is true or not true.
$\frac{5}{7} = \frac{25}{35}$

17. Solve the proportion. Round to the nearest hundredth.

$$\frac{24}{11} = \frac{n}{30}$$

18. Write "100 miles in 3 hours" as a rate in simplest form.

19. In 5 years, the price of a calculator went from $40 to $24. What is the ratio, as a fraction in simplest form, of the decrease in price to the original price?

20. The property tax on a $45,000 home is $900. At the same rate, what is the property tax on a home valued at $120,000?

21. The high temperature during a 24-hour period was 84 degrees, and the low temperature was 42 degrees. Write the ratio, as a fraction in simplest form, of the high temperature to the low temperature for the 24-hour period.

22. The total cost of manufacturing 1000 radios was $36,600. Of the radios made, 24 did not pass inspection. Find the cost per radio that the manufacturer must now assume in pricing the radios. That is, what is the cost per radio of the radios that *did* pass inspection?

23. A brick wall 40 feet in length contains 448 concrete blocks. At the same rate, how many blocks would it take to build a wall that is 120 feet in length?

24. A retail computer store spends $30,000 a year on TV advertising and $12,000 on newspaper advertising. Find the ratio, as a fraction in simplest form, of TV advertising to newspaper advertising.

25. A 15-pound turkey costs $10.20. What is the cost per pound?

26. Mahesh drove 198.8 miles in 3.5 hours. Find the average number of miles he drove per hour.

27. An insurance policy costs $3.87 for every $1000 of insurance. At this rate, what is the cost of $50,000 of insurance?

28. Pascal Hollis purchased 80 shares of stock for $3580. What was the cost per share?

29. Monique uses 1.5 pounds of fertilizer for every 200 square feet of lawn. How many pounds of fertilizer will she have to use on a lawn that measures 3000 square feet?

30. A house had an original value of $80,000, but its value increased to $120,000 in 2 years. Find the ratio, as a fraction in simplest form, of the increase to the original value.

Chapter Test

1. Write "\$22,036.80 earned in 12 months" as a unit rate.

2. Write the comparison 40 miles to 240 miles as a ratio in simplest form using a fraction, a colon (:), and the word *to*.

3. Write "18 supports for every 8 feet" as a rate in simplest form.

4. Determine whether the proportion is true or not true.
$$\frac{40}{125} = \frac{5}{25}$$

5. Write the comparison 12 days to 8 days as a ratio in simplest form using a fraction, a colon (:), and the word *to*.

6. Solve the proportion.
$$\frac{5}{12} = \frac{60}{n}$$

7. Write "256.2 miles on 8.4 gallons of gas" as a unit rate.

8. Write the comparison 27 dollars to 81 dollars as a ratio in simplest form using a fraction, a colon (:), and the word *to*.

9. Determine whether the proportion is true or not true.
$$\frac{5}{14} = \frac{25}{70}$$

10. Solve the proportion.
$$\frac{n}{18} = \frac{9}{4}$$

11. Write "\$81 for 12 boards" as a rate in simplest form.

12. Write the comparison 18 feet to 30 feet as a ratio in simplest form using a fraction, a colon (:), and the word *to*.

13. Fifty shares of a utility stock pay a dividend of \$62.50. At the same rate, find the dividend paid on 500 shares of the utility stock.

14. The average summer temperature in a California desert is 112 degrees. In a city 100 miles away, the average summer temperature is 86 degrees. Write the ratio, as a fraction in simplest form, of the average city temperature to the average desert temperature.

15. A plane travels 2421 miles in 4.5 hours. Find the plane's speed in miles per hour.

16. A research scientist estimates that the human body contains 88 pounds of water for every 100 pounds of body weight. At this rate, estimate the number of pounds of water in a college student who weighs 150 pounds.

17. If 40 feet of lumber costs \$69.20, what is the per-foot cost of the lumber?

18. The dosage of a certain medication is $\frac{1}{4}$ ounce for every 50 pounds of body weight. How many ounces of this medication are required for a person who weighs 175 pounds? Write the answer as a decimal.

19. An automobile sales company spends \$25,000 each month for television advertising and \$40,000 each month for radio advertising. Write, as a fraction in simplest form, the ratio of the cost of radio advertising to the total cost of advertising.

20. The property tax on a house valued at \$175,000 is \$1500. At the same rate, find the property tax on a house valued at \$140,000.

Cumulative Review

1. Subtract: $\begin{array}{r} 20{,}095 \\ -\ 10{,}937 \\ \hline \end{array}$

2. Write $2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$ in exponential notation.

3. Simplify: $4 - (5 - 2)^2 \div 3 + 2$

4. Find the prime factorization of 160.

5. Find the LCM of 9, 12, and 18.

6. Find the GCF of 28 and 42.

7. Write $\frac{40}{64}$ in simplest form.

8. Find $4\frac{7}{15}$ more than $3\frac{5}{6}$.

9. What is $4\frac{5}{9}$ less than $10\frac{1}{6}$?

10. Multiply: $\frac{11}{12} \times 3\frac{1}{11}$

11. Find the quotient of $3\frac{1}{3}$ and $\frac{5}{7}$.

12. Simplify: $\left(\frac{2}{5} + \frac{3}{4}\right) \div \frac{3}{2}$

13. Write 4.0709 in words.

14. Round 2.09762 to the nearest hundredth.

15. Divide: $8.09\overline{)16.0976}$
Round to the nearest thousandth.

16. Convert $0.06\frac{2}{3}$ to a fraction.

17. Write the comparison 25 miles to 200 miles as a ratio in simplest form using a fraction.

18. Write “87 cents for 6 pencils” as a rate in simplest form.

19. Write “250.5 miles on 7.5 gallons of gas” as a unit rate.

20. Solve $\frac{40}{n} = \frac{160}{17}$.

21. A car traveled 457.6 miles in 8 hours. Find the car’s speed in miles per hour.

22. Solve the proportion.
$\frac{12}{5} = \frac{n}{15}$

23. You had \$1024 in your checking account. You then wrote checks for \$192 and \$88. What is your new checking account balance?

24. Malek Khatri buys a tractor for \$22,760. A down payment of \$5000 is required. The balance remaining is paid in 48 equal monthly installments. What is the monthly payment?

25. Yuko is assigned to read a book containing 175 pages. She reads $\frac{2}{5}$ of the book during Thanksgiving vacation. How many pages of the assignment remain to be read?

26. A building contractor bought $2\frac{1}{3}$ acres of land for \$84,000. What was the cost of each acre?

27. Benjamin Eli bought a shirt for \$22.79 and a tie for \$9.59. He used a \$50 bill to pay for the purchases. Find the amount of change.

28. If you earn an annual salary of \$41,619, what is your monthly salary?

29. A soil conservationist estimates that a river bank is eroding at the rate of 3 inches every 6 months. At this rate, how many inches will be eroded in 50 months?

30. The dosage of a certain medication is $\frac{1}{2}$ ounce for every 50 pounds of body weight. How many ounces of this medication are required for a person who weighs 160 pounds? Write the answer as a decimal.

Chapter 5 Percents

Objectives

Section 5.1

A To write a percent as a fraction or a decimal

B To write a fraction or a decimal as a percent

Section 5.2

A To find the amount when the percent and the base are given

B To solve application problems

Section 5.3

A To find the percent when the base and amount are given

B To solve application problems

Section 5.4

A To find the base when the percent and amount are given

B To solve application problems

Section 5.5

A To solve percent problems using proportions

B To solve application problems

The number of children in this kindergarten class represents a percent of the population of a school district. Local populations and estimates of growth or decline are used to predict future job markets in that area. For example, population growth may indicate a need for more teachers in the local schools. The demography **project on page 219** describes different segments of the U.S. population as percents of the total population.

Need help? For on-line student resources, such as section quizzes, visit this textbook's web site at **math.college.hmco.com/students.**

Prep Test

For Exercises 1 to 6, multiply or divide.

1. $19 \times \frac{1}{100}$

2. 23×0.01

3. 0.47×100

4. $0.06 \times 47{,}500$

5. $60 \div 0.015$

6. $8 \div \frac{1}{4}$

7. Multiply $\frac{5}{8} \times 100$. Write the answer as a decimal.

8. Write $\frac{200}{3}$ as a mixed number.

9. Divide $28 \div 16$. Write the answer as a decimal.

Go Figure

A palindrome is a whole number that remains unchanged when its digits are written in reverse order. For example, 818 is a palindrome.

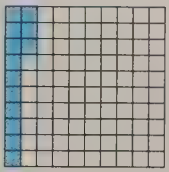
a. Find the smallest three-digit multiple of 6 that is a palindrome.

b. Many nonpalindrome numbers can be converted into palindromes: reverse the digits of the number and add the result to the original; continue until a palindrome is achieved. Using this process, what is the palindrome created from 874?

5.1 Introduction to Percents

Objective A To write a percent as a fraction or a decimal

Percent means "parts of 100." In the figure at the right, there are 100 parts. Because 13 of the 100 parts are shaded, 13% of the figure is shaded.



In most applied problems involving percents, it is necessary either to rewrite a percent as a fraction or a decimal or to rewrite a fraction or a decimal as a percent.

To write a percent as a fraction, remove the percent sign and multiply by $\frac{1}{100}$.

$$13\% = 13 \times \frac{1}{100} = \frac{13}{100}$$

To rewrite a percent as a decimal, remove the percent sign and multiply by 0.01.

$$13\% = 13 \times 0.01 = 0.13$$

Move the decimal point two places to the left. Then remove the percent sign.

Example 1 Write 120% as a fraction and as a decimal.

Solution

$$120\% = 120 \times \frac{1}{100} = \frac{120}{100} = 1\frac{1}{5}$$

$$120\% = 120 \times 0.01 = 1.2$$

Note that percents larger than 100 are greater than 1.

You Try It 1 Write 125% as a fraction and as a decimal.

Your solution

Example 2 Write $16\frac{2}{3}\%$ as a fraction.

Solution

$$16\frac{2}{3}\% = 16\frac{2}{3} \times \frac{1}{100} = \frac{50}{3} \times \frac{1}{100} = \frac{50}{300} = \frac{1}{6}$$

You Try It 2 Write $33\frac{1}{3}\%$ as a fraction.

Your solution

Example 3 Write 0.5% as a decimal.

Solution $0.5\% = 0.5 \times 0.01 = 0.005$

You Try It 3 Write 0.25% as a decimal.

Your solution

Solutions on p. S12

Objective D To write a fraction or a decimal as a percent

A fraction or a decimal can be written as a percent by multiplying by 100%.

Write $\frac{3}{8}$ as a percent.

$$\frac{3}{8} = \frac{3}{8} \times 100\% = \frac{3}{8} \times \frac{100}{1}\% = \frac{300}{8}\% = 37\frac{1}{2}\% \text{ or } 37.5\%$$

Write 0.37 as a percent.

$$0.37 = 0.37 \times 100\% = 37\%$$

Move the decimal point two places to the right. Then write the percent sign.

Example 4 Write 0.015 as a percent.

Solution $0.015 = 0.015 \times 100\%$
$= 1.5\%$

You Try It 4 Write 0.048 as a percent.

Your solution

Example 5 Write 2.15 as a percent.

Solution $2.15 = 2.15 \times 100\% = 215\%$

You Try It 5 Write 3.67 as a percent.

Your solution

Example 6 Write $0.33\frac{1}{3}$ as a percent.

Solution $0.33\frac{1}{3} = 0.33\frac{1}{3} \times 100\%$
$= 33\frac{1}{3}\%$

You Try It 6 Write $0.62\frac{1}{2}$ as a percent.

Your solution

Example 7 Write $\frac{2}{3}$ as a percent.
Write the remainder in fractional form.

Solution $\frac{2}{3} = \frac{2}{3} \times 100\% = \frac{200}{3}\%$
$= 66\frac{2}{3}\%$

You Try It 7 Write $\frac{5}{6}$ as a percent.
Write the remainder in fractional form.

Your solution

Example 8 Write $2\frac{2}{7}$ as a percent.
Round to the nearest tenth.

Solution $2\frac{2}{7} = \frac{16}{7} = \frac{16}{7} \times 100\%$
$= \frac{1600}{7}\% \approx 228.6\%$

You Try It 8 Write $1\frac{4}{9}$ as a percent.
Round to the nearest tenth.

Your solution

Solutions on pp. S12–S13

5.1 Exercises

Objective A

Write as a fraction and as a decimal.

1. 25%
2. 40%
3. 130%
4. 150%
5. 100%
6. 87%
7. 73%
8. 45%
9. 383%
10. 425%
11. 70%
12. 55%
13. 88%
14. 64%
15. 32%
16. 18%

Write as a fraction.

17. $66\frac{2}{3}\%$
18. $12\frac{1}{2}\%$
19. $83\frac{1}{3}\%$
20. $3\frac{1}{8}\%$
21. $11\frac{1}{9}\%$
22. $\frac{3}{8}\%$
23. $45\frac{5}{11}\%$
24. $15\frac{3}{8}\%$
25. $4\frac{2}{7}\%$
26. $5\frac{3}{4}\%$
27. $6\frac{2}{3}\%$
28. $8\frac{2}{3}\%$

Write as a decimal.

29. 6.5%
30. 9.4%
31. 12.3%
32. 16.7%
33. 0.55%
34. 0.45%
35. 8.25%
36. 6.75%
37. 5.05%
38. 3.08%
39. 2%
40. 7%
41. 80.4%
42. 36.2%
43. 4.9%

Objective B

Write as a percent.

44. 0.16
45. 0.73
46. 0.05
47. 0.01
48. 1.07
49. 2.94
50. 0.004
51. 0.006
52. 1.012
53. 3.106
54. 0.8
55. 0.7

Write as a percent. Round to the nearest tenth.

56. $\frac{27}{50}$

57. $\frac{37}{100}$

58. $\frac{1}{3}$

59. $\frac{2}{5}$

60. $\frac{5}{8}$

61. $\frac{1}{8}$

62. $\frac{1}{6}$

63. $1\frac{1}{2}$

64. $\frac{7}{40}$

65. $1\frac{2}{3}$

66. $1\frac{7}{9}$

67. $\frac{7}{8}$

Write as a percent. Write the remainder in fractional form.

68. $\frac{15}{50}$

69. $\frac{12}{25}$

70. $\frac{7}{30}$

71. $\frac{1}{3}$

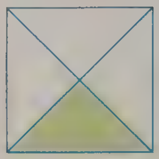
72. $2\frac{3}{8}$

73. $1\frac{2}{3}$

74. $2\frac{1}{6}$

75. $\frac{7}{8}$

76. Write the part of the square that is shaded as a fraction, as a decimal, and as a percent. Write the part of the square that is not shaded as a fraction, as a decimal, and as a percent.



APPLYING THE CONCEPTS

77. Determine whether the statement is true or false. If the statement is false, give an example to show that the statement is false.
  a. Multiplying a number by a percent always decreases the number.
  b. Dividing by a percent always increases the number.
  c. The word *percent* means "per hundred."

78. Explain in your own words how to change a percent to a decimal and how to change a decimal to a percent.

79. A sale on computers advertised $\frac{1}{3}$ off the regular price. What percent of the regular price does this represent?

80. A suit was priced at 50% off the regular price. What fraction of the regular price does this represent?

81. If $\frac{2}{5}$ of the population voted in an election, what percent of the population did not vote?

82. How can you recognize a fraction that represents a number that is less than 1%?

5.2 Percent Equations: Part I

Objective A To find the amount when the percent and the base are given

A real estate broker receives a payment that is 4% of an $85,000 sale. To find the amount the broker receives requires answering the question "4% of $85,000 is what?"

This sentence can be written using mathematical symbols and then solved for the unknown number.

$$\begin{array}{ccccc} 4\% & \text{of} & \$85{,}000 & \text{is} & \text{what?} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \boxed{\begin{array}{c}\text{percent}\\ 4\%\end{array}} & \times & \boxed{\begin{array}{c}\text{base}\\ 85{,}000\end{array}} & = & \boxed{\begin{array}{c}\text{amount}\\ n\end{array}} \end{array}$$

of is written as × (times)
is is written as = (equals)
what is written as n (the unknown number)

$$\begin{aligned} 0.04 \times 85{,}000 &= n \\ 3400 &= n \end{aligned}$$

Note that the percent is written as a decimal.

The broker receives a payment of $3400.

The solution was found by solving the basic percent equation for amount.

The Basic Percent Equation

$$\boxed{\text{Percent}} \times \boxed{\text{base}} = \boxed{\text{amount}}$$

In most cases, the percent is written as a decimal before the basic percent equation is solved. However, some percents are more easily written as a fraction than as a decimal. For example,

memorize

$$33\frac{1}{3}\% = \frac{1}{3} \qquad 66\frac{2}{3}\% = \frac{2}{3} \qquad 16\frac{2}{3}\% = \frac{1}{6} \qquad 83\frac{1}{3}\% = \frac{5}{6}$$

Example 1 Find 5.7% of 160.

Solution

$n = 0.057 \times 160$
$n = 9.12$

Note that the words "what is" are missing from the problem but are implied by the word "Find."

You Try It 1 Find 6.3% of 150.

Your solution

Example 2 What is $33\frac{1}{3}\%$ of 90?

Solution

$n = \frac{1}{3} \times 90$
$n = 30$

You Try It 2 What is $16\frac{2}{3}\%$ of 66?

Your solution

Solutions on p. S13

Objective B To solve application problems

Solving percent problems requires identifying the three elements of the basic percent equation. Recall that these three parts are the *percent,* the *base,* and the *amount.* Usually the base follows the phrase "percent of."

During a recent year, charitable contributions totaled $190 billion. The table at the right shows where this money was donated. Use these data for Example 3 and You Try It 3.

Recipients	*Percent of Total Charitable Contributions*
Religion	43%
Education	14%
Health	9%
Other	34%

Source: Giving USA 2000/AAFRC Trust for Philanthropy

Example 3
How much money was donated to religious organizations?

Strategy
To find the amount donated, write and solve the basic percent equation using n to represent the amount. The percent is 43%. The base is $190 billion.

Solution

$$43\% \times 190 = n$$
$$0.43 \times 190 = n$$
$$81.7 = n$$

$87.1 billion was donated to religious organizations.

You Try It 3
Determine how much money was donated to educational organizations.

Your strategy

Your solution

Example 4
A quality control inspector found that 1.2% of 2500 telephones inspected were defective. How many telephones inspected were not defective?

Strategy
To find the number of nondefective phones:

- Find the number of defective phones. Write and solve the basic percent equation using n to represent the number of defective phones (amount). The percent is 1.2% and the base is 2500.
- Subtract the number of defective phones from the number of phones inspected (2500).

Solution

$$1.2\% \times 2500 = n$$
$$0.012 \times 2500 = n$$
$$30 = n \text{ defective phones}$$

$$2500 - 30 = 2470$$

2470 telephones were not defective.

You Try It 4
An electrician's hourly wage was $23.50 before an 8% raise. What is the new hourly wage?

Your strategy

Your solution

Solutions on p. S13

5.2 Exercises

Objective A

Solve.

1. 8% of 100 is what?
2. 16% of 50 is what?
3. 27% of 40 is what?
4. 52% of 95 is what?
5. 0.05% of 150 is what?
6. 0.075% of 625 is what?
7. 125% of 64 is what?
8. 210% of 12 is what?
9. Find 10.7% of 485.
10. Find 12.8% of 625.
11. What is 0.25% of 3000?
12. What is 0.06% of 250?
13. 80% of 16.25 is what?
14. 26% of 19.5 is what?
15. What is $1\frac{1}{2}\%$ of 250?
16. What is $5\frac{3}{4}\%$ of 65?
17. $16\frac{2}{3}\%$ of 120 is what?
18. $83\frac{1}{3}\%$ of 246 is what?
19. What is $33\frac{1}{3}\%$ of 630?
20. What is $66\frac{2}{3}\%$ of 891?
21. Which is larger: 5% of 95, or 75% of 6?
22. Which is larger: 112% of 5, or 0.45% of 800?
23. Which is smaller: 79% of 16, or 20% of 65?
24. Which is smaller: 15% of 80, or 95% of 15?
25. Which is smaller: 2% of 1500, or 72% of 40?
26. Which is larger: 22% of 120, or 84% of 32?
27. Find 31.294% of 82,460.
28. Find 123.94% of 275,976.

Objective B *Application Problems*

29. Approximately 30% of the 44 million people in the United States who do not have health insurance are between the ages of 18 and 24. (Source: Census Bureau) About how many people in the United States aged 18 to 24 do not have health insurance?

30. During January of 2000, approximately 28.4% of the total auto industry sales of 1,212,321 vehicles were General Motors sales. (Source: Autodata) Find the approximate number of vehicles sold by General Motors in January of 2000.

During the first quarter of 2000, approximately 11 million personal computers were sold in the United States. The table at the right shows the leading PC makers and their percents of those sales. Use these data for Exercises 31 and 32.

PC Makers	*Percent of Sales*
Dell	17%
Compaq	16%
Hewlett-Packard	13%
Gateway	9%

GartnerGroup's Dataquest, April 2000

31. How many of the 11 million personal computers were sold by Dell?

32. How many more personal computers were sold by Compaq than by Gateway? Give the answer in thousands.

33. A sales tax of 6% of the cost of a car was added to the purchase price of $19,500.
a. How much was the sales tax?
b. What is the total cost of the car, including sales tax?

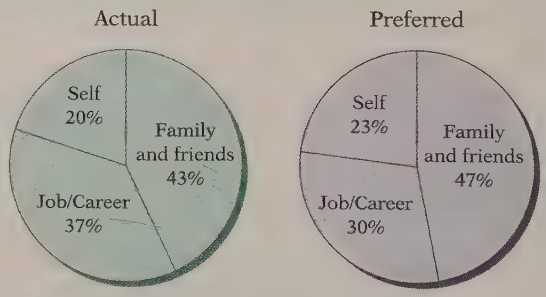
34. During the packaging process for oranges, spoiled oranges are discarded by an inspector. In one day an inspector found that 4.8% of the 20,000 pounds of oranges inspected were spoiled.
a. How many pounds of oranges were spoiled?
b. How many pounds of oranges were not spoiled?

35. In 1995, Americans ate an average of 236 eggs during the year. In 2000, that number increased by approximately 9%. (Source: USDA Economic Research Service)
a. Find the increase in the number of eggs eaten annually from 1995 to 2000. Round to the nearest whole number.
b. Americans ate an average of how many eggs during 2000? 1995 was + [illegible] 2000

36. Funtimes Amusement Park has 550 employees and must hire an additional 22% for the vacation season. What is the total number of employees needed for the vacation season?

APPLYING THE CONCEPTS

The two circle graphs at the right show how surveyed employees actually spend their time and the way they would prefer to spend their time. Assuming that employees have 112 hours a week of time that is not spent sleeping, answer Exercises 37 to 39. Round to the nearest tenth of an hour.



Source: WSJ Supplement, Work & Family; from *Families and Work Institute*

37. What is the actual number of hours per week that employees spend with family and friends?

38. What is the number of hours per week that employees would prefer to spend on job/career?

39. What is the difference between the number of hours an employee preferred to spend on self and the actual amount of time the employee spent on self?

5.3 Percent Equations: Part II

Objective A

To find the percent when the base and amount are given

A recent promotional game at a grocery store listed the probability of winning a prize as "1 chance in 2." A percent can be used to describe the chance of winning. This requires answering the question "What percent of 2 is 1?"

The chance of winning can be found by solving the basic percent equation for *percent*.

$$\begin{array}{ccccc} \text{What percent} & \text{of} & 2 & \text{is} & 1? \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \boxed{\begin{array}{c}\text{percent}\\ n\end{array}} & \times & \boxed{\begin{array}{c}\text{base}\\ 2\end{array}} & = & \boxed{\begin{array}{c}\text{amount}\\ 1\end{array}} \end{array}$$

$$\begin{aligned} n \times 2 &= 1 \\ n &= 1 \div 2 \\ n &= 0.5 \\ n &= 50\% \end{aligned}$$

- **The solution must be written as a percent to answer the question.**

There is a 50% chance of winning a prize.

Point of Interest

According to the U.S. Department of Agriculture, of the 356 billion pounds of food produced in the United States annually, approximately 96 billion pounds are wasted. That is approximately 27% of all the food produced in the United States.

Example 1 What percent of 40 is 30?

Solution

$$\begin{aligned} n \times 40 &= 30 \\ n &= 30 \div 40 \\ n &= 0.75 \\ n &= 75\% \end{aligned}$$

You Try It 1 What percent of 32 is 16?

Your solution

Example 2 What percent of 12 is 27?

Solution

$$\begin{aligned} n \times 12 &= 27 \\ n &= 27 \div 12 \\ n &= 2.25 \\ n &= 225\% \end{aligned}$$

You Try It 2 What percent of 15 is 48?

Your solution

Example 3 25 is what percent of 75?

Solution

$$\begin{aligned} 25 &= n \times 75 \\ 25 \div 75 &= n \\ \frac{1}{3} &= n \\ 33\frac{1}{3}\% &= n \end{aligned}$$

You Try It 3 30 is what percent of 45?

Your solution

Solutions on p. S13

Objective B To solve application problems

To solve percent problems, remember that it is necessary to identify the percent, base, and amount. Usually the base follows the phrase "percent of."

Example 4

The monthly house payment for the Kaminski family is \$787.50. What percent of the Kaminskis' monthly income of \$3750 is the house payment?

Strategy

To find what percent of the income the house payment is, write and solve the basic percent equation using n to represent the percent. The base is \$3750 and the amount is \$787.50.

Solution

$$\begin{aligned} n \times 3750 &= 787.50 \\ n &= 787.50 \div 3750 \\ n &= 0.21 = 21\% \end{aligned}$$

The house payment is 21% of the monthly income.

You Try It 4

Tomo Nagata had an income of \$33,500 and paid \$5025 in income tax. What percent of the income is the income tax?

Your strategy

Your solution

Example 5

On one Monday night, 31.39 million of the approximately 40.76 million households watching television were not watching David Letterman. What percent of these households were watching David Letterman? Round to the nearest percent.

Strategy

To find the percent of households watching David Letterman:

- Subtract to find the number of households that were watching David Letterman (40.76 million − 31.39 million).
- Write and solve the basic percent equation using n to represent the percent. The base is 40.76, and the amount is the number of households watching David Letterman.

Solution

40.76 million − 31.39 million = 9.37 million

9.37 million households watching David Letterman

$$\begin{aligned} n \times 40.76 &= 9.37 \\ n &= 9.37 \div 40.76 \\ n &\approx 0.23 \end{aligned}$$

Approximately 23% of the households were watching David Letterman.

You Try It 5

According to the U.S. Department of Defense, of the 518,921 enlisted personnel in the U.S. Army in 1950, 512,370 people were men. What percent of the enlisted personnel in the U.S. Army in 1950 were women? Round to the nearest tenth of a percent.

Your strategy

Your solution

Solutions on p. S13

5.3 Exercises

Objective A

Solve.

1. What percent of 75 is 24?

2. What percent of 80 is 20?

3. 15 is what percent of 90?

4. 24 is what percent of 60?

5. What percent of 12 is 24?

6. What percent of 6 is 9?

7. What percent of 16 is 6?

8. What percent of 24 is 18?

9. 18 is what percent of 100?

10. 54 is what percent of 100?

11. 5 is what percent of 2000?

12. 8 is what percent of 2500?

13. What percent of 6 is 1.2?

14. What percent of 2.4 is 0.6?

15. 16.4 is what percent of 4.1?

16. 5.3 is what percent of 50?

17. 1 is what percent of 40?

18. 0.3 is what percent of 20?

19. What percent of 48 is 18?

20. What percent of 11 is 88?

21. What percent of 2800 is 7?

22. What percent of 400 is 12?

23. 4.2 is what percent of 175?

24. 41.79 is what percent of 99.5?

25. What percent of 86.5 is 8.304?

26. What percent of 1282.5 is 2.565?

Objective B *Application Problems*

27. Seven in ten couples disagree about financial issues. (Source: Yankelovich Partners for Lutheran Brotherhood) What percent of couples disagree about financial matters?

28. In a survey, 1236 adults nationwide were asked, "What irks you most about the actions of other motorists?" The response "tailgaters" was given by 293 people. (Source: Reuters/Zogby) What percent of those surveyed were most irked by tailgaters? Round to the nearest tenth of a percent.

29. According to the U.S. Department of Agriculture, of the 63 billion pounds of vegetables produced in the United States in one year, 16 billion pounds were wasted. What percent of vegetables produced were wasted? Round to the nearest tenth of a percent.

30. In a recent year, 2400 billion cigarettes were produced. China National Tobacco Corporation produced the largest number, approximately 590.4 billion. (Source: World Tobacco File 1998–2000) What percent of the total cigarette production did China National Tobacco Corporation produce?

31. The typical American household spends \$1355 a year on energy utilities. Of this amount, approximately \$81.30 is spent on lighting. (Source: Department of Energy, Owens Corning) What percent of the total amount spent on energy utilities is spent on lighting?

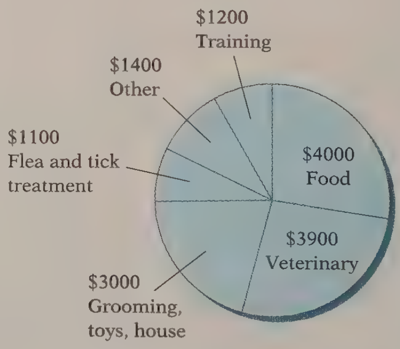
32. To receive a license to sell insurance, an insurance account executive must answer correctly 70% of the 250 questions on a test. Nicholas Mosley answered 177 questions correctly. Did he pass the test?

33. On average, an employee in the United States spends 3.1 hours per workday at a computer. (Source: John H. Heldrich Center for Workforce Development, Rutgers University; Center for Survey Research and Analysis, University of Connecticut; *Work Trends*, February 10, 2000) What percent of the workday is not spent at a computer? Use an 8-hour workday.

34. In a test of the breaking strength of concrete slabs for freeway construction, 3 of the 200 slabs tested did not meet safety requirements. What percent of the slabs did meet safety requirements?

APPLYING THE CONCEPTS

The graph at the right shows several categories of average lifetime costs of dog ownership. Use this graph for Exercises 35 and 36. Round answers to the nearest tenth of a percent.



35. What percent of the total amount is spent on food?

36. What percent of the total is spent on veterinary care?

37. Public utility companies will provide consumers with an analysis of their energy bills. For one customer, it was determined that \$76 of a total bill of \$134 was spent for home heating. What percent (to the nearest tenth) of the total bill was for home heating?

38. The Fun in the Sun organization claims to have taken a survey of 350 people, asking them to give their favorite outdoor temperature for hiking. The results are given in the table at the right. Explain why these results are not possible.

Favorite Temperature	*Percent*
Greater than 90	5%
80–89	28%
70–79	35%
60–69	32%
Below 60	13%

5.4 Percent Equations: Part III

Objective A To find the base when the percent and amount are given

Point of Interest

In 2000, food and drink sales at U.S. restaurants totaled approximately $376 billion. This was a 213% increase over the sales in 1980. (Source: National Restaurant Association)

In 1780, the population of Virginia was approximately 538,000. This was 19% of the total population of the United States at that time. To find the total population at that time, you must answer the question "19% of what number is 538,000?"

The population of the United States in 1780 can be found by solving the basic percent equation for the base.

$$\underset{\boxed{\begin{matrix}\text{percent}\\ 19\%\end{matrix}}}{\underset{\downarrow}{19\%}} \quad \text{of} \quad \underset{\boxed{\begin{matrix}\text{base}\\ n\end{matrix}}}{\underset{\downarrow}{\text{what}}} \quad \text{is} \quad \underset{\boxed{\begin{matrix}\text{amount}\\ 538{,}000\end{matrix}}}{\underset{\downarrow}{538{,}000?}}$$

$$\begin{aligned} 0.19 \times n &= 538{,}000 \\ n &= 538{,}000 \div 0.19 \\ n &\approx 2{,}832{,}000 \end{aligned}$$

The population of the United States in 1780 was approximately 2,832,000.

Example 1 18% of what is 900?

Solution

$$\begin{aligned} 0.18 \times n &= 900 \\ n &= 900 \div 0.18 \\ n &= 5000 \end{aligned}$$

You Try It 1 86% of what is 215?

Your solution

Example 2 30 is 1.5% of what?

Solution

$$\begin{aligned} 30 &= 0.015 \times n \\ 30 \div 0.015 &= n \\ 2000 &= n \end{aligned}$$

You Try It 2 15 is 2.5% of what?

Your solution

Example 3 $33\frac{1}{3}\%$ of what is 7?

Solution

$$\begin{aligned} \frac{1}{3} \times n &= 7 \\ n &= 7 \div \frac{1}{3} \\ n &= 21 \end{aligned}$$

- **Note that the percent is written as a fraction.**

You Try It 3 $16\frac{2}{3}\%$ of what is 5?

Your solution

Solutions on pp. S13–S14

Objective B To solve application problems

To solve percent problems, it is necessary to identify the percent, base, and amount. Usually the base follows the phrase "percent of."

Example 4

A business office bought a used copy machine for $900, which was 75% of the original cost. What was the original cost of the copier?

Strategy

To find the original cost of the copier, write and solve the basic percent equation, using n to represent the original cost (base). The percent is 75% and the amount is $900.

Solution

$$\begin{aligned} 75\% \times n &= 900 \\ 0.75 \times n &= 900 \\ n &= 900 \div 0.75 \\ n &= 1200 \end{aligned}$$

The original cost of the copier was $1200.

You Try It 4

A used car has a value of $10,458, which is 42% of the car's original value. What was the car's original value?

Your strategy

Your solution

Example 5

A carpenter's wage this year is $26.40 per hour, which is 110% of last year's wage. What was the increase in the hourly wage over last year?

Strategy

To find the increase in the hourly wage over last year:

- Find last year's wage. Write and solve the basic percent equation, using n to represent last year's wage (base). The percent is 110% and the amount is $26.40.
- Subtract last year's wage from this year's wage (26.40).

Solution

$$\begin{aligned} 110\% \times n &= 26.40 \\ 1.10 \times n &= 26.40 \\ n &= 26.40 \div 1.10 \\ n &= 24.00 \end{aligned}$$

- Last year's wage

$$26.40 - 24.00 = 2.40$$

The increase in the hourly wage was $2.40.

You Try It 5

Chang's Sporting Goods has a tennis racket on sale for $89.60, which is 80% of the original price. What is the difference between the original price and the sale price?

Your strategy

Your solution

Solutions on p. S14

5.4 Exercises

Objective A

Solve. Round to the nearest hundredth.

1. 12% of what is 9?

2. 38% of what is 171?

3. 8 is 16% of what?

4. 54 is 90% of what?

5. 10 is 10% of what?

6. 37 is 37% of what?

7. 30% of what is 25.5?

8. 25% of what is 21.5?

9. 2.5% of what is 30?

10. 10.4% of what is 52?

11. 125% of what is 24?

12. 180% of what is 21.6?

13. 18 is 240% of what?

14. 24 is 320% of what?

15. 4.8 is 15% of what?

16. 87.5 is 50% of what?

17. 25.6 is 12.8% of what?

18. 45.014 is 63.4% of what?

19. 0.7% of what is 0.56?

20. 0.25% of what is 1?

21. 30% of what is 2.7?

22. 78% of what is 3.9?

23. 84 is $16\frac{2}{3}\%$ of what?

24. 120 is $33\frac{1}{3}\%$ of what?

25. $66\frac{2}{3}\%$ of what is 72?

26. $83\frac{1}{3}\%$ of what is 13.5?

Objective B *Application Problems*

27. Of the travelers who, during a recent year, allowed their children to miss school to go along on a trip, approximately 1.738 million allowed their children to miss school for more than a week. This represented 11% of the travelers who allowed their children to miss school. (Source: Travel Industry Association) About how many travelers allowed their children to miss school to go along on a trip?

28. In the United States today, 23.1% of the women and 27.5% of the men have attained a bachelor's or graduate degree. (Source: Census Bureau) How many women in the United States have earned a bachelor's or graduate degree?

29. The average cost of attending a four-year public college in 1999–2000 was $3356. This was 103.4% of the cost for the 1998–1999 school year. (Source: College Board) What was the average cost of attending a four-year public college in 1998–1999? Round to the nearest dollar.

30. The Internal Revenue Service says that the average deduction for medical expenses for taxpayers in the $40,000–$50,000 bracket is $4500. This is 26% of the medical expenses claimed by taxpayers in the over-$200,000 bracket. How much is the average deduction for medical expenses claimed by taxpayers in the over-$200,000 bracket? Round to the nearest thousand.

31. During a quality control test, Micronics found that 24 computer boards were defective. This amount was 0.8% of the computer boards tested.
- **a.** How many computer boards were tested?
- **b.** How many computer boards tested were not defective?

32. Of the calls a directory assistance operator received, 441 were requests for telephone numbers listed in the current directory. This accounted for 98% of the calls for assistance that the operator received.
- **a.** How many calls did the operator receive?
- **b.** How many telephone numbers requested were not listed in the current directory?

33. In the New Hampshire presidential primary election on February 1, 2000, Al Gore received 76,897 votes. This represented approximately 52% of the Democratic votes cast. (Source: www.state.nh.us) How many of the Democratic votes were not for Al Gore?

APPLYING THE CONCEPTS

The table at the right contains nutrition information about a breakfast cereal. Solve Exercises 34 and 35 using information from this table.

NUTRITION INFORMATION

SERVING SIZE: 1.4 OZ WHEAT FLAKES WITH 0.4 OZ. RAISINS: 39.4 g. ABOUT 1/2 CUP

SERVINGS PER PACKAGE: 14

PERCENTAGE OF U.S. RECOMMENDED DAILY ALLOWANCES (U.S. RDA)	CEREAL & RAISINS	WITH 1/2 CUP VITAMINS A & D SKIM MILK
PROTEIN	4	15
VITAMIN A	15	20
VITAMIN C	**	2
THIAMIN	25	30
RIBOFLAVIN	25	35
NIACIN	25	35
CALCIUM	**	15
IRON	100	100
VITAMIN D	10	25
VITAMIN B_6	25	25
FOLIC ACID..............	25	25
VITAMIN B_{12}	25	30
PHOSPHOROUS.........	10	15
MAGNESIUM	10	20
ZINC	25	30
COPPER..................	2	4

* 2% MILK SUPPLIES AN ADDITIONAL 20 CALORIES, 2 g FAT, AND 10 mg CHOLESTEROL

** CONTAINS LESS THAN 2% OF THE U.S. RDA OF THIS NUTRIENT

34. The recommended daily amount of thiamin for an adult is 1.5 milligrams. Find the amount of thiamin in one serving of this cereal with skim milk.

35. The amount of copper in one serving of cereal with skim milk is 0.08 milligram. Find the recommended daily amount of copper for an adult.

36. Increase a number by 10%. Now decrease the number by 10%. Is the result the original number? Explain.

37. When a company goes bankrupt, another company may offer to purchase the assets of the company for "25 cents on the dollar." What percent of the value of the company is the buyer paying?

5.5 Percent Problems: Proportion Method

Objective A To solve percent problems using proportions

Problems that can be solved using the basic percent equation can also be solved using proportions.

The proportion method is based on writing two ratios. One ratio is the percent ratio, written as $\frac{\text{percent}}{100}$. The second ratio is the amount-to-base ratio, written as $\frac{\text{amount}}{\text{base}}$. These two ratios form the proportion

$$\frac{\textbf{Percent}}{\textbf{100}} = \frac{\textbf{amount}}{\textbf{base}}$$

To use the proportion method, first identify the percent, the amount, and the base (the base usually follows the phrase "percent of").

What is 23% of 45?

$$\begin{aligned} \frac{23}{100} &= \frac{n}{45} \\ 23 \times 45 &= 100 \times n \\ 1035 &= 100 \times n \\ 1035 \div 100 &= n \\ 10.35 &= n \end{aligned}$$

What percent of 25 is 4?

$$\begin{aligned} \frac{n}{100} &= \frac{4}{25} \\ n \times 25 &= 100 \times 4 \\ n \times 25 &= 400 \\ n &= 400 \div 25 \\ n &= 16\% \end{aligned}$$

12 is 60% of what number?

$$\begin{aligned} \frac{60}{100} &= \frac{12}{n} \\ 60 \times n &= 100 \times 12 \\ 60 \times n &= 1200 \\ n &= 1200 \div 60 \\ n &= 20 \end{aligned}$$

Example 1 15% of what is 7? Round to the nearest hundredth.

Solution

$$\begin{aligned} \frac{15}{100} &= \frac{7}{n} \\ 15 \times n &= 100 \times 7 \\ 15 \times n &= 700 \\ n &= 700 \div 15 \\ n &\approx 46.67 \end{aligned}$$

You Try It 1 26% of what is 22? Round to the nearest hundredth.

Your solution

Example 2 30% of 63 is what?

Solution

$$\begin{aligned} \frac{30}{100} &= \frac{n}{63} \\ 30 \times 63 &= 100 \times n \\ 1890 &= 100 \times n \\ 1890 \div 100 &= n \\ 18.90 &= n \end{aligned}$$

You Try It 2 16% of 132 is what?

Your solution

Solutions on p. S14

Objective B To solve application problems

Example 3

An antiques dealer found that 86% of the 250 items that were sold during one month sold for under \$1000. How many items sold for under \$1000?

Strategy

To find the number of items that sold for under \$1000, write and solve a proportion, using n to represent the number of items sold (amount) for less than \$1000. The percent is 86% and the base is 250.

Solution

$$\begin{aligned} \frac{86}{100} &= \frac{n}{250} \\ 86 \times 250 &= 100 \times n \\ 21{,}500 &= 100 \times n \\ 21{,}500 \div 100 &= n \\ 215 &= n \end{aligned}$$

215 items sold for under \$1000.

You Try It 3

Last year it snowed 64% of the 150 days of the ski season at a resort. How many days did it snow?

Your strategy

Your solution

Example 4

In a test of the strength of nylon rope, 5 pieces of the 25 pieces tested did not meet the test standards. What percent of the nylon ropes tested did meet the standards?

Strategy

To find the percent of ropes tested that met the standards:

- Find the number of ropes that met the test standards (25 − 5).
- Write and solve a proportion, using n to represent the percent of ropes that met the test standards. The base is 25. The amount is the number of ropes that met the standards.

Solution

$25 - 5 = 20$ ropes met test standards

$$\begin{aligned} \frac{n}{100} &= \frac{20}{25} \\ n \times 25 &= 100 \times 20 \\ n \times 25 &= 2000 \\ n &= 2000 \div 25 \\ n &= 80\% \end{aligned}$$

80% of the ropes tested did meet the test standards.

You Try It 4

Five ballpoint pens in a box of 200 were found to be defective. What percent of the pens were not defective?

Your strategy

Your solution

Solutions on p. S14

5.5 Exercises

Objective A

Solve.

1. 26% of 250 is what?

2. What is 18% of 150?

3. 37 is what percent of 148?

4. What percent of 150 is 33?

5. 68% of what is 51?

6. 126 is 84% of what?

7. What percent of 344 is 43?

8. 750 is what percent of 50?

9. 82 is 20.5% of what?

10. 2.4% of what is 21?

11. What is 6.5% of 300?

12. 96% of 75 is what?

13. 7.4 is what percent of 50?

14. What percent of 1500 is 693?

15. 50.5% of 124 is what?

16. What is 87.4% of 255?

17. 120% of what is 6?

18. 14 is 175% of what?

19. What is 250% of 18?

20. 325% of 4.4 is what?

21. 33 is 220% of what?

22. 160% of what is 40?

Objective B *Application Problems*

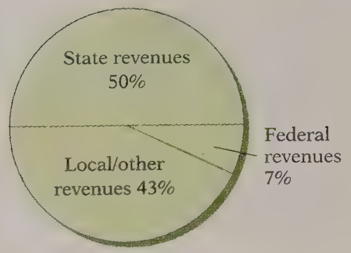
23. A charitable organization spent $2940 for administrative expenses. This amount is 12% of the money it collected. What is the total amount of money that the organization collected?

24. A manufacturer of an anti-inflammatory drug claims that the drug will be effective for 6 hours. An independent testing service determined that the drug was effective for only 80% of the length of time claimed by the manufacturer. Find the length of time the drug will be effective as determined by the testing service.

25. The land area of North America is approximately 9,400,000 square miles. This represents approximately 16% of the total land area of the world. What is the approximate total land area of the world?

26. The Rincon Fire Department received 24 false alarms out of a total of 200 alarms received. What percent of the alarms received were false alarms?

27. According to the National Education Association, elementary and secondary school revenues in 2000 totaled $335 billion. Use the figure at the right to determine how much of that amount came from the federal government.



Elementary and Secondary School Revenues in 2000

Source: U.S. Census Bureau

28. In a recent year, North Carolina produced 1,300,000,000 pounds of turkey. This was 18.6% of the U.S. total in that year. Calculate the U.S. total turkey production for that year. Round to the nearest billion.

29. During one year, approximately 2,240,000 ounces of gold went into the manufacturing of electronic equipment in the United States. This is 16% of all the gold mined in the United States that year. How many ounces of gold were mined in the United States that year?

30. The table at the right shows the predicted increase in population from 2000 to 2040 for each of four counties in the Central Valley of California.

a. What percent of the 2000 population of Sacramento County is the increase in population?

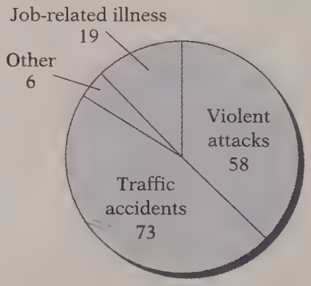
b. What percent of the 2000 population of Kern County is the increase in population? Round to the nearest tenth of a percent.

County	*2000 Population*	*Projected Increase*
Sacramento	1,200,000	900,000
Kern	651,700	948,300
Fresno	794,200	705,800
San Joaquin	562,000	737,400

Source: California Department of Finance

APPLYING THE CONCEPTS

31. The graph at the right shows the causes of death for all police officers killed in the line of duty during a recent year. What percent of the deaths were due to traffic accidents? Round to the nearest tenth of a percent.



Causes of Death for Police Officers Killed in the Line of Duty

Source: International Union of Police Associations

32. A certain solution of iodine is used to study the function of the thyroid gland. Approximately every 8 days, the potency of the solution is reduced by 50% of the amount present at that time. By what percent has the potency decreased after 24 days?

33. A certain solution of iron is used to study the formation of blood cells. Approximately every 15 hours, the potency of the solution is reduced by 50% of the amount present at that time. By what percent has the potency decreased after 60 hours?

Focus on Problem Solving

Using a Calculator as a Problem-Solving Tool

A calculator is an important tool for problem solving. Here are a few problems to solve with a calculator. You may need to research some of the questions to find information you do not know.

1. Choose any single-digit positive number. Multiply the number by 1507 and 7373. What is the answer? Choose another positive single-digit number and again multiply by 1507 and 7373. What is the answer? What pattern do you see? Why does this work?

2. The gross domestic product in 2000 was \$9,963,100,000,000. Is this more or less than the amount of money that would be placed on the last square of a standard checkerboard if 1 cent were placed on the first square, 2 cents were placed on the second square, 4 cents were placed on the third square, 8 cents were placed on the fourth square, and so on until the 64th square was reached?

3. Which of the reciprocals of the first 16 natural numbers have a terminating-decimal representation and which have a repeating-decimal representation?

4. What is the largest natural number n for which $4^n > 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot \cdots \cdot n$?

5. If \$1000 bills are stacked one on top of another, is the height of \$1 billion less than or greater than the height of the Washington Monument?

6. What is the value of $1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + 1}}}}$?

7. Calculate 15^2, 35^2, 65^2, and 85^2. Study the results. Make a conjecture about a relationship between a number ending in 5 and its square. Use your conjecture to find 75^2 and 95^2. Does your conjecture work for 125^2?

8. Find the sum of the first 1000 natural numbers. (*Hint:* You could just start adding $1 + 2 + 3 + \cdots$, but even if you performed one operation every 3 seconds, it would take you an hour to find the sum. Instead, try pairing the numbers and then adding the pairs. Pair 1 and 1000, 2 and 999, 3 and 998, and so on. What is the sum of each pair? How many pairs are there? Use this information to answer the original question.)

9. To qualify for a home loan, a bank requires that the monthly mortgage payment be less than 25% of a borrower's monthly take-home income. A laboratory technician has deductions for taxes, insurance, and retirement that amount to 25% of the technician's monthly gross income. What minimum monthly income must this technician earn to receive a bank loan that has a mortgage payment of \$1200 per month?

Projects and Group Activities

Consumer Price Index

The Consumer Price Index (CPI) is a percent that is written without the percent sign. For instance, a CPI of 160.1 means 160.1%. This number means that an item that cost \$100 between 1982 and 1984 (the base years) would cost \$160.10 today. Determining the cost is an application of the basic percent equation.

Percent × base = amount

CPI × cost in base year = cost today

$1.601 \times 100 = 160.1$ • $160.1\% = 1.601$

The table below gives the CPI for various products in June of 2000. If you have Internet access, you can obtain current data for the items below, as well as other items not on this list, by visiting the web site of the Bureau of Labor Statistics.

Product	*CPI*
All items	172.3
Food and beverages	167.9
Housing	169.4
Clothes	128.3
Transportation	155.7
Medical care	260.5
Entertainment	103.4
Education	101.5

1. Of the items listed, are there any items that in 2000 cost more than twice as much as they cost during the base year? If so, which items?
2. Of the items listed, are there any items that in 2000 cost more than one-and-one-half times as much as they cost during the base years but less than twice as much as they cost during the base years? If so, which items?
3. If the cost for textbooks for one semester was \$120 in the base years, how much did similar textbooks cost in 2000? Use the "Education" category.
4. If a new car cost \$20,000 in 2000, what would a comparable new car have cost during the base years? Use the "Transportation" category.
5. If a movie ticket cost \$8 in 2000, what would a comparable movie ticket have cost during the base years? Use the "Entertainment" category.
6. The base year for the CPI was 1967 before the change to 1982–1984. If 1967 were still used as the base year, the CPI for all items in 2000 (not just those listed above) would be 516.1.
   a. Using the base year of 1967, explain the meaning of a CPI of 516.1.
   b. Using the base year of 1967 and a CPI of 516.1, if textbooks cost \$75 for one semester in 1967, how much did similar textbooks cost in 2000?
   c. Using the base year of 1967 and a CPI of 516.1, if a family's food budget in 2000 is \$800 per month, what would a comparable family budget have been in 1967?

Demography

Demography is the statistical study of human populations. Many groups are interested in the size of certain segments of the population and in projections of population growth. For example, public school administrators want estimates of the number of school-age children who will be living in their districts 10 years from now.

The U.S. government provides estimates of what the population in the United States will be in the future. You can find these projections at the Census Bureau web site at www.census.gov. The table below contains data from that web site.

	Age:	*Under 5*	*5–17*	*18–24*	*25–34*	*35–44*	*45–54*	*55–64*	*65 and Older*
2010	Male	10,272	26,639	15,388	19,286	19,449	21,706	16,973	16,966
	Female	9,827	25,363	14,774	19,565	19,993	22,455	18,457	22,749
2050	Male	13,748	35,227	18,734	25,034	24,550	22,340	21,127	36,289
	Female	13,165	33,608	18,069	25,425	25,039	23,106	22,517	45,710

Population Projections for the United States, by Age and Sex (in thousands)

For these exercises, round all percents to the nearest tenth of a percent.

1. Which of the age groups listed are of interest to public school officials?
2. Which age group is of most interest to retirement home administrators?
3. Which age groups are of concern to accountants determining future benefits to be paid out by the Social Security Administration?
4. The population of which age group is of interest to manufacturers of disposable diapers?
5. The population of which age group is of primary concern to college and university admissions officers?
6. In which age groups do the males outnumber the females? In which do the females outnumber the males?
7. What percent of the projected population aged 65 and older in the year 2010 is female? Does this percent decrease in the projected population in 2050? If so, by how much?
8. Find the difference between the percent of the population that will be under 18 in 2010 and the percent that will be under 18 in 2050.
9. Assume that the work force consists of people aged 25 to 64. Find the decrease in the percents of the population for that group from 2010 to 2050.
10. Find the increase in the percents of the population for those aged 65 and over from 2010 to 2050.
11. Why are the sizes of the work force and the population aged 65 and over of special concern to the Social Security Administration?
12. Describe any patterns you see in the table.
13. Calculate a statistic based on the data in the table and explain why it would be of interest to an institution (such as a school system) or a manufacturer of consumer goods (such as baby food).

Health The American College of Sports Medicine (ACSM) recommends that you know how to determine your target heart rate in order to get the full benefit of exercise. Your **target heart rate** is the rate at which your heart should beat during any aerobic exercise such as running, cycling, fast walking, or participating in an aerobics class. According to the ACSM, you should reach your target rate and then maintain it for 20 minutes or more to achieve cardiovascular fitness. The intensity level varies for different individuals. A sedentary person might begin at the 60% level and gradually work up to 70%, whereas athletes and very fit individuals might work at the 85% level. The ACSM suggests that you calculate both 50% and 85% of your maximum heart rate. This will give you the low and high ends of the range within which your heart rate should stay.

To calculate your target heart rate:

	Example
Subtract your age from 220. This is your maximum heart rate.	$220 - 20 = 200$
Multiply your maximum heart rate by 50%. This is the low end of your range.	$200(0.50) = 100$
Divide the low end by 6. This is your low 10-second heart rate.	$100 \div 6 \approx 17$
Multiply your maximum heart rate by 85%. This is the high end of your range.	$200(0.85) = 170$
Divide the high end by 6. This is your high 10-second heart rate.	$170 \div 6 \approx 28$

1. Why are the low end and high end divided by 6 in order to detemine the low and high 10-second heart rates?
2. Calculate your target heart rate, both the low and high end of your range.
3. What is the percent increase in your 10-second heart rate from the low end to the high end?

Chapter Summary

Key Words *Percent* means "parts of 100." [p. 197]

Essential Rules

To Write a Percent as a Fraction
To write a percent as a fraction, remove the percent sign and multiply by $\frac{1}{100}$. [p. 197]

To Write a Percent as a Decimal
To write a percent as a decimal, remove the percent sign and multiply by 0.01. [p. 197]

To Write a Decimal as a Percent
To write a decimal as a percent, multiply by 100%. [p. 198]

To Write a Fraction as a Percent
To write a fraction as a percent, multiply by 100%. [p. 198]

Basic Percent Equation Percent × base = amount [p. 201]

Proportion Method to Solve Percent Equations $\frac{\text{Percent}}{100} = \frac{\text{amount}}{\text{base}}$ [p. 213]

Chapter Review

1. What is 30% of 200?

2. 16 is what percent of 80?

3. Write $1\frac{3}{4}$ as a percent.

4. 20% of what is 15?

5. Write 12% as a fraction.

6. Find 22% of 88.

7. What percent of 20 is 30?

8. $16\frac{2}{3}\%$ of what is 84?

9. Write 42% as a decimal.

10. What is 7.5% of 72?

11. $66\frac{2}{3}\%$ of what is 105?

12. Write 7.6% as a decimal.

13. Find 125% of 62.

14. Write $16\frac{2}{3}\%$ as a fraction.

15. Use the proportion method to find what percent of 25 is 40.

16. 20% of what number is 15? Use the proportion method.

17. Write 0.38 as a percent.

18. 78% of what is 8.5? Round to the nearest tenth.

19. What percent of 30 is 2.2? Round to the nearest tenth of a percent.

20. What percent of 15 is .92? Round to the nearest tenth of a percent.

21. Trent missed 9 out of 60 questions on a history exam. What percent of the questions did he answer correctly? Use the proportion method.

22. A company used 7.5% of its $60,000 advertising budget for TV advertising. How much of the advertising budget was spent for TV advertising?

23. In a two-year period, the population of Houston, Texas, increased from 1,600,000 to 1,700,000. What percent increase does this represent?

24. Joshua purchased a video camera for $980 and paid a sales tax of 6.25% of the cost. What was the total cost of the video camera?

25. In a survey of 350 women and 420 men, 275 of the women and 300 of the men reported that they wore sunscreen often. To the nearest tenth of a percent, what percent of the women wore sunscreen often?

26. It is estimated that the world's population will be 9,100,000,000 by the year 2050. This is 149% of the population in 2000. (Source: U.S. Bureau of the Census) What was the world's population in 2000? Round to the nearest hundred million.

27. A computer system can be purchased for $1800. This is 60% of what the computer cost 4 years ago. What was the cost of the computer 4 years ago? Use the proportion method.

28. In one basketball game, Michael Jordan scored 33 points. This was 30% of all the points scored by the team. How many points were scored by the team?

Chapter Test

1. Write 97.3% as a decimal.

2. Write $83\frac{1}{3}\%$ as a fraction.

3. Write 0.3 as a percent.

4. Write 1.63 as a percent.

5. Write $\frac{3}{2}$ as a percent.

6. Write $\frac{2}{3}$ as a percent.

7. What is 77% of 65?

8. 47.2% of 130 is what?

9. Which is larger:
7% of 120, or 76% of 13?

10. Which is smaller:
13% of 200, or 212% of 12?

11. A fast-food company uses 6% of its $75,000 budget for advertising. What amount of the budget is spent on advertising?

12. During the packaging process for vegetables, spoiled vegetables are discarded by an inspector. In one day an inspector found that 6.4% of the 1250 pounds of vegetables were spoiled. How many pounds of vegetables were not spoiled?

The table at the right contains nutrition information about a breakfast cereal. Solve Exercises 13 and 14 with information taken from this table.

13. The recommended amount of potassium per day for an adult is 3000 milligrams (mg). What percent, to the nearest tenth of a percent, of the daily recommended amount of potassium is provided by one serving of this cereal with skim milk?

14. The daily recommended number of calories for a 190-pound man is 2200 calories. What percent, to the nearest tenth of a percent, of the daily recommended number of calories is provided by one serving of this cereal with 2% milk?

NUTRITION INFORMATION

SERVING SIZE: 1.4 OZ WHEAT FLAKES WITH 0.4 OZ. RAISINS: 39.4 g. ABOUT 1/2 CUP
SERVINGS PER PACKAGE: 14

	CEREAL & RAISINS	WITH 1/2 CUP VITAMINS A & D SKIM MILK
CALORIES	120	180
PROTEIN, g	3	7
CARBOHYDRATE, g	28	34
FAT, TOTAL, g	1	1*
UNSATURATED, g	1	
SATURATED, g	0	
CHOLESTEROL, mg	0	0*
SODIUM, mg	125	190
POTASSIUM, mg	240	440

* 2% MILK SUPPLIES AN ADDITIONAL 20 CALORIES 2 g FAT, AND 10 mg CHOLESTEROL.
** CONTAINS LESS THAN 2% OF THE U.S. RDA OF THIS NUTRIENT

15. The Urban Center Department Store has 125 permanent employees and must hire an additional 20 temporary employees for the holiday season. What percent of the permanent employees is the number hired as temporary employees for the holiday season?

16. Conchita missed 7 out of 80 questions on a math exam. What percent of the questions did she answer correctly? (Round to the nearest tenth of a percent.)

17. 12 is 15% of what?

18. 42.5 is 150% of what? Round to the nearest tenth.

19. A manufacturer of transistors found 384 defective transistors during a quality control study. This amount was 1.2% of the transistors tested. Find the number of transistors tested.

20. A new house was bought for $95,000. Five years later the house sold for $152,000. The increase was what percent of the original price?

21. 123 is 86% of what number? Use the proportion method. Round to the nearest tenth.

22. What percent of 12 is 120? Use the proportion method.

23. A secretary receives a wage of $12.88 per hour. This amount is 112% of last year's wage. What is the dollar increase in the hourly wage over last year? Use the proportion method.

24. A city has a population of 71,500. Ten years ago the population was 32,500. The population now is what percent of what the population was 10 years ago? Use the proportion method.

25. The annual license fee on a car is 1.4% of the value of the car. If the license fee during a year is $91.00, what is the value of the car?

Cumulative Review

1. Simplify: $18 \div (7 - 4)^2 + 2$.

2. Find the LCM of 16, 24, and 30.

3. Find the sum of $2\frac{1}{3}$, $3\frac{1}{2}$, and $4\frac{5}{8}$.

4. Subtract: $27\frac{5}{12} - 14\frac{9}{16}$

5. Multiply: $7\frac{1}{3} \times 1\frac{5}{7}$

6. What is $\frac{14}{27}$ divided by $1\frac{7}{9}$?

7. Simplify: $\left(\frac{3}{4}\right)^3 \cdot \left(\frac{8}{9}\right)^2$

8. Simplify: $\left(\frac{2}{3}\right)^2 - \left(\frac{3}{8} - \frac{1}{3}\right) \div \frac{1}{2}$

9. Round 3.07973 to the nearest hundredth.

10. Subtract:
$$\begin{array}{r} 3.0902 \\ -\,1.9706 \\ \hline \end{array}$$

11. Divide: $0.032\overline{)1.097}$
Round to the nearest ten-thousandth.

12. Convert $3\frac{5}{8}$ to a decimal.

13. Convert 1.75 to a fraction.

14. Place the correct symbol, < or >, between the two numbers.
$\frac{3}{8}$ 0.87

15. Solve the proportion $\frac{3}{8} = \frac{20}{n}$. Round to the nearest tenth.

16. Write "$76.80 earned in 8 hours" as a unit rate.

17. Write $18\frac{1}{3}\%$ as a fraction.

18. Write $\frac{5}{6}$ as a percent.

19. 16.3% of 120 is what? Round to the nearest hundredth.

20. 24 is what percent of 18?

21. 12.4 is 125% of what?

22. What percent of 35 is 120? Round to the nearest tenth.

23. Sergio has an income of $740 per week. One-fifth of his income is deducted for income tax payments. Find his take-home pay.

24. Eunice bought a used car for $8353, with a down payment of $1000. The balance was paid in 36 equal monthly payments. Find the monthly payment.

25. The gasoline tax is $0.19 a gallon. Find the number of gallons of gasoline used during a month in which $79.80 was paid in gasoline taxes.

26. The real estate tax on a $72,000 home is $1440. At the same rate, find the real estate tax on a home valued at $150,000.

27. Ken purchased a stereo set for $490 and paid $29.40 in sales tax. What percent of the purchase price was the sales tax?

28. A survey of 300 people showed that 165 people favored a certain candidate for mayor. What percent of the people surveyed did not favor this candidate?

29. According to the Cabletelevision Advertising Bureau, cable households watch television 36.5% of the time. On average, how many hours per week do cable households spend watching TV? Round to the nearest tenth.

30. The Environmental Protection Agency found that 990 out of 5500 children tested had levels of lead in their blood exceeding federal guidelines. What percent of the children tested had levels of lead in the blood that exceeded federal standards?

Chapter 6

Applications for Business and Consumers

Your favorite cereal comes in two sizes, 12 ounces and 18 ounces. Would you save money in the long run by purchasing the larger size, even though it costs more than the smaller size? Calculating and comparing the unit price of a product enables you to make the more economical purchase, as shown in **Exercises 13 to 24 on pages 231 and 232.**

Objectives

Section 6.1

A To find unit cost
B To find the most economical purchase
C To find total cost

Section 6.2

A To find percent increase
B To apply percent increase to business—markup
C To find percent decrease
D To apply percent decrease to business—discount

Section 6.3

A To calculate simple interest
B To calculate finance charges on a credit card bill
C To calculate compound interest

Section 6.4

A To calculate the initial expenses of buying a home
B To calculate ongoing expenses of owning a home

Section 6.5

A To calculate the initial expenses of buying a car
B To calculate ongoing expenses of owning a car

Section 6.6

A To calculate commissions, total hourly wages, and salaries

Section 6.7

A To calculate checkbook balances
B To balance a checkbook

Need help? For on-line student resources, such as section quizzes, visit this textbook's web site at **math.college.hmco.com/students.**

Prep Test

For Exercises 1 to 6, add, subtract, multiply, or divide.

1. $3.75 \div 5$

2. 3.47×15

3. $874.50 - 369.99$

4. $0.065 \times 150{,}000$

5. $1500 \times 0.06 \times 0.5$

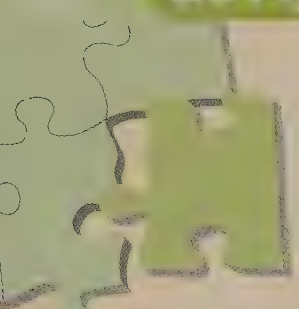
6. $1372.47 + 36.91 + 5.00 + 2.86$

7. Divide $10 \div 3$. Round to the nearest hundredth.

8. Divide $345 \div 570$. Round to the nearest thousandth.

9. Place the correct symbol, $<$ or $>$, between the two numbers.
   0.379   0.397

Go Figure



A store manager priced a pair of earrings in dollars and cents such that when 4% sales tax was added, the result was a whole number of dollars and 0 cents. Find the smallest possible number of dollars the items sold for, including the sales tax.

6.1 Applications to Purchasing

Objective A To find unit cost

Frequently, stores promote items for purchase by advertising, say, 2 Red Baron Bake to Rise Pizzas for $10.50 or 5 cans of StarKist tuna for $4.25.

The **unit cost** is the cost of *one* Red Baron Pizza or of *one* can of StarKist tuna. To find the unit cost, divide the total cost by the number of units.

2 pizzas for $10.50	5 cans for $4.25
10.50 ÷ 2 = 5.25	4.25 ÷ 5 = 0.85
$5.25 is the cost of one pizza.	$.85 is the cost of one can.
Unit cost: $5.25 per pizza	Unit cost: $.85 per can

Example 1
Find the unit cost. Round to the nearest tenth of a cent.
a. 3 gallons of mint chip ice cream for $17
b. 4 ounces of Crest toothpaste for $2.29

Strategy
To find the unit cost, divide the total cost by the number of units.

Solution
a. 17 ÷ 3 ≈ 5.667
$5.667 per gallon
b. 2.29 ÷ 4 = 0.5725
$.573 per ounce

You Try It 1
Find the unit cost. Round to the nearest tenth of a cent.
a. 8 size-AA Energizer batteries for $7.67
b. 15 ounces of Revlon shampoo for $2.29

Your strategy

Your solution

Solution on p. S15

Objective B To find the most economical purchase

Comparison shoppers often find the most economical buy by comparing unit costs.

One store is selling 6 twelve-ounce cans of ginger ale for $2.99, and a second store is selling 24 twelve-ounce cans of ginger ale for $11.79. To find the better buy, compare the unit costs.

2.99 ÷ 6 ≈ 0.498	11.79 ÷ 24 ≈ 0.491
Unit cost: $.498 per can	Unit cost: $.491 per can

Because $.491 < $.498, the better buy is 24 cans for $11.79.

Example 2
Find the more economical purchase: 5 pounds of nails for $4.80, or 4 pounds of nails for $3.78.

Strategy
To find the more economical purchase, compare the unit costs.

Solution
$4.80 \div 5 = 0.96$
$3.78 \div 4 = 0.945$
$\$.945 < \$.96$

The more economical purchase is 4 pounds for $3.78.

You Try It 2
Find the more economical purchase: 6 cans of fruit for $5.70, or 4 cans of fruit for $3.96.

Your strategy

Your solution

Solution on p. S15

Objective C To find total cost

An installer of floor tile found the unit cost of identical floor tiles at three stores.

Store 1	*Store 2*	*Store 3*
$1.22 per tile	$1.18 per tile	$1.28 per tile

By comparing the unit costs, the installer determined that store 2 would provide the most economical purchase.

The installer also uses the unit cost to find the total cost of purchasing 300 floor tiles at store 2. The **total cost** is found by multiplying the unit cost by the number of units purchased.

$$\boxed{\text{Unit cost}} \times \boxed{\text{number of units}} = \boxed{\text{total cost}}$$

$$1.18 \times 300 = 354$$

The total cost is $354.

Example 3
Clear redwood lumber costs $5.43 per foot. How much would 25 feet of clear redwood cost?

Strategy
To find the total cost, multiply the unit cost (5.43) by the number of units (25).

Solution

$$\boxed{\begin{matrix}\text{Unit}\\\text{cost}\end{matrix}} \times \boxed{\begin{matrix}\text{number}\\\text{of units}\end{matrix}} = \boxed{\begin{matrix}\text{total}\\\text{cost}\end{matrix}}$$

$$5.43 \times 25 = 135.75$$

The total cost is $135.75.

You Try It 3
Pine saplings cost $9.96 each. How much would 7 pine saplings cost?

Your strategy

Your solution

Solution on p. S15

6.1 Exercises

Objective A Application Problems

Find the unit cost. Round to the nearest tenth of a cent.

1. Heinz B·B·Q sauce, 18 ounces for $.99

2. Birds-eye maple, 6 feet for $18.75

3. Diamond walnuts, $2.99 for 8 ounces

4. A&W root beer, 6 cans for $2.99

5. Ibuprofen, 50 tablets for $3.99

6. Visine eye drops, 0.5 ounce for $3.89

7. Adjustable wood clamps, 2 for $13.95

8. Corn, 6 ears for $1.85

9. Cheerios cereal, 15 ounces for $2.99

10. Doritos Cool Ranch chips, 14.5 ounces for $2.99

11. Sheet metal screws, 8 for $.95

12. Folgers coffee, 11.5 ounces for $2.79

Objective B Application Problems

Suppose your local supermarket offers the following products at the given prices. Find the more economical purchase.

13. Sutter Home pasta sauce, 25.5 ounces for $3.29, or Muir Glen Organic pasta sauce, 26 ounces for $3.79

14. Kraft mayonnaise, 40 ounces for $2.98, or Springfield mayonnaise, 32 ounces for $2.39

15. Ortega salsa, 20 ounces for $3.29 or 12 ounces for $1.99

16. L'Oreal shampoo, 13 ounces for $3.99, or Cortexx shampoo, 12 ounces for $3.69

17. Golden Sun vitamin E, 200 tablets for $7.39 or 400 tablets for $12.99

18. Ultra Mr. Clean, 20 ounces for $2.67, or Ultra Spic and Span, 14 ounces for $2.19

19. 16 ounces of Kraft cheddar cheese, $4.37, or 9 ounces of Land to Lake cheddar cheese, $2.29

20. Bertolli olive oil, 34 ounces for $9.49, or Pompeian olive oil, 8 ounces for $2.39

21. Maxwell House coffee, 4 ounces for $3.99, or Sanka coffee, 2 ounces for $2.39

22. Wagner's vanilla extract, $3.29 for 1.5 ounces, or Durkee vanilla extract, 1 ounce for $2.74

23. Purina Cat Chow, $4.19 for 56 ounces, or Friskies Chef's Blend, $3.37 for 50.4 ounces

24. Kleenex tissues, $1.73 for 250 tissues, or Puffs tissues, $1.23 for 175 tissues

Objective C *Application Problems*

25. If sliced bacon costs $4.59 per pound, find the total cost of 3 pounds.

26. Used red brick costs $.98 per brick. Find the total cost of 75 bricks.

27. Kiwi fruit cost $.23 each. Find the total cost of 8 kiwi.

28. Boneless chicken filets cost $4.69 per pound. Find the cost of 3.6 pounds. Round to the nearest cent.

29. Herbal tea costs $.98 per ounce. Find the total cost of 6.5 ounces.

30. If Stella Swiss Lorraine cheese costs $5.99 per pound, find the total cost of 0.65 pound. Round to the nearest cent.

31. Red Delicious apples cost $1.29 per pound. Find the total cost of 2.1 pounds. Round to the nearest cent.

32. Choice rib eye steak costs $4.49 per pound. Find the total cost of 2.8 pounds. Round to the nearest cent.

33. If Godiva chocolate costs $7.95 per pound, find the total cost of $\frac{3}{4}$ pound. Round to the nearest cent.

34. Color photocopying costs $.89 per page. Find the total cost for photocopying 120 pages.

APPLYING THE CONCEPTS

35. Explain in your own words the meaning of unit pricing.

36. What is the UPC (Universal Product Code) and how is it used?

6.2 Percent Increase and Percent Decrease

Objective A To find percent increase

Point of Interest

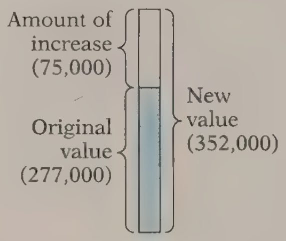
According to the U.S. Census Bureau, the number of persons aged 65 and over in the United States will increase to about 82.0 million by 2050, a 136% increase from 2000.

Percent increase is used to show how much a quantity has increased over its original value. The statements "food prices increased by 2.3% last year" and "city council members received a 4% pay increase" are examples of percent increase.

According to the Energy Information Administration, the number of alternative-fuel vehicles increased from approximately 277,000 to 352,000 in four years. Find the percent increase in alternative-fuel vehicles. Round to the nearest percent.

$$\boxed{\begin{array}{c}\text{New}\\\text{value}\end{array}} - \boxed{\begin{array}{c}\text{original}\\\text{value}\end{array}} = \boxed{\begin{array}{c}\text{amount of}\\\text{increase}\end{array}}$$

$$352{,}000 - 277{,}000 = 75{,}000$$



Now solve the basic percent equation for percent.

$$\text{Percent} \times \text{base} = \text{amount}$$

$$\boxed{\begin{array}{c}\text{Percent}\\\text{increase}\end{array}} \times \boxed{\begin{array}{c}\text{original}\\\text{value}\end{array}} = \boxed{\begin{array}{c}\text{amount of}\\\text{increase}\end{array}}$$

$$\begin{aligned} n \times 277{,}000 &= 75{,}000 \\ n &= 75{,}000 \div 277{,}000 \\ n &\approx 0.27 \end{aligned}$$

The number of alternative-fuel vehicles increased by approximately 27%.

Example 1

The average wholesale price of coffee increased from \$2 per pound to \$3 per pound in one year. What was the percent increase in the price of 1 pound of coffee?

Strategy

To find the percent increase:

- Find the amount of the increase.
- Solve the basic percent equation for *percent*.

Solution

$$\boxed{\begin{array}{c}\text{New}\\\text{value}\end{array}} - \boxed{\begin{array}{c}\text{original}\\\text{value}\end{array}} = \boxed{\begin{array}{c}\text{amount of}\\\text{increase}\end{array}}$$

$$3 - 2 = 1$$

$$\begin{aligned} \text{Percent} \times \text{base} &= \text{amount} \\ n \times 2 &= 1 \\ n &= 1 \div 2 \\ n &= 0.5 = 50\% \end{aligned}$$

The percent increase was 50%.

You Try It 1

The average price of gasoline rose from \$1.46 to \$1.83 in 5 months. What was the percent increase in the price of gasoline? Round to the nearest percent.

Your strategy

Your solution

Solution on p. S15

Example 2

Chris Carley was earning \$9.50 an hour as a nursing assistant before receiving a 10% increase in pay. What is Chris's new hourly pay?

Strategy

To find the new hourly wage:

- Solve the basic percent equation for *amount*.
- Add the amount of the increase to the original wage.

Solution

$$\begin{aligned} \text{Percent} \times \text{base} &= \text{amount} \\ 0.10 \times 9.50 &= n \\ 0.95 &= n \end{aligned}$$

The amount of the increase was \$.95.

$9.50 + 0.95 = 10.45$

The new hourly wage is \$10.45.

You Try It 2

Yolanda Liyama was making a wage of \$12.50 an hour as a baker before receiving a 14% increase in hourly pay. What is Yolanda's new hourly wage?

Your strategy

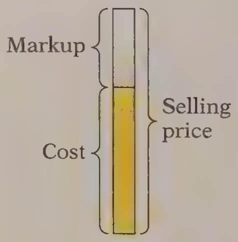
Your solution

Solution on p. S15

Objective B To apply percent increase to business—markup

Some of the expenses involved in operating a business are salaries, rent, equipment, and utilities. To pay these expenses and earn a profit, a business must sell a product at a higher price than it paid for the product.

Cost is the price a business pays for a product, and **selling price** is the price at which a business sells a product to a customer. The difference between selling price and cost is called **markup.**



$$\boxed{\text{Selling price}} - \boxed{\text{cost}} = \boxed{\text{markup}}$$

or

$$\boxed{\text{Cost}} + \boxed{\text{markup}} = \boxed{\text{selling price}}$$

Markup is frequently expressed as a percent of a product's cost. This percent is called the **markup rate.**

$$\boxed{\text{Markup rate}} \times \boxed{\text{cost}} = \boxed{\text{markup}}$$

Point of Interest

According to *Managing a Small Business*, from Liraz Publishing Company, goods in a store are often marked up 50% to 100% of the cost. This allows a business to make a profit of 5% to 10%.

⇨ Suppose Bicycles Galore purchases an AMP Research B4 bicycle for \$2119.20 and sells it for \$2649. What markup rate does Bicycles Galore use?

$$\boxed{\text{Selling price}} - \boxed{\text{cost}} = \boxed{\text{markup}}$$

$$2649.00 - 2119.20 = 529.80$$

- **First find the markup.**

$$\text{Percent} \times \text{base} = \text{amount}$$

$$\boxed{\text{Markup rate}} \times \boxed{\text{cost}} = \boxed{\text{markup}}$$

$$n \times 2119.20 = 529.80$$

$$n = 529.80 \div 2119.20 = 0.25$$

- **Then solve the basic percent equation for *percent*.**

The markup rate is 25%.

Example 3

The manager of a sporting goods store determines that a markup rate of 36% is necessary to make a profit. What is the markup on a pair of skis that costs the store $225?

Strategy

To find the markup, solve the basic percent equation for *amount.*

Solution

$$\text{Percent} \times \text{base} = \text{amount}$$

$$\boxed{\begin{matrix}\text{Markup}\\ \text{rate}\end{matrix}} \times \boxed{\text{cost}} = \boxed{\text{markup}}$$

$$0.36 \times 225 = n$$

$$81 = n$$

The markup is $81.

You Try It 3

A bookstore manager determines that a markup rate of 20% is necessary to make a profit. What is the markup on a book that costs the bookstore $8?

Your strategy

Your solution

Example 4

A plant nursery bought a yellow twig dogwood for $4.50 and used a markup rate of 46%. What is the selling price?

Strategy

To find the selling price:

- Find the markup by solving the basic percent equation for *amount.*
- Add the markup to the cost.

Solution

$$\text{Percent} \times \text{base} = \text{amount}$$

$$\boxed{\begin{matrix}\text{Markup}\\ \text{rate}\end{matrix}} \times \boxed{\text{cost}} = \boxed{\text{markup}}$$

$$0.46 \times 4.50 = n$$

$$2.07 = n$$

$$\boxed{\text{Cost}} + \boxed{\text{markup}} = \boxed{\text{selling price}}$$

$$4.50 + 2.07 = 6.57$$

The selling price is $6.57.

You Try It 4

A clothing store bought a suit for $72 and used a markup rate of 55%. What is the selling price?

Your strategy

Your solution

Solutions on p. S15

Objective C

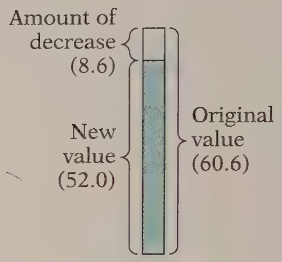
To find percent decrease

Percent decrease is used to show how much a quantity has decreased from its original value. The statements "the number of family farms decreased by 2% last year" and "there has been a 50% decrease in the cost of a Pentium chip" are examples of percent decrease.

During a two-year period, the value of U.S. agricultural products exported decreased from approximately $60.6 billion to $52.0 billion. Find the percent decrease in the value of agricultural exports. Round to the nearest tenth of a percent.

$$\boxed{\text{Original value}} - \boxed{\text{new value}} = \boxed{\text{amount of decrease}}$$

$$60.6 - 52.0 = 8.6$$



Now solve the basic percent equation for percent.

$$\text{Percent} \times \text{base} = \text{amount}$$

$$\boxed{\text{Percent decrease}} \times \boxed{\text{original value}} = \boxed{\text{amount of decrease}}$$

$$\begin{aligned} n \times 60.6 &= 8.6 \\ n &= 8.6 \div 60.6 \\ n &\approx 0.142 \end{aligned}$$

The value of agricultural exports decreased approximately 14.2%.

Point of Interest

A *bear market* is one in which the total value of all stocks decreases. During 1972 and 1973, the stock market experienced a 55% decrease. This means that a $10,000 investment would have decreased to $4500 during that period.

Example 5

During an 8-year period, Baltimore, Maryland's population decreased from approximately 736,000 to 646,000. Find the percent decrease in Baltimore's population. Round to the nearest tenth of a percent.

Strategy

To find the percent decrease:

- Find the amount of decrease.
- Solve the basic percent equation for *percent*.

Solution

$$\boxed{\text{Original value}} - \boxed{\text{new value}} = \boxed{\text{amount of decrease}}$$

$$736{,}000 - 646{,}000 = 90{,}000$$

$$\begin{aligned} \text{Percent} \times \text{base} &= \text{amount} \\ n \times 736{,}000 &= 90{,}000 \\ n &= 90{,}000 \div 736{,}000 \\ n &\approx 0.122 \end{aligned}$$

Baltimore's population decreased approximately 12.2%.

You Try It 5

During an 8-year period, Norfolk, Virginia's population decreased from approximately 261,000 to 215,000. Find the percent decrease in Norfolk's population. Round to the nearest tenth of a percent.

Your strategy

Your solution

Solution on p. S15

Example 6

The total sales for December for a stationery store were \$16,000. For January, total sales showed an 8% decrease from December's sales. What were the total sales for January?

Strategy

To find the total sales for January:

- Find the amount of decrease by solving the basic percent equation for *amount*.
- Subtract the amount of decrease from the December sales.

Solution

$$\begin{aligned} \text{Percent} \times \text{base} &= \text{amount} \\ 0.08 \times 16{,}000 &= n \\ 1280 &= n \end{aligned}$$

The decrease in sales was \$1280.

$16{,}000 - 1280 = 14{,}720$

The total sales for January were \$14,720.

You Try It 6

Fog decreased the normal 5-mile visibility at an airport by 40%. What was the visibility in the fog?

Your strategy

Your solution

Solution on p. S15

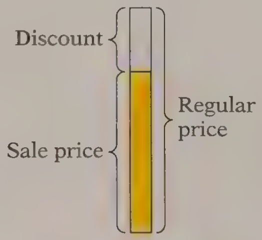
Objective D To apply percent decrease to business—discount

To promote sales, a store may reduce the regular price of some of its products temporarily. The reduced price is called the **sale price.** The difference between the regular price and the sale price is called the **discount.**

Regular price − sale price = discount

or

Regular price − discount = sale price



Discount is frequently stated as a percent of a product's regular price. This percent is called the **discount rate.**

Discount rate × regular price = discount

Example 7

A GE 25-inch stereo television that regularly sells for $299 is on sale for $250. Find the discount rate. Round to the nearest tenth of a percent.

Strategy

To find the discount rate:

- Find the discount.
- Solve the basic percent equation for *percent*.

Solution

$$\boxed{\begin{array}{c}\text{Regular}\\ \text{price}\end{array}} - \boxed{\begin{array}{c}\text{sale}\\ \text{price}\end{array}} = \boxed{\text{discount}}$$

$$299 - 250 = 49$$

$$\text{Percent} \times \text{base} = \text{amount}$$

$$\boxed{\begin{array}{c}\text{Discount}\\ \text{rate}\end{array}} \times \boxed{\begin{array}{c}\text{regular}\\ \text{price}\end{array}} = \boxed{\text{discount}}$$

$$n \times 299 = 49$$

$$n = 49 \div 299$$

$$n \approx 0.1639$$

The discount rate is 16.4%.

You Try It 7

A white azalea that regularly sells for $12.50 is on sale for $10.99. Find the discount rate. Round to the nearest tenth of a percent.

Your strategy

Your solution

Example 8

A 20-horsepower lawn mower is on sale for 25% off the regular price of $1125. Find the sale price.

Strategy

To find the sale price:

- Find the discount by solving the basic percent equation for *amount*.
- Subtract to find the sale price.

Solution

$$\text{Percent} \times \text{base} = \text{amount}$$

$$\boxed{\begin{array}{c}\text{Discount}\\ \text{rate}\end{array}} \times \boxed{\begin{array}{c}\text{regular}\\ \text{price}\end{array}} = \boxed{\text{discount}}$$

$$0.25 \times 1125 = n$$

$$281.25 = n$$

$$\boxed{\begin{array}{c}\text{Regular}\\ \text{price}\end{array}} - \boxed{\text{discount}} = \boxed{\begin{array}{c}\text{sale}\\ \text{price}\end{array}}$$

$$1125 - 281.25 = 843.75$$

The sale price is $843.75.

You Try It 8

A hardware store is selling a Newport security door for 15% off the regular price of $110. Find the sale price.

Your strategy

Your solution

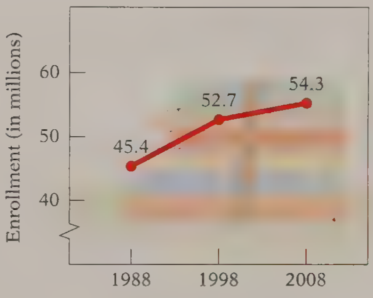
Solutions on p. S16

6.2 Exercises

Objective A Application Problems

Solve. Round percents to the nearest tenth of a percent.

1. The figure at the right shows the actual and projected enrollments in grades K–12 in the United States. What percent increase is expected from 1988 to 2008?



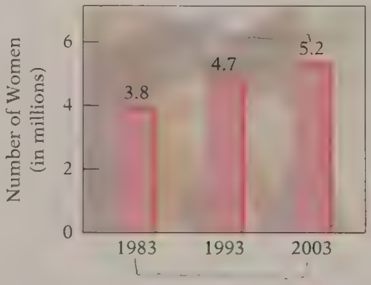
Elementary and Secondary School Enrollments

Source: National Center for Education

2. An automobile manufacturer increased the average mileage on a car from 17.5 miles per gallon to 18.2 miles per gallon. Find the percent increase in mileage.

3. In the 1990s, the number of Target stores increased from 420 stores to 914 stores. (Source: Target) What was the percent increase in the number of Target stores in the 1990s?

4. The graph at the right shows the number of women attending four-year colleges. Find the percent increase in the number of women at four-year colleges from 1983 to 2003.



Women Attending Four-Year Colleges

Source: Copyright © 2000 *USA Today.* Reprinted with permission.

5. In 1924, the number of events in the Winter Olympics was 14. The 2002 Winter Olympics in Salt Lake City included 78 medal events. (Source: David Wallenchinsky's *The Complete Book of the Winter Olympics*) Find the percent increase in the number of events in the Winter Olympics from 1924 to 2002.

6. The table at the right shows the estimated number of millionaire households in the United States. Find the percent increase in the estimated number of households containing millionaires from 1975 to 2005.

Year	*Number of Households Containing Millionaires*
1975	350,000
1997	3,500,000
2005	5,600,000

Source: Affluent Market Institute

7. The population of Boise City, Idaho, increased 24.3% over an 8-year period. The population initially was approximately 127,000. (Source: The Census Bureau) Find the population of Boise City 8 years later.

8. During one year, the number of people subscribing to direct broadcasting satellite systems increased 87%. If the number of subscribers at the beginning of the year was 2.3 million, how many subscribers were there at the end of the year?

9. From 1998 to 2000, the amount of time the average Internet user spent online each week rose 72.7%. If 4.4 hours were spent online in 1998, how many hours per week were spent online in 2000? Round to the nearest tenth. (Source: *Time*, February 21, 2000)

Objective B Application Problems

10. A window air conditioner cost AirRite Air Conditioning Systems $285. Find the markup on the air conditioner if the markup rate is 25% of the cost.

11. The owner of Kerr Electronics purchased 300 Craig portable CD players at a cost of $85 each. If the owner uses a markup rate of 42%, what is the markup on each of the Craig CD players?

12. The manager of Brass Antiques has determined that a markup rate of 38% is necessary for a profit to be made. What is the markup on a brass doorknob that costs $45?

13. Computer Inc. uses a markup of $975 on a computer system that costs $3250. What is the markup rate on this system?

14. Saizon Pen & Office Supply uses a markup of $12 on a calculator that costs $20. What markup rate does this amount represent?

15. Giant Photo Service uses a markup rate of 48% on its Model ZA cameras, which cost the shop $162.
a. What is the markup?
b. What is the selling price?

16. The Circle R golf pro shop uses a markup rate of 45% on a set of Tour Pro golf clubs that costs the shop $210.
a. What is the markup?
b. What is the selling price?

17. Harvest Time Produce Inc. uses a 55% markup rate and pays $2 for a box of strawberries.
a. What is the markup?
b. What is the selling price of a box of strawberries?

18. Resner Builders' Hardware uses a markup rate of 42% for a table saw that costs $160. What is the selling price of the table saw?

19. Brad Burt's Magic Shop uses a markup rate of 48%. What is the selling price of a telescoping sword that costs $50?

Objective C Application Problems

20. A new bridge reduced the normal 45-minute travel time between two cities by 18 minutes. What percent decrease does this represent?

21. By installing energy-saving equipment, the Pala Rey Youth Camp reduced its normal $800-per-month utility bill by $320. What percent decrease does this amount represent?

22. It is estimated that the value of a new car is reduced 30% after 1 year of ownership. Using this estimate, find how much value a $18,200 new car loses after 1 year.

23. A department store employs 1200 people during the holiday. At the end of the holiday season, the store reduces the number of employees by 45%. What is the decrease in the number of employees?

24. Because of a decrease in demand for super-8 video cameras, Kit's Cameras reduced the orders for these models from 20 per month to 8 per month.
a. What is the amount of the decrease?
b. What percent decrease does this amount represent?

25. A new computer system reduced the time for printing the payroll from 52 minutes to 39 minutes.
a. What is the amount of the decrease?
b. What percent decrease does this amount represent?

26. Juanita's average monthly expense for gasoline was $76. After joining a car pool, she was able to reduce the expense by 20%.
a. What was the amount of the decrease?
b. What is the average monthly gasoline bill now?

27. An oil company paid a dividend of $1.60 per share. After a reorganization, the company reduced the dividend by 37.5%.
a. What was the amount of the decrease?
b. What is the new dividend?

28. Because of an improved traffic pattern at a sports stadium, the average amount of time a fan waits to park decreased from 3.5 minutes to 2.8 minutes. What percent decrease does this amount represent?

29. One configuration of the Boeing 777-300 has a seating capacity of 394. This is 26 more than that of the corresponding 777-200 jet. What is the percent decrease in capacity from the 777-300 to the 777-200 model?

30. In 2000, the Pentagon revised its account of the number of Americans killed in the Korean War from 54,246 to 36,940. (Source: *Time*, June 12, 2000) What is the percent decrease in the reported number of military personnel killed in the Korean War?

Objective D *Application Problems*

31. The Austin College Bookstore is giving a discount of $8 on calculators that normally sell for $24. What is the discount rate?

32. A discount clothing store is selling a $72 sport jacket for $24 off the regular price. What is the discount rate?

33. A disk player that regularly sells for $340 is selling for 20% off the regular price. What is the discount?

34. Dacor Appliances is selling its $450 washing machine for 15% off the regular price. What is the discount?

35. An electric grill that regularly sells for $140 is selling for $42 off the regular price. What is the discount rate?

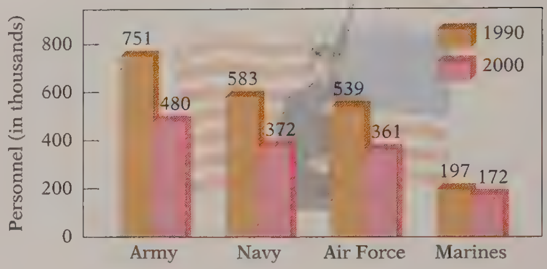
36. Quick Service Gas Station has its regularly priced \$45 tune-up on sale for 16% off the regular price.
a. What is the discount?
b. What is the sale price?

37. Tomatoes that regularly sell for \$.85 per pound are on sale for 20% off the regular price.
a. What is the discount?
b. What is the sale price?

38. An outdoor supply store has its regularly priced \$160 sleeping bags on sale for \$120.
a. What is the discount?
b. What is the discount rate?

39. Standard Brands paint that regularly sells for \$20 per gallon is on sale for \$16 per gallon.
a. What is the discount?
b. What is the discount rate?

40. The graph at the right shows the number of active-duty U.S. military personnel in 1990 and in 2000. Which branch of the military had the greatest percent decrease in personnel from 1990 to 2000? What was the percent decrease for this branch of the service?



Number of Active-Duty U.S. Military Personnel

Source: Fiscal 2000 Annual Report to the President and Congress by the Secretary of Defense

APPLYING THE CONCEPTS

41. A welder earning \$12 per hour is given a 10% raise. To find the new wage, we can multiply \$12 by 0.10 and add the product to \$12. Can the new wage be found by multiplying \$12 by 1.10? Try both methods and compare your answers.

42. Grocers, florists, bakers, and other businesses must consider spoilage when deciding the markup of a product. For instance, suppose a florist purchased 200 roses at a cost of \$.86 per rose. The florist wants a markup rate of 50% of the total cost of all the roses and expects 7% of the roses to wilt and therefore not be salable. Find the selling price per rose by answering each of the following questions.
a. What is the florist's total cost for the 200 roses?
b. Find the total selling price without spoilage.
c. Find the number of roses the florist expects to sell. *Hint:* The number of roses the florist expects to sell is

% of salable roses × number of roses purchased

d. To find the selling price per rose, divide the total selling price without spoilage by the number of roses the florist expects to sell. Round to the nearest cent.

43. A promotional sale at a department store offers 25% off the sale price. The sale price is 25% off the regular price. Is this the same as a sale that offers 50% off the regular price? If not, which sale gives the better price? Explain your answer.

6.3

Interest

Objective A

To calculate simple interest

When you deposit money in a bank—for example, in a savings account—you are permitting the bank to use your money. The bank may use the deposited money to lend customers the money to buy cars or make renovations on their homes. The bank pays you for the privilege of using your money. The amount paid to you is called **interest.** If you are the one borrowing money from the bank, the amount you pay for the privilege of using that money is also called interest.

TAKE NOTE

If you deposit $1000 in a savings account paying 5% interest, the $1000 is the principal and 5% is the interest rate.

The original amount deposited or borrowed is called the **principal.** The amount of interest paid is usually given as a percent of the principal. The percent used to determine the amount of interest is the **interest rate.**

Interest paid on the original principal is called **simple interest.** To calculate simple interest, multiply the principal by the interest rate per period by the number of time periods. In this objective, we are working with annual interest rates, so the time periods are years. The simple interest formula for an annual interest rate is given below.

Simple Interest Formula for Annual Interest Rates

Principal × annual interest rate × time (in years) = interest

Interest rates are generally given as percents. Before performing calculations involving an interest rate, write the interest rate as a decimal.

➡ Calculate the simple interest due on a 2-year loan of $1500 that has an annual interest rate of 7.5%.

$$\boxed{\text{Principal}} \times \boxed{\text{annual interest rate}} \times \boxed{\text{time (in years)}} = \boxed{\text{interest}}$$

$$1500 \times 0.075 \times 2 = 225$$

The simple interest due is $225.

When we borrow money, the total amount to be repaid to the lender is the sum of the principal and the interest. This amount is called the **maturity value** of the loan.

Maturity Value Formula for Simple Interest Loans

Principal + interest = maturity value

In the example above, the simple interest due on the loan of $1500 was $225. The maturity value of the loan is therefore $1500 + $225 = $1725.

Calculate the maturity value of a simple interest, 8-month loan of \$8000 if the annual interest rate is 9.75%.

First find the interest due on the loan.

$$\boxed{\text{Principal}} \times \boxed{\text{annual interest rate}} \times \boxed{\text{time (in years)}} = \boxed{\text{interest}}$$

$$8000 \times 0.0975 \times \frac{8}{12} = 520$$

Find the maturity value.

Principal + interest = maturity value
8000 + 520 = 8520

The maturity value of the loan is \$8520.

TAKE NOTE
The time of the loan must be in years. Eight months is $\frac{8}{12}$ of a year.

See Example 1. The time of the loan must be in years. 180 days is $\frac{180}{365}$ of a year.

The monthly payment on a loan can be calculated by dividing the maturity value by the length of the loan in months.

Monthly Payment on a Simple Interest Loan

Maturity value ÷ length of the loan in months = monthly payment

In the example above, the maturity value of the loan is \$8520. To find the monthly payment on the 8-month loan, divide 8520 by 8.

$$\boxed{\text{Maturity value}} \div \boxed{\text{length of the loan in months}} = \boxed{\text{monthly payment}}$$

$$8520 \div 8 = 1065$$

The monthly payment on the loan is \$1065.

Example 1
Kamal borrowed \$500 from a savings and loan association for 180 days at an annual interest rate of 7%. What is the simple interest due on the loan?

Strategy
To find the simple interest due, multiply the principal (500) times the annual interest rate (7% = 0.07) times the time, in years (180 days = $\frac{180}{365}$ year).

Solution

$$\boxed{\text{Principal}} \times \boxed{\begin{array}{c}\text{annual}\\\text{interest}\\\text{rate}\end{array}} \times \boxed{\begin{array}{c}\text{time}\\\text{(in years)}\end{array}} = \boxed{\text{interest}}$$

$$500 \times 0.07 \times \frac{180}{365} \approx 17.26$$

The simple interest due is \$17.26.

You Try It 1
A company borrowed \$15,000 from a bank for 18 months at an annual interest rate of 8%. What is the simple interest due on the loan?

Your strategy

Your solution

Solution on p. S16

Example 2

Calculate the maturity value of a simple interest, nine-month loan of $4000 if the annual interest rate is 8.75%.

Strategy

To find the maturity value:

- Use the simple interest formula to find the simple interest due.
- Find the maturity value by adding the principal and the interest.

Solution

$$\boxed{\text{Principal}} \times \boxed{\begin{matrix}\text{annual}\\ \text{interest}\\ \text{rate}\end{matrix}} \times \boxed{\begin{matrix}\text{time}\\ \text{(in years)}\end{matrix}} = \boxed{\text{interest}}$$

$$4000 \times 0.0875 \times \frac{9}{12} = 262.5$$

$$\text{Principal} + \text{interest} = \text{maturity value}$$
$$4000 + 262.50 = 4262.50$$

The maturity value is $4262.50.

You Try It 2

Calculate the maturity value of a simple interest, 90-day loan of $3800. The annual interest rate is 6%.

Your strategy

Your solution

Example 3

The simple interest due on a 3-month loan of $1400 is $26.25. Find the monthly payment on the loan.

Strategy

To find the monthly payment:

- Find the maturity value by adding the principal and the interest.
- Divide the maturity value by the length of the loan in months (3).

Solution

$$\text{Principal} + \text{interest} = \text{maturity value}$$
$$1400 + 26.25 = 1426.25$$

$$\text{Maturity value} \div \text{length of the loan} = \text{payment}$$
$$1426.25 \div 3 \approx 475.42$$

The monthly payment is $475.42.

You Try It 3

The simple interest due on a 1-year loan of $1900 is $152. Find the monthly payment on the loan.

Your strategy

Your solution

Solutions on p. S16

Objective B

To calculate finance charges on a credit card bill

When a customer uses a credit card to make a purchase, the customer is actually receiving a loan. Therefore, there is frequently an added cost to the consumer who purchases on credit. This may be in the form of an annual fee and interest charges on purchases. The interest charges on purchases are called **finance charges.**

The finance charge on a credit card bill is calculated using the simple interest formula. In the last objective, the interest rates were annual interest rates. However, credit card companies generally issue *monthly* bills and express interest rates on credit card purchases as *monthly* interest rates. Therefore, when using the simple interest formula to calculate finance charges on credit card purchases, use a monthly interest rate and express the time in months.

Note: In the simple interest formula, the time must be expressed in the same period as the rate. For an *annual* interest rate, the time must be expressed in years. For a *monthly* interest rate, the time must be in months.

Example 4

A credit card company charges a customer 1.5% per month on the unpaid balance of charges on the credit card. What is the finance charge in a month when the customer has an unpaid balance of $254?

Strategy

To find the finance charge, multiply the principal, or unpaid balance (254), times the monthly interest rate (1.5%) times the number of months (1).

Solution

$$\boxed{\text{Principal}} \times \boxed{\begin{array}{c}\text{monthly}\\ \text{interest}\\ \text{rate}\end{array}} \times \boxed{\begin{array}{c}\text{time}\\ \text{(in months)}\end{array}}$$

$$254 \times 0.015 \times 1 = 3.81$$

The simple interest due is $3.81.

You Try It 4

The credit card that Francesca uses charges her 1.6% per month on her unpaid balance. Find the finance charge when her unpaid balance one month is $1250.

Your strategy

Your solution

Solution on p. S16

Objective 6 To calculate compound interest

Usually, the interest paid on money deposited or borrowed is compound interest. **Compound interest** is computed not only on the original principal but also on interest already earned. Here is an illustration.

Suppose $1000 is invested for 3 years at an annual interest rate of 9% compounded annually. Because this is an *annual* interest rate, we will calculate the interest earned each year.

During the first year, the interest earned is calculated as follows:

$$\boxed{\text{Principal}} \times \boxed{\text{annual interest rate}} \times \boxed{\text{time (in years)}} = \boxed{\text{interest}}$$

$$1000 \times 0.09 \times 1 = 90$$

At the end of the first year, the total amount in the account is

$$1000 + 90 = 1090$$

During the second year, the interest earned is calculated on the amount in the account at the end of the first year.

$$\begin{array}{ccccccc} \boxed{\text{Principal}} & \times & \boxed{\text{annual interest rate}} & \times & \boxed{\text{time (in years)}} & = & \boxed{\text{interest}} \\ 1090 & \times & 0.09 & \times & 1 & = & 98.10 \end{array}$$

Note that the interest earned during the second year ($98.10) is greater than the interest earned during the first year ($90). This is because the interest earned during the first year was added to the original principal, and the interest for the second year was calculated using this sum. If the account earned simple interest, the interest earned would be the same every year ($90).

At the end of the second year, the total amount in the account is the sum of the amount in the account at the end of the first year and the interest earned during the second year.

$$1090 + 98.10 = 1188.10$$

The interest earned during the third year is calculated using the amount in the account at the end of the second year ($1188.10).

$$\begin{array}{ccccccc} \boxed{\text{Principal}} & \times & \boxed{\text{annual interest rate}} & \times & \boxed{\text{time (in years)}} & = & \boxed{\text{interest}} \\ 1188.10 & \times & 0.09 & \times & 1 & \approx & 106.93 \end{array}$$

TAKE NOTE
The interest earned each year keeps increasing. This is the effect of compound interest.

The amount in the account at the end of the third year is

$$1188.10 + 106.93 = 1295.03$$

To find the interest earned for the three years, subtract the original principal from the new principal.

$$\begin{array}{ccccc} \boxed{\text{New principal}} & - & \boxed{\text{original principal}} & = & \boxed{\text{interest earned}} \\ 1295.03 & - & 1000 & = & 295.03 \end{array}$$

Note that the compound interest earned is $295.03. The simple interest earned on the investment would have been only $1000 × 0.09 × 3 = $270.

In this example, the interest was compounded annually. However, compound interest can be compounded

annually (once a year)
semiannually (twice a year)
quarterly (four times a year)
monthly (twelve times a year)
daily (365 times a year)

The more frequent the compounding periods, the more interest the account earns. For example, if, in the above example, the interest had been compounded quarterly rather than annually, the interest earned would have been greater.

Calculating compound interest can be very tedious, so there are tables that can be used to simplify these calculations. A portion of a Compound Interest Table is given in the Appendix.

⇨ What is the value after 5 years of $1000 invested at 7% annual interest compounded quarterly?

To find the interest earned, multiply the original principal (1000) by the factor found in the Compound Interest Table. To find the factor, first find the table headed "Compounded Quarterly" in the Compound Interest Table in the Appendix. Then look at the number where the 7% column and the 5-year row meet.

	Compounded Quarterly						
	4%	*5%*	*6%*	*7%*	*8%*	*9%*	*10%*
1 year	1.04060	1.05094	1.06136	1.07186	1.08243	1.09308	1.10381
5 years	1.22019	1.28204	1.34686	**1.41478**	1.48595	1.56051	1.63862
10 years	1.48886	1.64362	1.81402	2.00160	2.20804	2.43519	2.68506
15 years	1.81670	2.10718	2.44322	2.83182	3.28103	3.80013	4.39979
20 years	2.21672	2.70148	3.29066	4.00639	4.87544	5.93015	7.20957

The factor is 1.41478.

$$1000 \times 1.41478 = 1414.78$$

The value of the investment after 5 years is $1414.78.

Example 5

An investment of $650 pays 8% annual interest compounded semiannually. What is the interest earned in 5 years?

Strategy

To find the interest earned:

- Find the new principal by multiplying the original principal (650) by the factor found in the Compound Interest Table (1.48024).
- Subtract the original principal from the new principal.

Solution

$650 \times 1.48024 \approx 962.16$

The new principal is $962.16.

$962.16 - 650 = 312.16$

The interest earned is $312.16.

You Try It 5

An investment of $1000 pays 6% annual interest compounded quarterly. What is the interest earned in 20 years?

Your strategy

Your solution

Solution on p. S16

6.3 Exercises

Objective A Application Problems

1. A 2-year student loan of \$10,000 is made at an annual simple interest rate of 10.25%. The simple interest on the loan is \$2050. Identify **a.** the principal, **b.** the interest, **c.** the interest rate, and **d.** the time period of the loan.

2. A contractor obtained a 9-month loan for \$80,000 at an annual simple interest rate of 9.75%. The simple interest on the loan is \$5850. Identify **a.** the principal, **b.** the interest, **c.** the interest rate, and **d.** the time period of the loan.

3. Find the simple interest Jacob Zucker owes on a 2-year student loan of \$8000 at an annual interest rate of 9%.

4. Find the simple interest Kara Tanamachi owes on a $1\frac{1}{2}$-year loan of \$1500 at an annual interest rate of 7.5%.

5. To finance the purchase of 15 new cars, the Tropical Car Rental Agency borrowed \$100,000 for 9 months at an annual interest rate of 9%. What is the simple interest due on the loan?

6. A home builder obtained a preconstruction loan of \$50,000 for 8 months at an annual interest rate of 9.5%. What is the simple interest due on the loan?

7. A bank lent Gloria Masters \$20,000 at an annual interest rate of 8.8%. The period of the loan was 9 months. Find the simple interest due on the loan.

8. Eugene Madison obtained an 8-month loan of \$4500 at an annual interest rate of 10.2%. Find the simple interest Eugene owes on the loan.

9. Shannon O'Hara borrowed \$5000 for 90 days at an annual simple interest rate of 7.5%. Find the simple interest due on the loan.

10. Jon McCloud borrowed \$8500 for 180 days at an annual simple interest rate of 9.25%. Find the simple interest due on the loan.

11. Jorge Elizondo took out a 75-day loan of \$7500 at an annual interest rate of 9.5%. Find the simple interest due on the loan.

12. Kristi Yang borrowed \$15,000. The term of the loan was 90 days, and the annual simple interest rate was 7.4%. Find the simple interest due on the loan.

13. The simple interest due on a 4-month loan of $4800 is $320. What is the maturity value of the loan?

14. The simple interest due on a 60-day loan of $6500 is $80.14. Find the maturity value of the loan.

15. An auto parts dealer borrowed $150,000 at a 9.5% annual simple interest rate for 1 year. Find the maturity value of the loan.

16. A corporate executive took out a $25,000 loan at an 8.2% annual simple interest rate for 1 year. Find the maturity value of the loan.

17. William Carey borrowed $12,500 for 8 months at an annual simple interest rate of 9.5%. Find the total amount due on the loan.

18. You arrange for a 9-month bank loan of $9000 at an annual simple interest rate of 8.5%. Find the total amount you must repay to the bank.

19. Capital City Bank approves a home-improvement loan application for $14,000 at an annual simple interest rate of 10.25% for 270 days. What is the maturity value of the loan?

20. A credit union lends a member $5000 for college tuition. The loan is made for 18 months at an annual simple interest rate of 6.9%. What is the maturity value of this loan?

21. Action Machining Company purchased a robot-controlled lathe for $225,000 and financed the full amount at 8% annual simple interest for 4 years. The simple interest on the loan is $72,000. Find the monthly payment.

22. For the purchase of an entertainment center, a $1900 loan is obtained for 2 years at an annual simple interest rate of 9.4%. The simple interest due on the loan is $357.20. What is the monthly payment on the loan?

23. To attract new customers, Heller Ford is offering car loans at an annual simple interest rate of 4.5%.
  a. Find the interest charged to a customer who finances a car loan of $12,000 for 2 years.
  b. Find the monthly payment.

24. Cimarron Homes Inc. purchased a small plane for $57,000 and financed the full amount for 5 years at an annual simple interest rate of 9%.
  a. Find the interest ~~due on~~ the loan.
  b. Find the monthly payment.

25. Dennis Pappas decided to build onto his present home instead of buying a new, larger house. He borrowed $42,000 for $3\frac{1}{2}$ years at an annual simple interest rate of 9.5%. Find the monthly payment.

26. Rosalinda Johnson took out a 6-month, $12,000 loan. The annual simple interest rate on the loan was 8.5%. Find the monthly payment.

Objective B Application Problems

27. A credit card company charges a customer 1.25% per month on the unpaid balance of charges on the credit card. What is the finance charge = I in a month when the customer has an unpaid balance of $118.72?

28. The credit card that Dee Brown uses charges her 1.75% per month on her unpaid balance. Find the finance charge when her unpaid balance one month is $391.64.

29. What is the finance charge on an unpaid balance of $12,368.92 on a credit card that charges 1.5% per month on any unpaid balance?

30. Suppose you have an unpaid balance of $995.04 on a credit card that charges 1.2% per month on any unpaid balance. What finance charge do you owe the company?

31. A credit card customer has an unpaid balance of $1438.20. What is the difference between monthly finance charges of 1.15% per month on the unpaid balance and monthly finance charges of 1.85% per month?

32. One credit card company charges 1.25% per month on any unpaid balance, and a second company charges 1.75%. What is the difference between the finance charges that these two companies assess on an unpaid balance of $687.45?

Objective C Application Problems

33. North Island Federal Credit Union pays 4% annual interest, compounded daily, on time savings deposits. Find the value of $750 deposited in this account after 1 year.

34. Tanya invested $2500 in a tax-sheltered annuity that pays 8% annual interest compounded daily. Find the value of her investment after 20 years.

35. Sal Travato invested $3000 in a corporate retirement account that pays 6% annual interest compounded semiannually. Find the value of his investment after 15 years.

36. To replace equipment, a farmer invested $20,000 in an account that pays 7% annual interest compounded monthly. What is the value of the investment after 5 years?

37. Green River Lodge invests $75,000 in a trust account that pays 8% interest compounded quarterly.

a. What will the value of the investment be in 5 years?
b. How much interest will be earned in the 5 years?

38. To save for retirement, a couple deposited $3000 in an account that pays 7% annual interest compounded daily.

a. What will the value of the investment be in 10 years?
b. How much interest will be earned in the 10 years?

39. To save for a child's education, the Petersens deposited $2500 into an account that pays 6% annual interest compounded daily. Find the amount of interest earned on this account over a 20-year period.

40. How much interest is earned in 2 years on $4000 deposited in an account that pays 6% interest, compounded quarterly?

APPLYING THE CONCEPTS

41. The Mission Valley Credit Union charges its customers an interest rate of 2% per month on money that is transferred into an account that is overdrawn. Find the interest owed to the credit union for 1 month when $800 is transferred into an overdrawn account.

42. Suppose you have a savings account that earns interest at the rate of 6% per year compounded monthly. On January 1, you open this acount with a deposit of $100.

a. On February 1, you deposit an additional $100 into the account. What is the value of the account after the deposit?
b. On March 1, you deposit an additional $100 into the account. What is the value of the account after the deposit?

Note: This type of savings plan, wherein equal amounts ($100) are saved at equal time intervals (every month), is called an annuity.

43. At 4 P.M. on July 31, you open a savings account that pays 5% annual interest and you deposit $500 in the account. Your deposit is credited as of August 1. At the beginning of September, you receive a statement from the bank that shows that during the month of August, you received $2.12 in interest. The interest has been added to your account, bringing the total on deposit to $502.12. At the beginning of October, you receive a statement from the bank that shows that during the month of September, you received $2.06 in interest on the $502.12 on deposit. Explain why you received less interest during the second month when there was more money on deposit.

6.4 Real Estate Expenses

Objective A

To calculate the initial expenses of buying a home

One of the largest investments most people ever make is the purchase of a home. The major initial expense in the purchase is the down payment. The amount of the down payment is normally a percent of the purchase price. This percent varies among banks, but it usually ranges from 5% to 25%.

The **mortgage** is the amount that is borrowed to buy real estate. The mortgage amount is the difference between the purchase price and the down payment.

⇨ A home is purchased for \$140,000, and a down payment of \$21,000 is made. Find the mortgage.

$$\begin{array}{ccccc} \boxed{\text{Purchase price}} & - & \boxed{\text{down payment}} & = & \boxed{\text{mortgage}} \\ 140{,}000 & - & 21{,}000 & = & 119{,}000 \end{array}$$

The mortgage is \$119,000.

TAKE NOTE
Because *points* means percent, a loan origination fee of $2\frac{1}{2}$ points $= 2\frac{1}{2}\% = 2.5\% = 0.025$.

Another initial expense in buying a home is the **loan origination fee,** which is a fee that the bank charges for processing the mortgage papers. The loan origination fee is usually a percent of the mortgage and is expressed in **points,** which is the term banks use to mean percent. For example, "5 points" means "5 percent."

$$\boxed{\text{Points}} \times \boxed{\text{mortgage}} = \boxed{\text{loan origination fee}}$$

Example 1
A house is purchased for \$125,000, and a down payment, which is 20% of the purchase price, is made. Find the mortgage.

Strategy
To find the mortgage:

- Find the down payment by solving the basic percent equation for *amount.*
- Subtract the down payment from the purchase price.

Solution

$$\begin{array}{ccccc} \text{Percent} & \times & \text{base} & = & \text{amount} \\ \boxed{\text{Percent}} & \times & \boxed{\begin{array}{c}\text{purchase}\\ \text{price}\end{array}} & = & \boxed{\begin{array}{c}\text{down}\\ \text{payment}\end{array}} \\ 0.20 & \times & 125{,}000 & = & n \\ & & 25{,}000 & = & n \end{array}$$

$$\begin{array}{ccccc} \boxed{\begin{array}{c}\text{Purchase}\\ \text{price}\end{array}} & - & \boxed{\begin{array}{c}\text{down}\\ \text{payment}\end{array}} & = & \boxed{\text{mortgage}} \\ 125{,}000 & - & 25{,}000 & = & 100{,}000 \end{array}$$

The mortgage is \$100,000.

You Try It 1
An office building is purchased for \$216,000, and a down payment, which is 25% of the purchase price, is made. Find the mortgage.

Your strategy

Your solution

Solution on p. S16

Example 2

A home is purchased with a mortgage of \$65,000. The buyer pays a loan origination fee of $3\frac{1}{2}$ points. How much is the loan origination fee?

Strategy

To find the loan origination fee, solve the basic percent equation for *amount*.

Solution

$$\text{Percent} \times \text{base} = \text{amount}$$
$$\boxed{\text{Points}} \times \boxed{\text{mortgage}} = \boxed{\text{fee}}$$
$$0.035 \times 65{,}000 = n$$
$$2275 = n$$

The loan origination fee is \$2275.

You Try It 2

The mortgage on a real estate investment is \$80,000. The buyer paid a loan origination fee of $4\frac{1}{2}$ points. How much was the loan origination fee?

Your strategy

Your solution

Solution on p. S16

Objective B To calculate ongoing expenses of owning a home

Point of Interest

The number-one response of adults asked what they would spend money on first if they suddenly became wealthy (for example, by winning the lottery) was a house; 31% gave this response. (Source: Yankelovich Partners for Lutheran Brotherhood)

Besides the initial expenses of buying a house, there are continuing monthly expenses involved in owning a home. The monthly mortgage payment, utilities, insurance, and taxes are some of these ongoing expenses. Of these expenses, the largest one is normally the monthly mortgage payment.

For a fixed-rate mortgage, the monthly mortgage payment remains the same throughout the life of the loan. The calculation of the monthly mortgage payment is based on the amount of the loan, the interest rate on the loan, and the number of years required to pay back the loan. Calculating the monthly mortgage payment is fairly difficult, so tables such as the one in the Appendix are used to simplify these calculations.

⇒ Find the monthly mortgage payment on a 30-year \$60,000 mortgage at an interest rate of 9%. Use the Monthly Payment Table in the Appendix.

$$60{,}000 \times \underset{\substack{\uparrow \\ \text{From the} \\ \text{table}}}{\underline{0.0080462}} \approx 482.77$$

The monthly mortgage payment is \$482.77.

The monthly mortgage payment includes the payment of both principal and interest on the mortgage. The interest charged during any one month is charged on the unpaid balance of the loan. Therefore, during the early years of the mortgage, when the unpaid balance is high, most of the monthly mortgage payment is interest charged on the loan. During the last few years of a mortgage, when the unpaid balance is low, most of the monthly mortgage payment goes toward paying off the loan.

Point of Interest

Home buyers rated the following characteristics "extremely important" in their purchase decision.

Natural, open space: 77%
Walking and biking paths: 74%
Gardens with native plants: 56%
Clustered retail stores: 55%
Wilderness area: 52%
Outdoor pool: 52%
Community recreation center: 52%
Interesting little parks: 50%

(Source: American Lives, Inc. and Intercommunications, Inc.)

Find the interest paid on a mortgage during a month when the monthly mortgage payment is \$186.26 and \$58.08 of that amount goes toward paying off the principal.

$$\boxed{\text{Monthly mortgage payment}} - \boxed{\text{principal}} = \boxed{\text{interest}}$$

$$186.26 - 58.08 = 128.18$$

The interest paid on the mortgage is \$128.18.

Property tax is another ongoing expense of owning a house. Property tax is normally an annual expense that may be paid on a monthly basis. The monthly property tax, which is determined by dividing the annual property tax by 12, is usually added to the monthly mortgage payment.

A homeowner must pay \$534 in property tax annually. Find the property tax that must be added each month to the homeowner's monthly mortgage payment.

$534 \div 12 = 44.5$

Each month, \$44.50 must be added to the monthly mortgage payment for property tax.

Example 3

Serge purchased some land for \$120,000 and made a down payment of \$25,000. The savings and loan association charges an annual interest rate of 8% on Serge's 25-year mortgage. Find the monthly mortgage payment.

Strategy

To find the monthly mortgage payment:

- Subtract the down payment from the purchase price to find the mortgage.
- Multiply the mortgage by the factor found in the Monthly Payment Table in the Appendix.

Solution

$$\boxed{\begin{array}{c}\text{Purchase}\\ \text{price}\end{array}} - \boxed{\begin{array}{c}\text{down}\\ \text{payment}\end{array}} = \boxed{\text{mortgage}}$$

$$120{,}000 - 25{,}000 = 95{,}000$$

$$95{,}000 \times \underset{\substack{\uparrow \\ \text{From the table}}}{0.0077182} \approx 733.23$$

The monthly mortgage payment is \$733.23.

You Try It 3

A new condominium project is selling townhouses for \$75,000. A down payment of \$15,000 is required, and a 20-year mortgage at an annual interest rate of 9% is available. Find the monthly mortgage payment.

Your strategy

Your solution

Solution on p. S17

Example 4

A home has a mortgage of $134,000 for 25 years at an annual interest rate of 7%. During a month when $375.88 of the monthly mortgage payment is principal, how much of the payment is interest?

Strategy

To find the interest:

- Multiply the mortgage by the factor found in the Monthly Payment Table in the Appendix to find the monthly mortgage payment.
- Subtract the principal from the monthly mortgage payment.

Solution

$$134{,}000 \times \underset{\substack{\uparrow \\ \text{From the} \\ \text{table}}}{0.0070678} \approx \underset{\substack{\uparrow \\ \text{Monthly mortgage} \\ \text{payment}}}{947.09}$$

$$\boxed{\begin{array}{c}\text{Monthly}\\ \text{mortgage}\\ \text{payment}\end{array}} - \boxed{\text{principal}} = \boxed{\text{interest}}$$

$$947.09 - 375.88 = 571.21$$

$571.21 of the payment is interest on the mortgage.

You Try It 4

An office building has a mortgage of $125,000 for 25 years at an annual interest rate of 9%. During a month when $492.65 of the monthly mortgage payment is principal, how much of the payment is interest?

Your strategy

Your solution

Example 5

The monthly mortgage payment for a home is $598.75. The annual property tax is $900. Find the total monthly payment for the mortgage and property tax.

Strategy

To find the monthly payment:

- Divide the annual property tax by 12 to find the monthly property tax.
- Add the monthly property tax to the monthly mortgage payment.

Solution

$900 \div 12 = 75$ monthly property tax

$598.75 + 75 = 673.75$

The total monthly payment is $673.75.

You Try It 5

The monthly mortgage payment for a home is $415.20. The annual property tax is $744. Find the total monthly payment for the mortgage and property tax.

Your strategy

Your solution

Solutions on p. S17

6.4 Exercises

Objective A *Application Problems*

1. A condominium at Mt. Baldy Ski Resort was purchased for $97,000, and a down payment of $14,550 was made. Find the mortgage.

2. An insurance business was purchased for $173,000, and a down payment of $34,600 was made. Find the mortgage.

3. A building lot was purchased for $25,000. The lender requires a down payment of 30% of the purchase price. Find the down payment.

4. Ian Goldman purchased a new home for $88,500. The lender requires a down payment of 20% of the purchase price. Find the down payment.

5. Brian Stedman made a down payment of 25% of the $850,000 purchase price of an apartment building. How much was the down payment?

6. A clothing store was purchased for $125,000, and a down payment that was 25% of the purchase price was made. How much was the down payment?

7. A loan of $150,000 is obtained to purchase a home. The loan origination fee is $2\frac{1}{2}$ points. Find the amount of the loan origination fee.

8. Security Savings & Loan requires a borrower to pay $3\frac{1}{2}$ points for a loan. Find the amount of the loan origination fee for a loan of $90,000.

9. Baja Construction Inc. is selling homes for $150,000. A down payment of 5% is required.
   a. Find the down payment.
   b. Find the mortgage.

10. A cattle rancher purchased some land for $240,000. The bank requires a down payment of 15% of the purchase price.
    a. Find the down payment.
    b. Find the mortgage.

11. Vivian Tom purchased a home for $210,000. Find the mortgage if the down payment Vivian made is 10% of the purchase price.

12. A mortgage lender requires a down payment of 5% of the $80,000 purchase price of a condominium. How much is the mortgage?

Objective B *Application Problems*

Solve. Use the Monthly Payment Table in the Appendix. Round to the nearest cent.

13. An investor obtained a loan of $150,000 to buy a car wash business. The monthly mortgage payment was based on 25 years at 8%. Find the monthly mortgage payment.

14. A beautician obtained a 20-year mortgage of $90,000 to expand the business. The credit union charges an annual interest rate of 9%. Find the monthly mortgage payment.

15. A couple interested in buying a home determines that they can afford a monthly mortgage payment of $800. Can they afford to buy a home with a 30-year $110,000 mortgage at 8% interest?

16. A lawyer is considering purchasing a new office building with a 15-year $400,000 mortgage at 8% interest. The lawyer can afford a monthly mortgage payment of $4000. Can the lawyer afford the monthly mortgage payment on the new office building?

17. The county tax assessor has determined that the annual property tax on a $125,000 house is $1348.20. Find the monthly property tax.

18. The annual property tax on a $155,000 home is $1992. Find the monthly property tax.

19. Abacus Imports Inc. has a warehouse with a 25-year mortgage of $200,000 at an annual interest rate of 9%.
a. Find the monthly mortgage payment.
b. During a month when $941.72 of the monthly mortgage payment is principal, how much of the payment is interest?

20. A vacation home has a mortgage of $135,000 for 30 years at an annual interest rate of 7%.
a. Find the monthly mortgage payment.
b. During a month when $392.47 of the monthly mortgage payment is principal, how much of the payment is interest?

21. The annual mortgage payment on a duplex is $10,844.40. The owner must pay an annual property tax of $948. Find the total monthly payment for the mortgage and property tax.

22. The monthly mortgage payment on a home is $716.40, and the homeowner pays an annual property tax of $792. Find the total monthly payment for the mortgage and property tax.

23. Maria Hernandez purchased a home for $210,000 and made a down payment of $15,000. The balance was financed for 15 years at an annual interest rate of 9%. Find the monthly mortgage payment.

24. A customer of a savings and loan purchased a $185,000 home and made a down payment of $20,000. The savings and loan charges its customers an annual interest rate of 7% for 30 years for a home mortgage. Find the monthly mortgage payment.

APPLYING THE CONCEPTS

25. A couple considering a mortgage of $100,000 have a choice of loans. One loan is an 8% loan for 20 years, and the other loan is at 8% for 30 years. Find the amount of interest that the couple can save by choosing the 20-year loan.

26. Find out what an adjustable-rate mortgage is. What is the difference between this type of loan and a fixed-rate mortgage? List some of the advantages and disadvantages of each.

6.5 Car Expenses

Objective A To calculate the initial expenses of buying a car

The initial expenses in the purchase of a car usually include the down payment, the license fees, and the sales tax. The down payment may be very small or as much as 25% or 30% of the purchase price of the car, depending on the lending institution. License fees and sales tax are regulated by each state, so these expenses vary from state to state.

Example 1
A car is purchased for \$18,500, and the lender requires a down payment of 15% of the purchase price. Find the amount financed.

Strategy
To find the amount financed:

- Find the down payment by solving the basic percent equation for *amount.*
- Subtract the down payment from the purchase price.

Solution

$$\text{Percent} \times \text{base} = \text{amount}$$
$$\boxed{\text{Percent}} \times \boxed{\text{purchase price}} = \boxed{\text{down payment}}$$
$$0.15 \times 18{,}500 = n$$
$$2775 = n$$

$18{,}500 - 2775 = 15{,}725$

The amount financed is \$15,725.

You Try It 1
A down payment of 20% of the \$19,200 purchase price of a new car is made. Find the amount financed.

Your strategy

Your solution

Example 2
A sales clerk purchases a car for \$16,500 and pays a sales tax that is 5% of the purchase price. How much is the sales tax?

Strategy
To find the sales tax, solve the basic percent equation for *amount.*

Solution

$$\text{Percent} \times \text{base} = \text{amount}$$
$$\boxed{\text{Percent}} \times \boxed{\text{purchase price}} = \boxed{\text{sales tax}}$$
$$0.05 \times 16{,}500 = n$$
$$825 = n$$

The sales tax is \$825.

You Try It 2
A car is purchased for \$17,350. The car license fee is 1.5% of the purchase price. How much is the license fee?

Your strategy

Your solution

Solutions on p. S17

Objective B To calculate ongoing expenses of owning a car

Besides the initial expenses of buying a car, there are continuing expenses involved in owning a car. These ongoing expenses include car insurance, gas and oil, general maintenance, and the monthly car payment. The monthly car payment is calculated in the same manner as monthly mortgage payments on a home loan. A monthly payment table, such as the one in the Appendix, is used to simplify the calculation of monthly car payments.

TAKE NOTE
The same formula that is used to calculate a monthly mortgage payment is used to calculate a monthly car payment.

Example 3
At a cost of \$.33 per mile, how much does it cost to operate a car during a year in which the car is driven 15,000 miles?

Strategy
To find the cost, multiply the cost per mile (0.33) by the number of miles driven (15,000).

Solution
$15{,}000 \times 0.33 = 4950$

The cost is \$4950.

You Try It 3
At a cost of \$.31 per mile, how much does it cost to operate a car during a year in which the car is driven 23,000 miles?

Your strategy

Your solution

Example 4
During one month, your total gasoline bill was \$84 and the car was driven 1200 miles. What was the cost per mile for gasoline?

Strategy
To find the cost per mile, divide the cost for gasoline (84) by the number of miles driven (1200).

Solution
$84 \div 1200 = 0.07$

The cost per mile was \$.07.

You Try It 4
In a year in which your total car insurance bill was \$360 and the car was driven 15,000 miles, what was the cost per mile for car insurance?

Your strategy

Your solution

Example 5
A car is purchased for \$18,500 with a down payment of \$3700. The balance is financed for 3 years at an annual interest rate of 9%. Find the monthly car payment.

Strategy
To find the monthly payment:

- Subtract the down payment from the purchase price to find the amount financed.
- Multiply the amount financed by the factor found in the Monthly Payment Table in the Appendix.

Solution
$18{,}500 - 3700 = 14{,}800$
$14{,}800 \times 0.0317997 \approx 470.64$

The monthly payment is \$470.64.

You Try It 5
A truck is purchased for \$25,900 with a down payment of \$6475. The balance is financed for 4 years at an annual interest rate of 8%. Find the monthly car payment.

Your strategy

Your solution

Solutions on p. S17

6.5 Exercises

Objective A *Application Problems*

1. Amanda has saved $780 to make a down payment on a used minivan that costs $7100. The car dealer requires a down payment of 12% of the purchase price. Has she saved enough money to make the down payment?

2. A sedan was purchased for $23,500. A down payment of 15% of the purchase price was required. How much was the down payment?

3. A drapery installer bought a van to carry drapery samples. The purchase price of the van was $26,500, and a 4.5% sales tax was paid. How much was the sales tax?

4. A & L Lumber Company purchased a delivery truck for $28,500. A sales tax of 4% of the purchase price was paid. Find the sales tax.

5. A license fee of 2% of the purchase price is paid on a pickup truck costing $22,500. Find the license fee for the truck.

6. Your state charges a license fee of 1.5% on the purchase price of a car. How much is the license fee for a car that costs $16,998?

7. An electrician bought a $32,000 flatbed truck. A state license fee of $275 and a sales tax of 3.5% of the purchase price are required.
   a. Find the sales tax.
   b. Find the total cost of the sales tax and the license fee.

8. A physical therapist bought a used car for $9375 and made a down payment of $1875. The sales tax is 5% of the purchase price.
   a. Find the sales tax.
   b. Find the total cost of the sales tax and the down payment.

9. Martin bought a motorcycle for $16,200 and made a down payment of 25% of the purchase price.
   a. Find the down payment.
   b. Find the amount financed.

10. A carpenter bought a utility van for $24,900 and made a down payment of 15% of the purchase price.
    a. Find the down payment.
    b. Find the amount financed.

11. An author bought a sports car for $35,000 and made a down payment of 20% of the purchase price. Find the amount financed.

12. Tania purchased a used car for $13,500 and made a down payment of 25% of the cost. Find the amount financed.

Objective B *Application Problems*

Solve. Use the Monthly Payment Table in the Appendix. Round to the nearest cent.

13. A rancher financed $14,000 for the purchase of a truck through a credit union at 9% interest for 4 years. Find the monthly truck payment.

14. A car loan of $8000 is financed for 3 years at an annual interest rate of 10%. Find the monthly car payment.

15. An estimate of the cost of owning a compact car is $.32 per mile. Using this estimate, find how much it costs to operate a car during a year in which the car is driven 16,000 miles.

16. An estimate of the cost of care and maintenance of automobile tires is $.015 per mile. Using this estimate, find how much it costs for care and maintenance of tires during a year in which the car is driven 14,000 miles.

17. A family spent $1600 on gas, oil, and car insurance during a period in which the car was driven 14,000 miles. Find the cost per mile for gas, oil, and car insurance.

18. Last year you spent $2100 for gasoline for your car. The car was driven 15,000 miles. What was your cost per mile for gasoline?

19. Elena's monthly car payment is $143.50. During a month in which $68.75 of the monthly payment is principal, how much of the payment is interest?

20. The cost for a pizza delivery truck for the year included $2868 in truck payments, $2400 for gasoline, and $675 for insurance. Find the total cost for truck payments, gasoline, and insurance for the year.

21. The city of Colton purchased a fire truck for $82,000 and made a down payment of $5400. The balance is financed for 5 years at an annual interest rate of 9%.
a. Find the amount financed.
b. Find the monthly truck payment.

22. A used car is purchased for $4995, and a down payment of $995 is made. The balance is financed for 3 years at an annual interest rate of 8%.
a. Find the amount financed.
b. Find the monthly car payment.

23. An artist purchased a new car costing $27,500 and made a down payment of $5500. The balance is financed for 3 years at an annual interest rate of 10%. Find the monthly car payment.

24. A camper is purchased for $19,500, and a down payment of $2500 is made. The balance is financed for 4 years at an annual interest rate of 9%. Find the monthly payment.

APPLYING THE CONCEPTS

25. One bank offers a 4-year car loan at an annual interest rate of 10% plus a loan application fee of $45. A second bank offers 4-year car loans at an annual interest rate of 11% but charges no loan application fee. If you need to borrow $5800 to purchase a car, which of the two bank loans has the lesser loan costs? Assume you keep the car for 4 years.

26. How much interest is paid on a 5-year car loan of $9000 if the interest rate is 9%?

6.6 Wages

Objective A

To calculate commissions, total hourly wages, and salaries

Commissions, hourly wage, and salary are three ways to receive payment for doing work.

Commissions are usually paid to salespersons and are calculated as a percent of total sales.

➡ As a real estate broker, Emma Smith receives a commission of 4.5% of the selling price of a house. Find the commission she earned for selling a home for $175,000.

To find the commission Emma earned, solve the basic percent equation for *amount*.

Percent × base = amount

$\boxed{\text{Commission rate}} \times \boxed{\text{total sales}} = \boxed{\text{commission}}$

0.045 × 175,000 = 7875

The commission is $7875.

An employee who receives an **hourly wage** is paid a certain amount for each hour worked.

➡ A plumber receives an hourly wage of $18.25. Find the plumber's total wages for working 37 hours.

To find the plumber's total wages, multiply the hourly wage by the number of hours worked.

$\boxed{\text{Hourly wage}} \times \boxed{\text{number of hours worked}} = \boxed{\text{total wages}}$

18.25 × 37 = 675.25

The plumber's total wages for working 37 hours are $675.25.

An employee who is paid a **salary** receives payment based on a weekly, biweekly (every other week), monthly, or annual time schedule. Unlike the employee who receives an hourly wage, the salaried worker does not receive additional pay for working more than the regularly scheduled workday.

➡ Ravi Basar is a computer operator who receives a weekly salary of $695. Find his salary for 1 month (4 weeks).

To find Ravi's salary for 1 month, multiply the salary per pay period by the number of pay periods.

$\boxed{\text{Salary per pay period}} \times \boxed{\text{number of pay periods}} = \boxed{\text{total salary}}$

695 × 4 = 2780

Ravi's total salary for 1 month is $2780.

Example 1
A pharmacist's hourly wage is $28. On Saturday, the pharmacist earns time and a half (1.5 times the regular hourly wage). How much does the pharmacist earn for working 6 hours on Saturday?

Strategy
To find the pharmacist's earnings:

- Find the hourly wage for working on Saturday by multiplying the hourly wage by 1.5.
- Multiply the hourly wage by the number of hours worked.

Solution
$28 \times 1.5 = 42 \qquad 42 \times 6 = 252$

The pharmacist earns $252.

You Try It 1
A construction worker, whose hourly wage is $18.50, earns double time (2 times the regular hourly wage) for working overtime. Find the worker's wages for working 8 hours of overtime.

Your strategy

Your solution

Example 2
An efficiency expert received a contract for $3000. The consultant spent 75 hours on the project. Find the consultant's hourly wage.

Strategy
To find the hourly wage, divide the total earnings by the number of hours worked.

Solution
$3000 \div 75 = 40$

The hourly wage was $40.

You Try It 2
A contractor for a bridge project receives an annual salary of $48,228. What is the contractor's salary per month?

Your strategy

Your solution

Example 3
Dani Greene earns $28,500 per year plus a 5.5% commission on sales over $100,000. During one year, Dani sold $150,000 worth of computers. Find Dani's total earnings for the year.

Strategy
To find the total earnings:

- Find the sales over $100,000.
- Multiply the commission rate by sales over $100,000.
- Add the commission to the annual pay.

Solution
$150{,}000 - 100{,}000 = 50{,}000$
$50{,}000 \times 0.055 = 2750$ Commission
$28{,}500 + 2750 = 31{,}250$

Dani earned $31,250.

You Try It 3
An insurance agent earns $27,000 per year plus a 9.5% commission on sales over $50,000. During one year, the agent's sales totaled $175,000. Find the agent's total earnings for the year.

Your strategy

Your solution

Solutions on pp. S17–S18

6.6 Exercises

Objective A Application Problems

1. Lewis works in a clothing store and earns $9.50 per hour. How much does he earn in a 40-hour week?

2. Sasha pays a gardener an hourly wage of $11. How much does she pay the gardener for working 25 hours?

3. A real estate agent receives a 3% commission for selling a house. Find the commission that the agent earned for selling a house for $131,000.

4. Ron Caruso works as an insurance agent and receives a commission of 40% of the first year's premium. Find Ron's commission for selling a life insurance policy with a first-year premium of $1050.

5. A stockbroker receives a commission of 1.5% of the price of stock that is bought or sold. Find the commission on 100 shares of stock that were bought for $5600.

6. The owner of the Carousel Art Gallery receives a commission of 20% on paintings that are sold on consignment. Find the commission on a painting that sold for $22,500.

7. Keisha Brown receives an annual salary of $38,928 as a teacher of Italian. How much does Keisha receive each month? (4 weeks)(12 mounths)

8. An apprentice plumber receives an annual salary of $27,900. How much does the plumber receive per month?

9. An electrician's hourly wage is $25.80. For working overtime, the electrician earns double time. What is the electrician's hourly wage for working overtime?

10. Carlos receives a commission of 12% of his weekly sales as a sales representative for a medical supply company. Find the commission he earned during a week in which sales were $4500.

11. A golf pro receives a commission of 25% for selling a golf set. Find the commission the pro earned for selling a golf set costing $450.

12. Steven receives $3.75 per square yard to install carpet. How much does he receive for installing 160 square yards of carpet?

13. A typist charges $2.75 per page for typing technical material. How much does the typist earn for typing a 225-page book?

14. A nuclear chemist received \$15,000 in consulting fees while working on a nuclear power plant. The chemist worked 120 hours on the project. Find the consultant's hourly wage.

15. Maxine received \$3400 for working on a project as a computer consultant for 40 hours. Find her hourly wage.

16. Gil Stratton's hourly wage is \$10.78. For working overtime, he receives double time.
a. What is Gil's hourly wage for working overtime?
b. How much does he earn for working 16 hours of overtime?

17. Mark is a lathe operator and receives an hourly wage of \$15.90. When working on Saturday, he receives time and a half.
a. What is Mark's hourly wage on Saturday?
b. How much does he earn for working 8 hours on Saturday?

18. A stock clerk at a supermarket earns \$8.20 an hour. For working the night shift, the clerk's wage increases by 15%.
a. What is the increase in hourly pay for working the night shift?
b. What is the clerk's hourly wage for working the night shift?

19. A nurse earns \$16.50 an hour. For working the night shift, the nurse receives a 10% increase in pay.
a. What is the increase in hourly pay for working the night shift?
b. What is the hourly pay for working the night shift?

20. Tony's hourly wage as a service station attendant is \$9.40. For working the night shift, his wage is increased 25%. What is Tony's hourly wage for working the night shift?

21. Nicole Tobin, a salesperson, receives a salary of \$250 per week plus a commission of 15% on all sales over \$1500. Find her earnings during a week in which sales totaled \$3000.

APPLYING THE CONCEPTS

The table at the right shows the top five and the bottom five starting salaries for recent college graduates. Use this table for Exercises 22 to 25.

Estimated Starting Salaries

Bachelor's Degree	*Estimated Starting Salary*	*% Change from Previous Year*
Top 5		
Chemical engineering	\$42,758	4.3%
Mechanical engineering	\$39,852	4.5%
Electrical engineering	\$38,811	4.0%
Industrial engineering	\$37,732	4.0%
Computer science	\$36,964	4.5%
Bottom 5		
Liberal arts	\$24,102	3.5%
Natural resources	\$22,950	3.5%
Home economics	\$22,916	3.5%
Telecommunications	\$22,447	4.0%
Journalism	\$22,102	4.0%

Source: Michigan State University

22. What was the amount of increase in the starting salary for journalism majors from the previous year? Round to the nearest dollar.

23. What was the starting salary in the previous year for an industrial engineer? Round to the nearest dollar.

24. Between electrical engineers and computer scientists, which received the larger amount of increase in starting salary from the previous year?

25. Between telecommunications majors and liberal arts majors, which received the smaller amount of increase in starting salary from the previous year?

6.7

Bank Statements

Objective A

To calculate checkbook balances

A checking account can be opened at most banks and savings and loan associations by depositing an amount of money in the bank. A checkbook contains checks and deposit slips and a checkbook register in which to record checks written and amounts deposited in the checking account. Each time a check is written, the amount of the check is subtracted from the amount in the account. When a deposit is made, the amount deposited is added to the amount in the account.

A portion of a checkbook register is shown below. The account holder had a balance of $587.93 before writing two checks, one for $286.87 and the other for $202.38, and making one deposit of $345.00.

RECORD ALL CHARGES OR CREDITS THAT AFFECT YOUR ACCOUNT

NUMBER	DATE	DESCRIPTION OF TRANSACTION	PAYMENT/DEBIT (–)		√ T	FEE (IF ANY) (–)	DEPOSIT/CREDIT (+)		BALANCE	
									$ 587	93
108	8/4	Plumber	$ 286	87		$	$		301	06
109	8/10	Car Payment	202	38					98	68
	8/14	Deposit					345	00	443	68

To find the current checking account balance, subtract the amount of each check from the previous balance. Then add the amount of the deposit.

The current checking account balance is $443.68.

Point of Interest

There are a number of computer programs that serve as "electronic" checkbooks. With these programs, you can pay your bills by using a computer to write the check and then transmit the check over telephone lines using a modem.

Example 1 A mail carrier had a checking account balance of $485.93 before writing two checks, one for $18.98 and another for $35.72, and making a deposit of $250. Find the current checking account balance.

Strategy To find the current balance:

- Subtract the amount of each check from the old balance.
- Add the amount of the deposit.

Solution

$$\begin{array}{r l} 485.93 & \\ -\ \ 18.98 & \text{first check} \\ \hline 466.95 & \\ -\ \ 35.72 & \text{second check} \\ \hline 431.23 & \\ +\ 250.00 & \text{deposit} \\ \hline 681.23 & \end{array}$$

The current checking account balance is $681.23.

You Try It 1 A cement mason had a checking account balance of $302.46 before writing a check for $20.59 and making two deposits, one in the amount of $176.86 and another in the amount of $94.73. Find the current checking account balance.

Your strategy

Your solution

Solution on p. S18

Objective B To balance a checkbook

Each month a bank statement is sent to the account holder. The bank statement shows the checks that the bank has paid, the deposits received, and the current bank balance.

A bank statement and checkbook register are shown on the next page.

Balancing the checkbook, or determining whether the checking account balance is accurate, requires a number of steps.

1. In the checkbook register, put a check mark (✓) by each check paid by the bank and by each deposit recorded by the bank.

RECORD ALL CHARGES OR CREDITS THAT AFFECT YOUR ACCOUNT

NUMBER	DATE	DESCRIPTION OF TRANSACTION	PAYMENT/DEBIT (-)		√ T	FEE (IF ANY) (–)	DEPOSIT/CREDIT (+)		BALANCE	
									$ 840	27
263	5/20	Dentist	$ 75	00	√	$	$		765	27
264	5/22	Post Office	33	61	√				731	66
265	5/22	Gas Company	67	14					664	52
	5/29	Deposit			√		192	00	856	52
266	5/29	Pharmacy	38	95	√				817	57
267	5/30	Telephone	63	85					753	72
268	6/2	Groceries	73	19	√				680	53
	6/3	Deposit			√		215	00	895	53
269	6/7	Insurance	103	00	√				792	53
	6/10	Deposit					225	00	1017	53
270	6/15	Photo Shop	16	63	√				1000	90
271	6/18	Newspaper	27	00					973	90

CHECKING ACCOUNT Monthly Statement Account Number: 924-297-8

Date	Transaction	Amount	Balance
5/20	OPENING BALANCE		840.27
5/21	CHECK	75.00	765.27
5/23	CHECK	33.61	731.66
5/29	DEPOSIT	192.00	923.66
6/1	CHECK	38.95	884.71
6/1	INTEREST	4.47	889.18
6/3	CHECK	73.19	815.99
6/3	DEPOSIT	215.00	1030.99
6/9	CHECK	103.00	927.99
6/16	CHECK	16.63	911.36
6/20	SERVICE CHARGE	3.00	908.36
6/20	CLOSING BALANCE		908.36

2. Add to the current checkbook balance all checks that have been written but have not yet been paid by the bank and any interest paid on the account.

Current checkbook balance:	973.90
Checks: 265	67.14
267	63.85
271	27.00
Interest:	+ 4.47
	1136.36

3. Subtract any service charges and any deposits not yet recorded by the bank. This is the checkbook balance.

Service charge:	− 3.00
	1133.36
Deposit:	− 225.00
Checkbook balance:	908.36

4. Compare the balance with the bank balance listed on the bank statement. If the two numbers are equal, the bank statement and the checkbook "balance."

Closing bank balance from bank statement		Checkbook balance
$908.36	=	$908.36

The bank statement and checkbook balance.

RECORD ALL CHARGES OR CREDITS THAT AFFECT YOUR ACCOUNT

NUMBER	DATE	DESCRIPTION OF TRANSACTION	PAYMENT/DEBIT (−)		√ T	FEE (IF ANY) (−)	DEPOSIT/CREDIT (+)		BALANCE	
									$ 1620	42
413	3/2	Car Payment	$ 232	15	√	$	$		1388	27
414	3/2	Utility	67	14	√				1321	13
415	3/5	Restaurant	78	14					1242	99
	3/8	Deposit			√		1842	66	3085	65
416	3/10	House Payment	672	14	√				2413	51
417	3/14	Insurance	177	10					2236	41

CHECKING ACCOUNT Monthly Statement — Account Number: 924-297-8

Date	Transaction	Amount	Balance
3/1	OPENING BALANCE		1620.42
3/4	CHECK	232.15	1388.27
3/5	CHECK	67.14	1321.13
3/8	DEPOSIT	1842.66	3163.79
3/10	INTEREST	6.77	3170.56
3/12	CHECK	672.14	2498.42
3/25	SERVICE CHARGE	2.00	2496.42
3/30	CLOSING BALANCE		2496.42

Balance the bank statement shown above.

1. In the checkbook register, put a check mark (√) by each check paid by the bank and by each deposit recorded by the bank.
2. Add to the current checkbook balance all checks that have been written but have not yet been paid by the bank and any interest paid on the account.
3. Subtract any service charges and any deposits not yet recorded by the bank. This is the checkbook balance.
4. Compare the balance with the bank balance listed on the bank statement. If the two numbers are equal, the bank statement and the checkbook "balance."

Current checkbook balance:	2236.41
Checks: 415	78.14
417	177.10
Interest:	+ 6.77
	2498.42
Service charge:	− 2.00
Checkbook balance:	2496.42

Closing bank balance from bank statement	Checkbook balance
$2496.42	= $2496.42

The bank statement and checkbook balance.

Example 2 Balance the bank statement shown below.

RECORD ALL CHARGES OR CREDITS THAT AFFECT YOUR ACCOUNT

NUMBER	DATE	DESCRIPTION OF TRANSACTION	PAYMENT/DEBIT (−)		√ T	FEE (IF ANY) (−)	DEPOSIT/CREDIT (+)		BALANCE	
									$ 412	64
345	1/14	Phone Bill	$ 54	75	√	$	$		357	89
346	1/19	News Shop	18	98	√				338	91
347	1/23	Theatre Tickets	95	00					243	91
	1/31	Deposit			√		947	00	1190	91
348	2/5	Cash	250	00	√				940	91
349	2/12	Rent	840	00					100	91

CHECKING ACCOUNT Monthly Statement Account Number: 924-297-8

Date	Transaction	Amount	Balance
1/10	OPENING BALANCE		412.64
1/18	CHECK	54.75	357.89
1/23	CHECK	18.98	338.91
1/31	DEPOSIT	947.00	1285.91
2/1	INTEREST	4.52	1290.43
2/10	CHECK	250.00	1040.43
2/10	CLOSING BALANCE		1040.43

Solution

Current checkbook balance:	100.91
Checks: 347	95.00
349	840.00
Interest:	+ 4.52
	1040.43
Service charge:	− 0.00
	1040.43
Deposit:	− 0.00
Checkbook balance:	1040.43

Closing bank balance from bank statement: $1040.43

Checkbook balance: $1040.43

The bank statement and the checkbook balance.

You Try It 2

Balance the bank statement shown below.

RECORD ALL CHARGES OR CREDITS THAT AFFECT YOUR ACCOUNT

NUMBER	DATE	DESCRIPTION OF TRANSACTION	PAYMENT/DEBIT (–)		√ T	FEE (IF ANY) (–)	DEPOSIT/CREDIT (+)		BALANCE	
									$ 903	17
	2/15	Deposit	$			$	$ 523	84	1427	01
234	2/20	Mortgage	773	21					653	80
235	2/27	Cash	200	00					453	80
	3/1	Deposit					523	84	977	64
236	3/12	Insurance	275	50					702	14
237	3/12	Telephone	78	73					623	41

CHECKING ACCOUNT Monthly Statement Account Number: 314-271-4

Date	Transaction	Amount	Balance
2/14	OPENING BALANCE		903.17
2/15	DEPOSIT	523.84	1427.01
2/21	CHECK	773.21	653.80
2/28	CHECK	200.00	453.80
3/1	INTEREST	2.11	455.91
3/14	CHECK	275.50	180.41
3/14	CLOSING BALANCE		180.41

Your solution

Solution on p. S18

6.7 Exercises

Objective A *Application Problems*

1. You had a checking account balance of $342.51 before making a deposit of $143.81. What is your new checking account balance?

2. Carmen had a checking account balance of $493.26 before writing a check for $48.39. What is the current checking account balance?

3. A real estate firm had a balance of $2431.76 in its rental-property checking account. What is the balance in this account after a check for $1209.29 has been written?

4. The business checking account for R and R Tires showed a balance of $1536.97. What is the balance in this account after a deposit of $439.21 has been made?

5. A nutritionist had a checking account balance of $1204.63 before writing one check for $119.27 and another check for $260.09. Find the current checkbook balance.

6. Sam had a checking account balance of $3046.93 before writing a check for $1027.33 and making a deposit of $150.00. Find the current checkbook balance.

7. The business checking account for Rachael's Dry Cleaning had a balance of $3476.85 before a deposit of $1048.53 was made. The store manager then wrote checks, one for $848.37 and another for $676.19. Find the current checkbook balance.

8. Joel had a checking account balance of $427.38 before a deposit of $127.29 was made. Joel then wrote two checks, one for $43.52 and one for $249.78. Find the current checkbook balance.

9. A carpenter had a checkbook balance of $404.96 before making a deposit of $350 and writing a check for $71.29. Is there enough money in the account to purchase a refrigerator for $675?

10. A taxi driver had a checkbook balance of $149.85 before making a deposit of $245 and writing a check for $387.68. Is there enough money in the account for the bank to pay the check?

11. A sporting goods store has the opportunity to buy downhill skis and cross-country skis at a manufacturer's closeout sale. The downhill skis will cost $3500, and the cross-country skis will cost $2050. There is currently $5625.42 in the sporting goods store's checking account. Is there enough money in the account to make both purchases by check?

12. A lathe operator's current checkbook balance is $1143.42. The operator wants to purchase a utility trailer for $525 and a used piano for $650. Is there enough money in the account to make the two purchases?

Objective B *Application Problems*

13. Balance the checkbook.

RECORD ALL CHARGES OR CREDITS THAT AFFECT YOUR ACCOUNT

NUMBER	DATE	DESCRIPTION OF TRANSACTION	PAYMENT/DEBIT (−)		√ T	FEE (IF ANY) (−)	DEPOSIT/CREDIT (+)		BALANCE	
									$ 2466	79
223	3/2	Groceries	$ 167	32		$	$		2299	47
	3/5	Deposit					960	70	3260	17
224	3/5	Rent	860	00					2400	17
225	3/7	Gas & Electric	142	35					2257	82
226	3/7	Cash	300	00					1957	82
227	3/7	Insurance	218	44					1739	38
228	3/7	Credit Card	419	32					1320	06
229	3/12	Dentist	92	00					1228	06
230	3/13	Drug Store	47	03					1181	03
	3/19	Deposit					960	70	2141	73
231	3/22	Car Payment	241	35					1900	38
232	3/25	Cash	300	00					1600	38
233	3/25	Oil Company	166	40					1433	98
234	3/28	Plumber	155	73					1278	25
235	3/29	Department Store	288	39					989	86

CHECKING ACCOUNT Monthly Statement Account Number: 122-345-1

Date	Transaction	Amount	Balance
3/1	OPENING BALANCE		2466.79
3/5	DEPOSIT	960.70	3427.49
3/7	CHECK	167.32	3260.17
3/8	CHECK	860.00	2400.17
3/8	CHECK	300.00	2100.17
3/9	CHECK	142.35	1957.82
3/12	CHECK	218.44	1739.38
3/14	CHECK	92.00	1647.38
3/18	CHECK	47.03	1600.35
3/19	DEPOSIT	960.70	2561.05
3/25	CHECK	241.35	2319.70
3/27	CHECK	300.00	2019.70
3/29	CHECK	155.73	1863.97
3/30	INTEREST	13.22	1877.19
4/1	CLOSING BALANCE		1877.19

14. Balance the checkbook.

RECORD ALL CHARGES OR CREDITS THAT AFFECT YOUR ACCOUNT

NUMBER	DATE	DESCRIPTION OF TRANSACTION	PAYMENT/DEBIT (–)		√ T	FEE (IF ANY) (–)	DEPOSIT/CREDIT (+)		BALANCE	
									$ 1219	43
	5/1	Deposit	$			$	$ 619	14	1838	57
515	5/2	Electric Bill	42	35					1796	22
516	5/2	Groceries	95	14					1701	08
517	5/4	Insurance	122	17					1578	91
518	5/5	Theatre Tickets	84	50					1494	41
	5/8	Deposit					619	14	2113	55
519	5/10	Telephone	37	39					2076	16
520	5/12	Newspaper	22	50					2053	66
	5/15	Interest					7	82	2061	48
	5/15	Deposit					619	14	2680	62
521	5/20	Computer Store	172	90					2507	72
522	5/21	Credit Card	313	44					2194	28
523	5/22	Eye Exam	82	00					2112	28
524	5/24	Groceries	107	14					2005	14
525	5/24	Deposit					619	14	2624	28
526	5/25	Oil Company	144	16					2480	12
527	5/30	Car Payment	288	62					2191	50
528	5/30	Mortgage Payment	877	42					1314	08

CHECKING ACCOUNT Monthly Statement Account Number: 122-345-1

Date	Transaction	Amount	Balance
5/1	OPENING BALANCE		1219.43
5/1	DEPOSIT	619.14	1838.57
5/3	CHECK	95.14	1743.43
5/4	CHECK	42.35	1701.08
5/6	CHECK	84.50	1616.58
5/8	CHECK	122.17	1494.41
5/8	DEPOSIT	619.14	2113.55
5/15	INTEREST	7.82	2121.37
5/15	CHECK	37.39	2083.98
5/15	DEPOSIT	619.14	2703.12
5/23	CHECK	82.00	2621.12
5/23	CHECK	172.90	2448.22
5/24	CHECK	107.14	2341.08
5/24	DEPOSIT	619.14	2960.22
5/30	CHECK	288.62	2671.60
6/1	CLOSING BALANCE		2671.60

15. Balance the checkbook.

RECORD ALL CHARGES OR CREDITS THAT AFFECT YOUR ACCOUNT

NUMBER	DATE	DESCRIPTION OF TRANSACTION	PAYMENT/DEBIT (–)		√ T	FEE (IF ANY) (–)	DEPOSIT/CREDIT (+)		BALANCE	
									$ 2035	18
218	7/2	Mortgage	$ 984	60	√	$	$		1050	58
219	7/4	Telephone	63	36	√				987	22
220	7/7	Cash	200	00	√				787	22
	7/12	Deposit			√		792	60	1579	82
221	7/15	Insurance	292	30	√				1287	52
222	7/18	Investment	500	00	√				787	52
223	7/20	Credit Card	414	83					372	69
	7/26	Deposit			√		792	60	1165	29
224	7/27	Department Store	113	37					1051	92

CHECKING ACCOUNT Monthly Statement — Account Number: 122-345-1

Date	Transaction	Amount	Balance
7/1	OPENING BALANCE		2035.18
7/1	INTEREST	5.15	2040.33
7/4	CHECK	984.60	1055.73
7/6	CHECK	63.36	992.37
7/12	DEPOSIT	792.60	1784.97
7/20	CHECK	292.30	1492.67
7/24	CHECK	500.00	992.67
7/26	DEPOSIT	792.60	1785.27
7/28	CHECK	200.00	1585.27
7/30	CLOSING BALANCE		1585.27

APPLYING THE CONCEPTS

16. When a check is written, the amount is ___________ from the balance.

17. When a deposit is made, the amount is ___________ to the balance.

18. In checking the bank statement, ___________ to the checkbook balance all checks that have been written but not processed.

19. In checking the bank balance, ___________ any service charge and any deposits not yet recorded.

20. Define the words *credit* and *debit* as they apply to checkbooks.

Focus on Problem Solving

Counterexamples

An example that is given to show that a statement is not true is called a **counterexample.** For instance, suppose someone makes the statement "All colors are red." A counterexample to that statement would be to show someone the color blue or some other color.

If a statement is *always* true, there are no counterexamples. The statement "All even numbers are divisible by 2" is always true. It is not possible to give an example of an even number that is not divisible by 2.

TAKE NOTE
Recall that a prime number is a natural number greater than 1 that can be divided by only itself and 1. For instance, 17 is a prime number. 12 is not a prime number because 12 is divisible by numbers other than 1 and 12—for example, 4.

In mathematics, statements that are always true are called *theorems,* and mathematicians are always searching for theorems. Sometimes a conjecture by a mathematician appears to be a theorem. That is, the statement appears to be always true, but later on someone finds a counterexample.

One example of this occurred when the French mathematician Pierre de Fermat (1601–1665) conjectured that $2^{(2^n)} + 1$ is always a prime number for any natural number n. For instance, when $n = 3$, we have $2^{(2^3)} + 1 = 2^8 + 1 = 257$, and 257 is a prime number. However, in 1732 Leonhard Euler (1707–1783) showed that when $n = 5$, $2^{(2^5)} + 1 = 4{,}294{,}967{,}297$ and that $4{,}294{,}967{,}297 = 641 \cdot 6{,}700{,}417$—without a calculator! Because 4,294,967,297 was the product of two numbers (other than itself and 1), it is not a prime number. This counterexample showed that Fermat's conjecture is not a theorem.

For Exercises 1 and 5, find at least one counterexample.

1. All composite numbers are divisible by 2.

2. All prime numbers are odd numbers.

3. The square of any number is always bigger than the number.

4. The reciprocal of a number is always less than 1.

5. A number ending in 9 is always larger than a number ending in 3.

When a problem is posed, it may not be known whether the problem statement is true or false. For instance, Christian Goldbach (1690–1764) stated that every even number greater than 2 can be written as the sum of two prime numbers. For example,

$$12 = 5 + 7 \qquad 32 = 3 + 29$$

Although this problem is approximately 250 years old, mathematicians have not been able to prove it is a theorem, nor have they been able to find a counterexample.

For Exercises 6 to 9, answer true if the statement is always true. If there is an instance in which the statement is false, give a counterexample.

6. The sum of two positive numbers is always larger than either of the two numbers.

7. The product of two positive numbers is always larger than either of the two numbers.

8. Percents always represent a number less than or equal to 1.

9. It is never possible to divide by zero.

Projects and Group Activities

Buying a Car

Suppose a student has an after-school job to earn money to buy and maintain a car. We will make assumptions about the monthly costs in several categories in order to determine how many hours per week the student must work to support the car. Assume the student earns $8.50 per hour.

1. Monthly payment

   Assume that the car cost $8500 with a down payment of $1020. The remainder is financed for 3 years at an annual simple interest rate of 9%.

   Monthly payment = ______________

2. Insurance

   Assume that insurance costs $1500 per year.

   Monthly insurance payment = ______________

3. Gasoline

   Assume that the student travels 750 miles per month, that the car travels 25 miles per gallon of gasoline, and that gasoline costs $1.50 per gallon.

   Number of gallons of gasoline purchased per month = ______________

   Monthly cost for gasoline = ______________

4. Miscellaneous

   Assume $.33 per mile for upkeep.

   Monthly expense for upkeep = ______________

5. Total monthly expenses for the monthly payment, insurance, gasoline, and miscellaneous = ______________

6. To find the number of hours per month that the student must work to finance the car, divide the total monthly expenses by the hourly rate.

   Number of hours per month = ______________

7. To find the number of hours per week that the student must work, divide the number of hours per month by 4.

   Number of hours per week = ______________

   The student has to work ______________ hours per week to pay the monthly car expenses.

If you own a car, make out your own expense record. If you do not own a car, make assumptions about the kind of car that you would like to purchase, and calculate the total monthly expenses that you would have. An insurance company will give you rates on different kinds of insurance. An automobile club can give you approximations of miscellaneous expenses.

Credit Card Charges

All credit card companies assess a finance charge when a credit card balance is not paid within a certain number of days of the *billing date* (the date the credit card bill is sent). There may also be an annual fee.

The table below shows the charges for five banks in September of 2000. (If you have Internet access, you can get current rates by visiting **http://www.bankrate.com**.)

Bank	*Annual Interest Rate*	*Annual Fee*
Pulaski Bank & Trust Co.	8.75%	\$35
Wachovia Bank Card Services	9.50%	\$88
USAA Savings Bank	10.50%	\$45
Metropolitan National Bank	10.98%	\$25
First Internet Bank of Indiana	12.00%	\$0

The amount of the monthly finance charge is usually based on the average daily balance. For instance, say your credit card was issued by the USAA Savings Bank. If your average daily balance for one month was \$375.89, then the monthly finance charge is calculated, using the simple interest formula, as follows:

TAKE NOTE
The monthly interest rate is the annual interest rate divided by 12.

$$\boxed{\text{Interest}} = \boxed{\text{principal}} \times \boxed{\text{monthly interest rate}} \times \boxed{\text{time (in months)}}$$

$$= 375.89 \times \frac{0.1050}{12} \times 1$$

$$\approx 3.29$$

The finance charge is \$3.29.

1. Suppose you have an average daily balance of \$421.65 for 12 months. With which bank will your *annual* charges (the finance charges and the annual fee) be least?
2. Suppose you have an average daily balance of \$1175.91 for 12 months. With which bank will your *annual* charges (the finance charges and the annual fee) be least?

Chapter Summary

Key Words

The *unit cost* is the cost of one item. [p. 229]

Percent increase is used to show how much a quantity has increased over its original value. [p. 233]

Cost is the price a business pays for a product. [p. 234]

Selling price is the price at which a business sells a product to a customer. [p. 234]

Markup is the difference between selling price and cost. [p. 234]

Markup rate is the markup expressed as a percent of a product's cost. [p. 234]

Percent decrease is used to show how much a quantity has decreased from its original value. [p. 236]

Sale price is the price after a reduction from the regular price. [p. 237]

Discount is the difference between the regular price and the sale price. [p. 237]

Discount rate is the discount as a percent of a product's regular price. [p. 237]

Interest is the amount of money paid for the privilege of using someone else's money. [p. 243]

Principal is the amount of money originally deposited or borrowed. [p. 243]

The percent used to determine the amount of interest is the *interest rate.* [p. 243]

Interest computed on the original principal is called *simple interest.* [p. 243]

Compound interest is computed not only on the original principal but also on interest already earned. [p. 246]

A *mortgage* is an amount that is borrowed to buy real estate. [p. 253]

The *loan origination fee* is usually a percent of the mortgage and is expressed in *points.* [p. 253]

Commissions are usually paid to salespersons and are calculated as a percent of total sales. [p. 263]

An employee who receives an *hourly wage* is paid a certain amount for each hour worked. [p. 263]

An employee who is paid a *salary* receives payment based on a weekly, biweekly, monthly, or annual time schedule. [p. 263]

To *balance a checkbook* means to determine whether the checkbook balance is accurate. [p. 268]

Essential Rules

To Find Unit Cost	To find the unit cost, divide the total cost by the number of units. [p. 229]
To Find Total Cost	To find the total cost, multiply the unit cost by the number of units. [p. 230]
Basic Markup Equations	Selling price = cost + markup Markup = markup rate × cost [p. 234]
Basic Discount Equations	Sale price = regular price − discount Discount = discount rate × regular price [p. 237]
Annual Simple Interest Formula	Principal × annual interest rate × time in years = interest [p. 243]
Maturity Value Formula for Simple Interest	Principal + interest = maturity value [p. 243]
Monthly Payment on a Simple Interest Loan	Maturity value ÷ length of the loan in months = monthly payment [p. 244]

Chapter Review

1. A 20-ounce box of cereal costs $3.90. Find the unit cost.

2. An account executive had car expenses of $1025.58 for insurance, $605.82 for gas, $37.92 for oil, and $188.27 for maintenance during a year in which 11,320 miles were driven. Find the cost per mile for these four items taken as a group. Round to the nearest tenth of a cent.

3. An oil stock was bought for $42.375 per share. Six months later, the stock was selling for $55.25 per share. Find the percent increase in the price of the stock for the 6 months. Round to the nearest tenth of a percent.

4. A sporting goods store uses a markup rate of 40%. What is the markup on a ski suit that costs the store $180?

5. A contractor borrowed $100,000 from a credit union for 9 months at an annual interest rate of 9%. What is the simple interest due on the loan?

6. A computer programmer invested $25,000 in a retirement account that pays 6% interest, compounded daily. What is the value of the investment in 10 years? Use the Compound Interest Table in the Appendix. Round to the nearest cent.

7. Last year an oil company had earnings of $4.12 per share. This year the earnings are $4.73 per share. What is the percent increase in earnings per share? Round to the nearest percent.

8. The monthly mortgage payment for a condominium is $523.67. The owner must pay an annual property tax of $658.32. Find the total monthly payment for the mortgage and property tax.

9. A used pickup truck is purchased for $14,450. A down payment of 8% is made, and the remaining cost is financed for 4 years at an annual interest rate of 9%. Find the monthly payment. Use the Monthly Payment Table in the Appendix. Round to the nearest cent.

10. A fast-food restaurant invested $50,000 in an account that pays 7% annual interest compounded quarterly. What is the value of the investment in 1 year? Use the Compound Interest Table in the Appendix.

11. Paula Mason purchased a home for $125,000. The lender requires a down payment of 15%. Find the amount of the down payment.

12. A plumber bought a truck for $18,500. A state license fee of $315 and a sales tax of 6.25% of the purchase price are required. Find the total cost of the sales tax and the license fee.

13. Techno-Center uses a markup rate of 35% on all computer systems. Find the selling price of a computer system that costs the store $1540.

14. Mien pays a monthly car payment of $122.78. During a month in which $25.45 is principal, how much of the payment is interest?

15. The manager of the retail store at a ski resort receives a commission of 3% on all sales at the alpine shop. Find the total commission received during a month in which the shop had $108,000 in sales.

16. A suit that regularly costs $235 is on sale for 40% off the regular price. Find the sale price.

17. Luke had a checking account balance of $1568.45 before writing checks for $123.76, $756.45, and $88.77. He then deposited a check for $344.21. Find Luke's current checkbook balance.

18. Pros' Sporting Goods borrowed $30,000 at an annual interest rate of 8% for 6 months. Find the maturity value of the loan.

19. A credit union requires a borrower to pay $2\frac{1}{2}$ points for a loan. Find the origination fee for a $75,000 loan.

20. Sixteen ounces of mouthwash cost $3.49. A 33-ounce container of the same brand of mouthwash costs $6.99. Which is the better buy?

21. The Sweeneys bought a home for $156,000. The family made a 10% down payment and financed the remainder with a 30-year loan at an annual interest rate of 7%. Find the monthly mortgage payment. Use the Monthly Payment Table in the Appendix. Round to the nearest cent.

22. Richard Valdez receives $12.60 per hour for working 40 hours a week and time and a half for working over 40 hours. Find his total income during a week in which he worked 48 hours.

23. The business checking account of a donut shop showed a balance of $9567.44 before checks of $1023.55, $345.44, and $23.67 were written and checks of $555.89 and $135.91 were deposited. Find the current checkbook balance.

24. The simple interest due on a 4-month loan of $55,000 is $1375. Find the monthly payment on the loan.

25. A credit card company charges a customer 1.25% per month on the unpaid balance of charges on the card. What is the finance charge in a month in which the customer has an unpaid balance of $576?

Chapter Test

1. Twenty feet of lumber cost \$138.40. What is the cost per foot?

2. Which is the more economical purchase: 3 pounds of tomatoes for \$7.49 or 5 pounds of tomatoes for \$12.59?

3. Red snapper costs \$4.15 per pound. Find the cost of $3\frac{1}{2}$ pounds. Round to the nearest cent.

4. An exercise bicycle increased in price from \$415 to \$498. Find the percent increase in the cost of the exercise bicycle.

5. A department store uses a 40% markup rate. Find the selling price of a compact disc player that the store purchased for \$215.

6. The price of gold dropped from \$390 per ounce to \$360 per ounce. What percent decrease does this amount represent? Round to the nearest tenth of a percent.

7. The price of a video camera dropped from \$1120 to \$896. What percent decrease does this price drop represent?

8. A corner hutch with a regular price of \$299 is on sale for 30% off the regular price. Find the sale price.

9. A box of stationery that regularly sells for \$9.50 is on sale for \$5.70. Find the discount rate.

10. A construction company borrowed \$75,000 for 4 months at an annual interest rate of 8%. Find the simple interest due on the loan.

11. Craig Allen borrowed \$25,000 at an annual interest rate of 9.2% for 9 months. Find the maturity value of the loan.

12. A credit card company charges a customer 1.2% per month on the unpaid balance of charges on the card. What is the finance charge in a month in which the customer has an unpaid balance of \$374.95?

13. Jorge, who is self-employed, placed \$30,000 in an account that pays 6% annual interest compounded quarterly. How much interest was earned in 10 years? Use the Compound Interest Table in the Appendix.

14. A savings and loan institution is offering mortgage loans that have a loan origination fee of $2\frac{1}{2}$ points. Find the loan origination fee when a home is purchased with a loan of \$134,000.

15. A new housing development offers homes with a mortgage of $222,000 for 25 years at an annual interest rate of 8%. Find the monthly mortgage payment. Use the Monthly Payment Table in the Appendix.

16. A Chevrolet was purchased for $23,750, and a 20% down payment was made. Find the amount financed.

17. A rancher purchased an SUV for $23,714 and made a down payment of 15% of the cost. The balance was financed for 4 years at an annual interest rate of 7%. Find the monthly truck payment. Use the Monthly Payment Table in the Appendix.

18. Shaney receives an hourly wage of $18.40 an hour as an emergency room nurse. When called in at night, she receives time and a half. How much does Shaney earn in a week when she works 30 hours at normal rates and 15 hours during the night?

19. The business checking account for a pottery store had a balance of $7349.44 before checks for $1349.67 and $344.12 were written. The store manager then made a deposit of $956.60. Find the current checkbook balance.

20. Balance the checkbook shown.

RECORD ALL CHARGES OR CREDITS THAT AFFECT YOUR ACCOUNT

NUMBER	DATE	DESCRIPTION OF TRANSACTION	PAYMENT/DEBIT (−)		√ T	FEE (IF ANY) (−)	DEPOSIT/CREDIT (+)		BALANCE	
									$ 1422	13
843	8/1	House Payment	$ 713	72		$	$		708	41
	8/4	Deposit					852	60	1561	01
844	8/5	Loan Payment	162	40					1398	61
845	8/6	Groceries	166	44					1232	17
846	8/10	Car Payment X	322	37					909	80
	8/15	Deposit					852	60	1762	40
847	8/16	Credit Card	413	45					1348	95
848	8/18	Pharmacy	92	14					1256	81
849	8/22	Utilities	72	30					1184	51
850	8/28	Telephone X	78	20					1106	31

CHECKING ACCOUNT Monthly Statement

Account Number: 122-345-1

Date	Transaction	Amount	Balance
8/1	OPENING BALANCE		1422.13
8/3	CHECK	713.72	708.41
8/4	DEPOSIT	852.60	1561.01 ✓
8/8	CHECK	166.44	1394.57 ✓
8/8	CHECK	162.40	1232.17 ✓
8/15	DEPOSIT	852.60	2084.77 ✓
8/23	CHECK	72.30	2012.47 ✓
8/24	CHECK	92.14	1920.33 ✓
9/1	CLOSING BALANCE		1920.33

Cumulative Review

1. Simplify: $12 - (10 - 8)^2 \div 2 + 3$

2. Add: $3\frac{1}{3} + 4\frac{1}{8} + 1\frac{1}{12}$

3. Find the difference between $12\frac{3}{16}$ and $9\frac{5}{12}$.

4. Find the product of $5\frac{5}{8}$ and $1\frac{9}{15}$.

5. Divide: $3\frac{1}{2} \div 1\frac{3}{4}$

6. Simplify $\left(\frac{3}{4}\right)^2 \div \left(\frac{3}{8} - \frac{1}{4}\right) + \frac{1}{2}$.

7. Divide: $0.059\overline{)3.0792}$
Round to the nearest tenth.

8. Convert $\frac{17}{12}$ to a decimal. Round to the nearest thousandth.

9. Write "\$410 in 8 hours" as a unit rate.

10. Solve the proportion $\frac{5}{n} = \frac{16}{35}$.
Round to the nearest hundredth.

11. Write $\frac{5}{8}$ as a percent.

12. Find 6.5% of 420.

13. Write 18.2% as a decimal.

14. What percent of 20 is 8.4?

15. 30 is 12% of what?

16. 65 is 42% of what? Round to the nearest hundredth.

17. A series of late-summer storms produced rainfall of $3\frac{3}{4}$, $8\frac{1}{2}$, and $1\frac{2}{3}$ inches during a 3-week period. Find the total rainfall during the 3 weeks.

18. The Homer family pays $\frac{1}{5}$ of its total monthly income for taxes. The family has a total monthly income of $4850. Find the amount of their monthly income that the Homers pay in taxes.

19. In 5 years, the cost of a scientific calculator went from $75 to $30. What is the ratio of the decrease in price to the original price?

20. A compact car was driven 417.5 miles on 12.5 gallons of gasoline. Find the number of miles driven per gallon of gasoline.

21. A 14-pound turkey costs $12.96. Find the unit cost. Round to the nearest cent.

22. Eighty shares of a stock paid a dividend of $112. At the same rate, find the dividend on 200 shares of the stock.

23. A video camera that regularly sells for $900 is on sale for 20% off the regular price. What is the sale price?

24. A department store bought a portable disc player for $85 and used a markup rate of 40%. Find the selling price of the disc player.

25. Sook Kim, an elementary school teacher, received an increase in salary from $2800 per month to $3024 per month. Find the percent increase in her salary.

26. A contractor borrowed $120,000 for 6 months at an annual interest rate of 10%. How much simple interest is due on the loan?

27. A red Ford Mustang was purchased for $26,900, and a down payment of $2000 was made. The balance is financed for 3 years at an annual interest rate of 9%. Find the monthly payment. Use the Monthly Payment Table in the Appendix. Round to the nearest cent.

28. A family had a checking account balance of $1846.78. A check of $568.30 was deposited into the account, and checks of $123.98 and $47.33 were written. Find the new checking account balance.

29. During one year, Anna Gonzalez spent $840 on gasoline and oil, $520 on insurance, $185 on tires, and $432 on repairs. Find the cost per mile to drive the car 10,000 miles during the year. Round to the nearest cent.

30. A house has a mortgage of $72,000 for 20 years at an annual interest rate of 11%. Find the monthly mortgage payment. Use the Monthly Payment Table in the Appendix. Round to the nearest cent.

Chapter 7

Statistics and Probability

Objectives

Section 7.1

- **A** To read a pictograph
- **B** To read a circle graph

Section 7.2

- **A** To read a bar graph
- **B** To read a broken-line graph

Section 7.3

- **A** To read a histogram
- **B** To read a frequency polygon

Section 7.4

- **A** To find the mean, median, and mode of a distribution
- **B** To draw a box-and-whiskers plot

Section 7.5

- **A** To calculate the probability of simple events

1600
1400
1200
1000
800
600
400
200
J F M A M J J A

When business people use graphs during presentations, the information they present is often more easily understood and evaluated. Data displays, such as circle graphs and bar graphs, are used to assist us in analyzing a lot of information at one time because they condense all data into a single figure. Line graphs, like the one shown here and in the **examples on page 298**, provide the advantage of illustrating the trend of data over time.

Need help? For on-line student resources, such as section quizzes, visit this textbook's web site at **math.college.hmco.com/students.**

Prep Test

1. Bill-related mail accounted for 49 billion of the 102 billion pieces of first-class mail handled by the U.S. Postal Service during a recent year. (Source: USPS) What percent of the pieces of first-class mail handled by the U.S. Postal Service was bill-related mail? Round to the nearest tenth of a percent.

2. The table at the right shows the estimated costs of funding an education at a public college. Between which two enrollment years is the increase in cost greatest? What is the increase between these two years?

Enrollment Year	*Cost of Public College*
2005	$70,206
2006	$74,418
2007	$78,883
2008	$83,616
2009	$88,633
2010	$93,951

Source: The College Board's Annual Survey of Colleges

3. During the 1924 Summer Olympics in Paris, France, the United States won 45 gold medals, 27 silver medals, and 27 bronze medals. (Source: The Ultimate Book of Sports Lists)

a. Find the ratio of gold medals won by the United States to silver medals won by the United States during the 1924 Summer Olympics. Write the ratio as a fraction in simplest form.

b. Find the ratio of silver medals won by the United States to bronze medals won by the United States during the 1924 Summer Olympics. Write the ratio using a colon.

4. The table below shows the number of television viewers, in millions, who watch pay-cable channels, such as HBO and Showtime, each night of the week. (Source: Neilsen Media Research analyzed by Initiative Media North America)

Mon	Tue	Wed	Thu	Fri	Sat	Sun
3.9	4.5	4.2	3.9	5.2	7.1	5.5

a. Arrange the numbers in the table from smallest to largest.
b. Find the average number of viewers per night.

5. Approximately 90,000 women serve in the U.S. military. Five percent of these women serve in the Marine Corps. (Source: U.S. Department of Defense)

a. Approximately how many women are in the Marine Corps?
b. What fractional amount of women in the military are in the Marine Corps?

Go Figure

I have 2 brothers and 1 sister. My father's parents have 10 grandchildren. My mother's parents have 11 grandchildren. If no divorces or remarriages occurred, how many first cousins do I have?

7.1 Pictographs and Circle Graphs

Objective A

To read a pictograph

Statistics is the branch of mathematics concerned with **data,** or numerical information. **Graphs** are displays that provide a pictorial representation of data. The advantage of graphs is that they present information in a way that is easily read. The disadvantage of graphs is that they can be misleading. (See "Projects and Group Activities" at the end of this chapter.)

A **pictograph** uses symbols to represent information. The pictograph in Figure 1 represents the net worth of America's richest billionaires. Each symbol represents ten billion dollars.

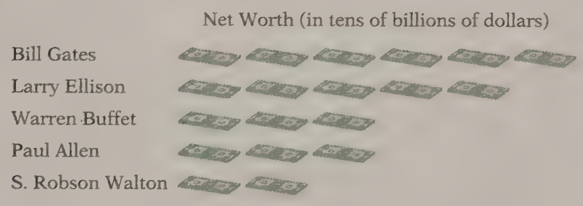


Figure 1 Net worth of America's richest billionaires
Source: Forbes

From the pictograph, we can determine that Bill Gates has the greatest net worth. Paul Allen's net worth is $10 billion more than S. Robson Walton's net worth.

The pictograph in Figure 2 represents the responses of 600 young Americans when asked what they would like to have with them on a desert island. "Books" was the response of what percent of the respondents?

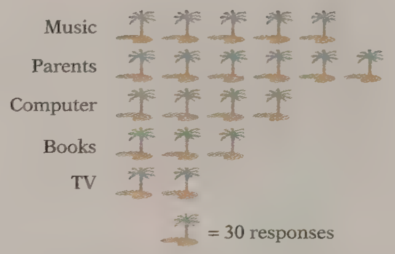


Figure 2 What 600 young Americans want on a desert island
Source: Time, June 30, 1997

Strategy
Use the basic percent equation. The base is 600 (the total number of responses), and the amount is 90 (the number responding "Books").

Solution

$$\text{Percent} \times \text{base} = \text{amount}$$

$$n \times 600 = 90$$

$$n = 90 \div 600$$

$$n = 0.15$$

15% of the respondents wanted books on a desert island.

The pictograph in Figure 3 shows the number of new cellular phones purchased in a particular city during a 4-month period.

The ratio of the number of cellular phones purchased in March to the number purchased in January is

$$\frac{3000}{4500} = \frac{2}{3}$$

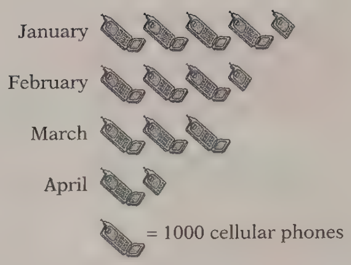


Figure 3 Monthly cellular phone purchases

Example 1

Use Figure 3 to find the total number of cellular phones purchased during the 4-month period.

Strategy

To find the total number of cellular phones purchased in the 4-month period:

- Read the pictograph to determine the number of cellular phones purchased each month.
- Add the four numbers.

Solution

Purchases for January: 4500
Purchases for February: 3500
Purchases for March: 3000
Purchases for April: 1500

Total purchases for the 4-month period:

$$\begin{array}{r} 4,500 \\ 3,500 \\ 3,000 \\ +\ 1,500 \\ \hline 12,500 \end{array}$$

There were 12,500 cellular phones purchased in the 4-month period.

You Try It 1

According to Figure 3, the number of cellular phones purchased in March represents what percent of the total number of cellular phones purchased in that city during that 4-month period?

Your strategy

Your solution

Solution on pp. S18–S19

Objective B

To read a circle graph

A **circle graph** represents data by the size of the sectors.

TAKE NOTE
One quadrillion is 1,000,000,000,000,000.

The circle graph in Figure 4 shows the consumption of energy sources in the United States during a recent year. The complete circle graph represents the total amount of energy consumed, 94.4 quadrillion Btu. Each sector of the circle represents the consumption of energy from a different source.

Point of Interest

Fossil fuels include coal, natural gas, and petroleum. Renewable energy includes hydroelectric power, solar energy, wood burning, and wind energy.

To find the percent of the total energy consumed that originated from nuclear power, solve the basic percent equation for percent. The base is 94.4 quadrillion Btu, and the amount is 7.2 quadrillion Btu.

Percent × base = amount

$$n \times 94.4 = 7.2$$

$$n = 7.2 \div 94.4$$

$$n \approx 0.076$$

To the nearest tenth of a percent, 7.6% of the energy consumed originated from nuclear power.

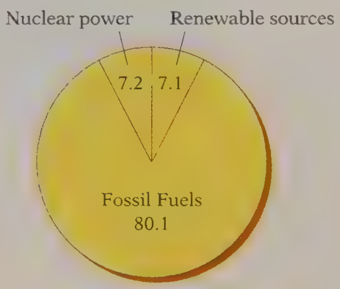


Figure 4 Annual energy consumption in quadrillion Btu in the United States
Source: The World Almanac and Book of Facts 2000

The circle graph in Figure 5 shows what percents of the $17.8 billion that the top 25 companies in the United States spent for national advertising in one year went to the various advertising media. The complete circle represents 100% of all the money spent by these companies. Each sector of the graph represents the percent of the total spent for a particular medium.

➡ According to Figure 5, how much money was spent for magazine advertising? Round to the nearest hundred million dollars.

Strategy
Use the basic percent equation. The base is $17.8 billion, and the percent is 16%.

Solution

Percent × base = amount

$$0.16 \times 17.8 = n$$

$$2.848 = n$$

2.848 billion = 2,848,000,000

To the nearest hundred million, the amount spent for magazine advertising was $2,800,000,000.

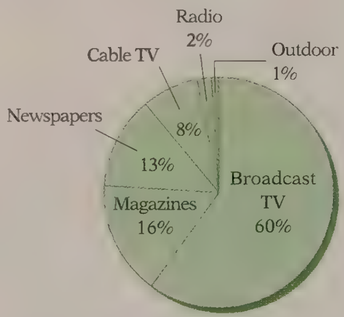


Figure 5 Distribution of advertising dollars for 25 companies
Source: Interep research

The circle graph in Figure 6 shows typical annual expenses of owning, operating, and financing a new car. Use this figure for Example 2 and You Try It 2.

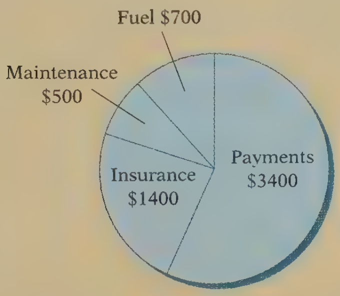


Figure 6 Annual expenses of $6000 for owning, operating, and financing a car
Source: Based on data from IntelliChoice

The circle graph in Figure 7 shows the distribution of an employee's gross monthly income. Use this figure for Example 3 and You Try It 3.

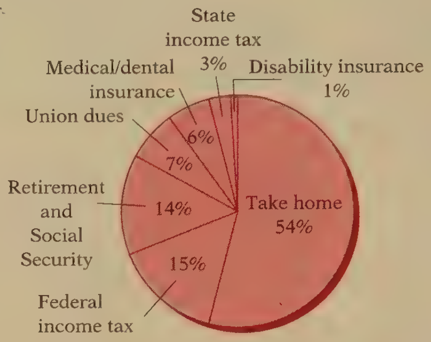


Figure 7 Distribution of gross monthly income of $2900

Example 2

Use Figure 6 to find the ratio of the annual insurance expense to the total annual cost of the car.

Strategy

To find the ratio:

- Locate the annual insurance expense in the circle graph.
- Write in simplest form the ratio of the annual insurance expense to the total annual cost of operating the car.

Solution

Annual insurance expense: $1400

$$\frac{1400}{6000} = \frac{7}{30}$$

The ratio is $\frac{7}{30}$.

You Try It 2

Use Figure 6 to find the ratio of the annual cost of fuel to the annual cost of maintenance.

Your strategy

Your solution

Example 3

Use Figure 7 to find the employee's take-home pay.

Strategy

To find the take-home pay:

- Locate the percent of the distribution that is take-home pay.
- Solve the basic percent equation for amount.

Solution

Take-home pay: 54%

$$\begin{aligned} \text{Percent} \times \text{base} &= \text{amount} \\ 0.54 \times 2900 &= n \\ 1566 &= n \end{aligned}$$

The employee's take-home pay is $1566.

You Try It 3

Use Figure 7 to find the amount paid for medical/dental insurance.

Your strategy

Your solution

Solutions on p. S19

7.1 Exercises

Objective A

The pictograph in Figure 8 shows the approximate gross revenues in the United States from four Walt Disney animated movies.

1. Find the total gross revenues from the four movies.

2. Find the ratio of the gross revenue of *Beauty and the Beast* to the gross revenue of *The Hunchback of Notre Dame.*

3. Find the percent of the total gross revenue that was earned by *The Lion King.*

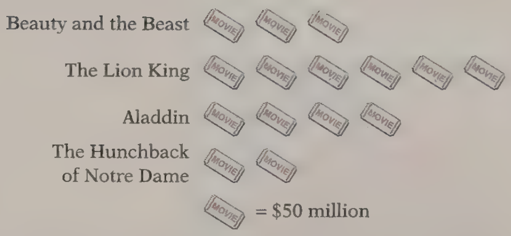


Figure 8 Gross revenues of four Walt Disney animated movies

Source: **www.worldboxoffice.com**

The pictograph in Figure 9 is based on a survey of adults who were asked whether they agreed with each statement.

4. Find the ratio of the number of people who agree that space exploration impacts daily life to the number of people who agree that space will be colonized in their lifetime.

5. How many more people agreed that humanity should explore planets than agreed that space exploration impacts daily life?

6. Is the number of people who said they would travel in space more than twice the number of people who agreed that space would be colonized in their lifetime?

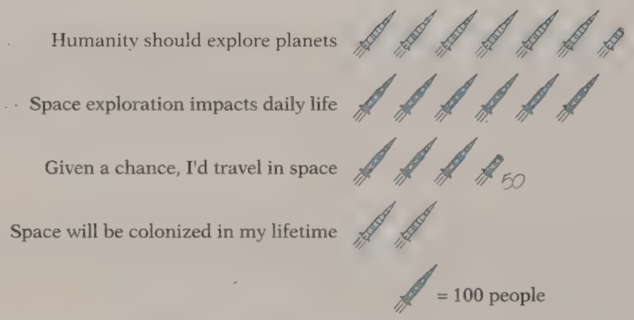


Figure 9 Number of adults who agree with the statement

Source: Opinion Research for Space Day Partners

The pictograph in Figure 10 is based on a survey of children aged 7 through 12. The percent of children's responses to the survey are shown. Assume that 500 children were surveyed.

7. Find the number of children who said they hid vegetables under a napkin.

8. What is the difference between the number of children who fed vegetables to the dog and the number who dropped them on the floor?

9. Were the responses given in the graph the only responses given by the children? Explain your answer.

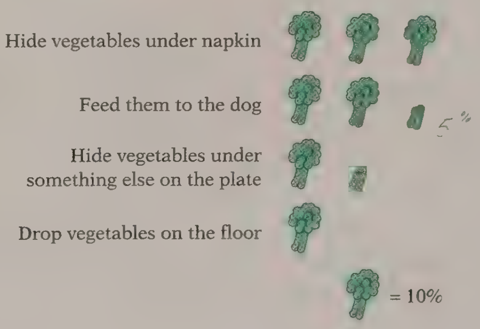


Figure 10 How children try to hide vegetables

Source: Strategic Consulting and Research for Del Monte

Objective B

An accounting major recorded the number of units required in each discipline to graduate with a degree in accounting. The results are shown in the circle graph in Figure 11.

10. How many units are required to graduate with a degree in accounting?

11. What is the ratio of the number of units in finance to the number of units in accounting?

12. What percent of the units required to graduate are taken in accounting? Round to the nearest tenth of a percent.

13. What percent of the units required to graduate are taken in mathematics? Round to the nearest tenth of a percent.

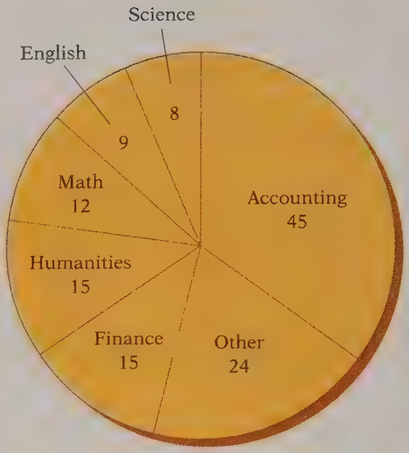


Figure 11 Number of units required to graduate with an accounting degree

The circle graph in Figure 12 shows the result of a survey in which people were asked, "What bothers you most about movie theaters?"

14. **a.** What complaint was mentioned the most often?
b. What complaint was mentioned the least often?

15. How many people were surveyed?

16. What is the ratio of the number of people responding "Dirty floors" to the number responding "High ticket prices"?

17. What percent of the respondents said that people talking bothered them most?

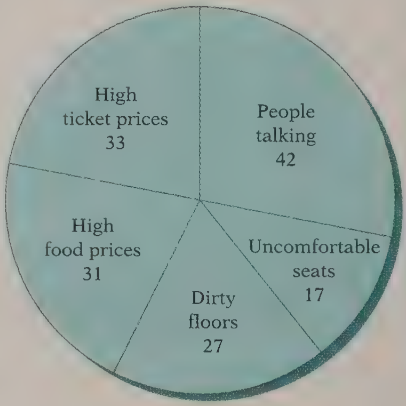


Figure 12 Distribution of responses in a survey

The circle graph in Figure 13 shows the breakdown of the approximately $3,100,000,000 that Americans spent on home video game equipment in one year.

18. Find the amount of money spent on TV game machines.

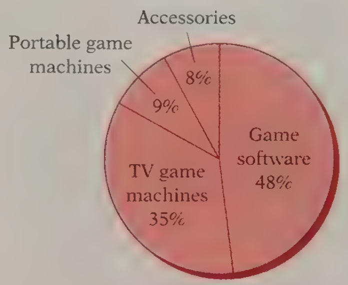


Figure 13 Percents of $3,100,000,000 spent annually on home video games

Source: The NPD Group, Toy Manufacturers of America

19. Find the amount of money spent on portable game machines.

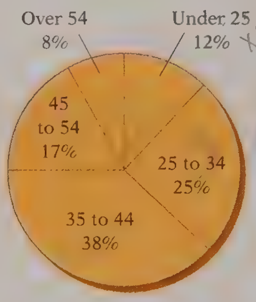
20. What fractional amount of the total money spent was spent on accessories?

21. Is the amount spent for TV game machines more than three times the amount spent for portable game machines?

The circle graph in Figure 14 shows a breakdown, according to age, of the homeless in America.

22. What age group represents the largest segment of the homeless population?

23. Is the number of homeless who are aged 25 to 34 more or less than twice the number who are under the age of 25?



x2=24.

Figure 14 Ages of the homeless in America

Source: The Department of Housing and Urban Development

24. What percent of the homeless population is under the age of 35?

25. On average, how many of every 100,000 homeless people in America are over the age of 54?

The circle graph in Figure 15 shows the land area of each of the seven continents in square miles.

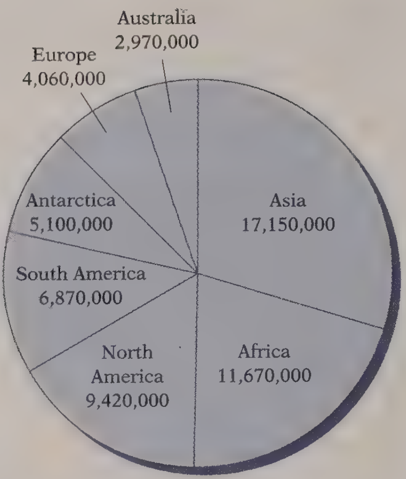


Figure 15 Land area of the seven continents (in square miles)

26. Find the total land area of the seven continents.

27. How much larger is North America than South America?

28. What percent of the total land area is the land area of Asia? Round to the nearest tenth of a percent.

29. What percent of the total land area is the land area of Australia? Round to the nearest tenth of a percent.

The circle graph in Figure 16 shows the average percent distribution of a family's income of $34,000 per year.

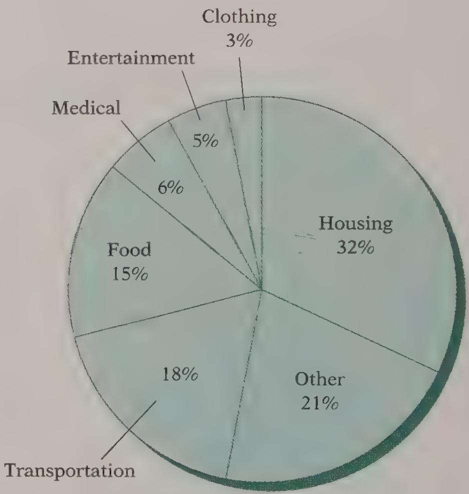


Figure 16 Average percent distribution of a family's income

Source: Consumer Expenditure Survey for the Bureau of Labor Statistics

30. How much of the family's income was spent on food?

31. How much of the family's income was spent on entertainment?

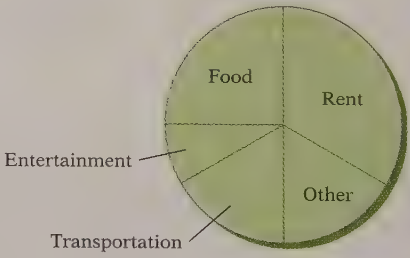
32. How much more of the family's income was spent on medical needs than was spent on clothing?

33. Was the amount spent for housing more than twice the amount spent for food?

APPLYING THE CONCEPTS

34. **a.** What are the advantages of presenting data in the form of a pictograph?
b. What are the disadvantages?

35. The circle graph at the right shows a couple's expenditures last month. Write two observations about this couple's expenses.



7.2 Bar Graphs and Broken-Line Graphs

Objective A To read a bar graph

Point of Interest

The first bar graph appeared in 1786 in the book *The Commercial and Political Atlas.* The author, William Playfair (1759–1823), was a pioneer in the use of graphical displays.

A **bar graph** represents data by the height of the bars. The bar graph in Figure 17 shows temperature data recorded for Cincinnati, Ohio, for the months March through November. For each month, the height of the bar indicates the normal daily high temperature during that month. The jagged line near the bottom of the graph indicates that the vertical scale is missing the numbers between 0 and 50.

The daily high temperature in September was 78°F. Because the bar for July is the tallest, the daily high temperature was highest in July.

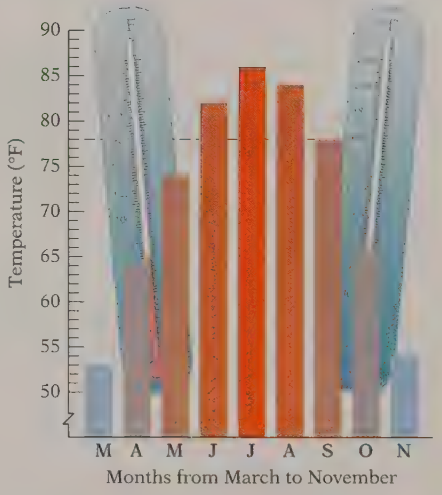


Figure 17 Daily high temperatures in Cincinnati, Ohio

Source: U.S. Weather Bureau

TAKE NOTE

The bar for athletic females is halfway between the marks for 50 and 60. Therefore, we estimate that the lung capacity is halfway between these two numbers, at 55.

A **double-bar graph** is used to display data for purposes of comparison. The double-bar graph in Figure 18 shows the lung capacity of inactive, versus that of athletic, 45-year-olds.

The lung capacity of an athletic female is 55 milliliters of oxygen per kilogram of body weight per minute.

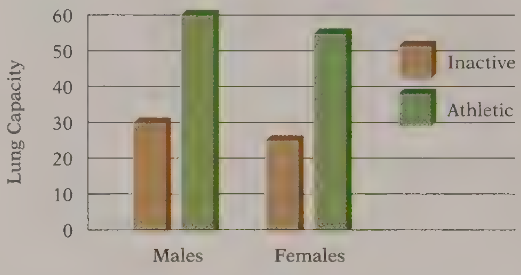


Figure 18 Lung capacity (in milliliters of oxygen per kilogram of body weight per minute)

Example 1

What is the ratio of the lung capacity of an inactive male to that of an athletic male?

Strategy

To write the ratio:

- Read the graph to find the lung capacity of an inactive male and of an athletic male.
- Write the ratio in simplest form.

Solution

Lung capacity of inactive male: 30

Lung capacity of athletic male: 60

$$\frac{30}{60} = \frac{1}{2}$$

The ratio is $\frac{1}{2}$.

You Try It 1

What is the ratio of the lung capacity of an inactive female to that of an athletic female?

Your strategy

Your solution

Solution on p. S19

Objective B To read a broken-line graph

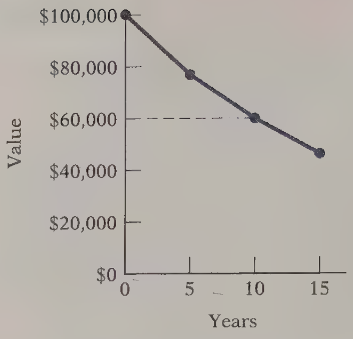


Figure 19 Effect of inflation on the value of a $100,000 life insurance policy

A **broken-line graph** represents data by the position of the lines. It is used to show trends.

The broken-line graph in Figure 19 shows the effect of inflation on the value of a $100,000 life insurance policy. The height of each dot indicates the value of the policy.

After 10 years, the purchasing power of the $100,000 has decreased to approximately $60,000.

Two broken-line graphs are often shown in the same figure for comparison. Figure 20 shows the net incomes of two software companies, Math Associates and MatheMentors, before their merger.

Several things can be determined from the graph:

The net income for Math Associates in 2004 was $12 million.

The net income for MatheMentors declined from 2000 to 2001.

The net income for Math Associates increased for each year shown.

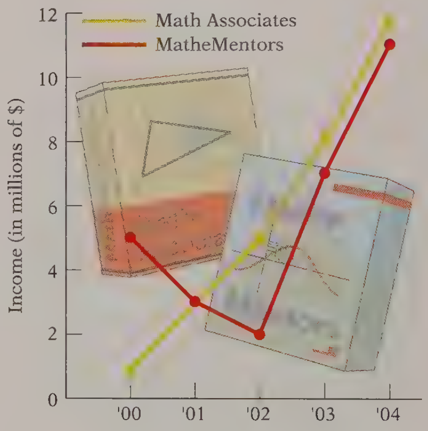


Figure 20 Net incomes of Math Associates and MatheMentors

Example 2

Use Figure 20 to approximate the difference between the net income of Math Associates and that of MatheMentors in 2002.

Strategy

To write the difference:

- Read the line graph to determine the net income of Math Associates and that of MatheMentors in 2002.
- Subtract to find the difference.

Solution

Net income for Math Associates: $5 million
Net income for MatheMentors: $2 million

5 − 2 = 3

The difference between the net incomes in 2002 was $3 million.

You Try It 2

Use Figure 20 to determine between which two years the net income of Math Associates increased the most.

Your strategy

Your solution

Solution on p. S19

7.2 Exercises

Objective A

The bar graph in Figure 21 shows the regions in which all the passenger cars were produced during a recent year.

1. How many passenger cars were produced worldwide?

2. What is the difference between the number of passenger cars produced in Western Europe and the number produced in North America?

3. What percent of the passenger cars were produced in Asia? Round to the nearest percent.

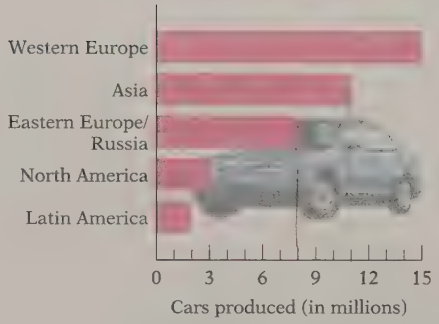


Figure 21 Number of passenger cars produced (in a recent year)
Source: Reprinted with permission.

The double-bar graph in Figure 22 shows the fuel efficiency of four vehicles, as rated by the Environmental Protection Agency. They are among the most fuel-efficient 2000 model-year cars for city and highway mileage.

4. Is the fuel efficiency of the Chevrolet Metro greater on the highway or in city driving?

5. Approximately how many more miles per gallon does the Honda Insight get while traveling on the highway than in the city?

6. Estimate the difference between the fuel efficiency of the Honda Insight in city driving and the fuel efficiency of the VW Beetle in city driving.

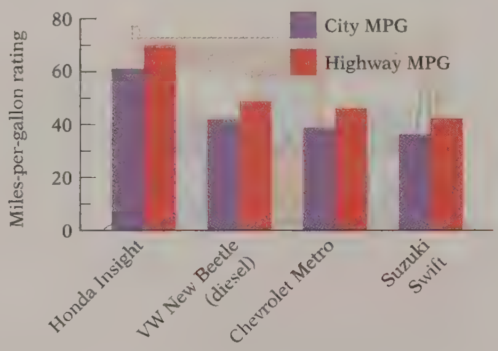


Figure 22 Fuel efficiency ratings
Source: Environmental Protection Agency

The double-bar graph in Figure 23 shows maximum salaries for police officers in selected cities and the corresponding maximum salaries for officers in the suburbs of that city.

7. Estimate the difference between the maximum salaries of police officers in the suburbs of New York City and in the city of New York.

8. Is there a city for which the maximum salary of a police officer in the city is greater than the salary in the suburbs?

9. For which city is the difference between the maximum salary in the suburbs and that in the city the greatest?

10. Of the cities shown on the graph, which city has the lowest maximum salary for police officers in the suburbs?

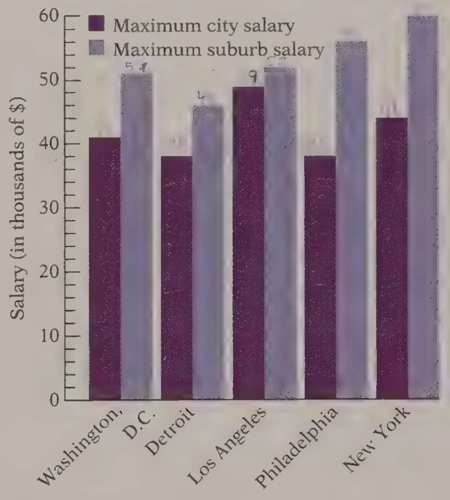


Figure 23 Maximum salaries of police officers in the city and the suburbs
Source: USA Today

Objective B

The broken-line graph in Figure 24 shows the average monthly snowfall during ski season around Aspen, Colorado.

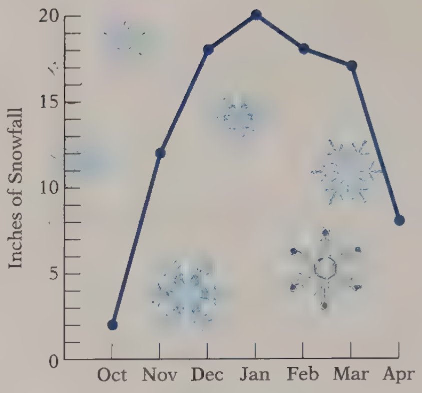


Figure 24 Average snowfall in Aspen, Colorado

Source: Weather America, by Alfred Garwood

11. What is the average snowfall during January?

12. During which month is the snowfall the greatest?

13. What is the total average snowfall during March and April?

14. Find the ratio of the average snowfall in November to the average snowfall in December.

The double-broken-line graph in Figure 25 shows the number of calories per day that should be consumed by women and men in various age groups.

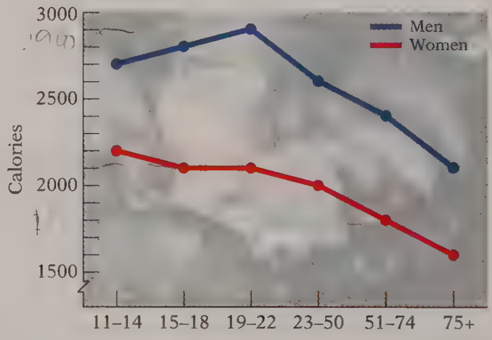


Figure 25 Recommended number of calories per day for women and men

Source: Numbers, by Andrea Sutcliffe (HarperCollins)

15. What is the difference between the number of calories recommended for men and the number recommended for women 19–22 years of age?

16. People of what age and gender have the lowest recommended number of calories?

17. Find the ratio of the number of calories recommended for women 15 to 18 years old to the number recommended for women 51 to 74 years old.

APPLYING THE CONCEPTS

The graph in Figure 26 shows the amount of money, in billions of dollars, spent by the United States government for drug prevention in the 1990s.

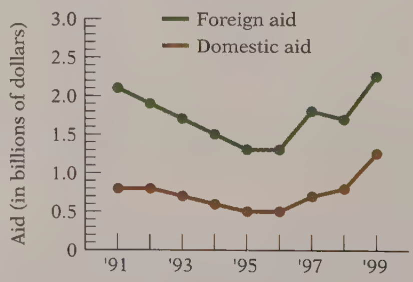


Figure 26 Aid provided by the U.S. government for drug prevention

Source: Reprinted by permission of the *San Diego Union-Tribune.*

18. Create a table that shows the total amount spent for foreign and domestic aid for each year from 1991 to 1999.

19. Create a table that shows the difference between the amount spent for foreign and for domestic aid in each year from 1991 to 1999.

7.3 Histograms and Frequency Polygons

Objective A To read a histogram

A research group measured the fuel usage of 92 cars. The results are recorded in the histogram in Figure 27. A **histogram** is a special type of bar graph. The width of each bar corresponds to a range of numbers called a **class interval.** The height of each bar corresponds to the number of occurrences of data in each class interval and is called the **class frequency.**

Class Intervals (miles per gallon)	*Class Frequencies (number of cars)*
18–20	12
20–22	19
22–24	24
24–26	17
26–28	15
28–30	5

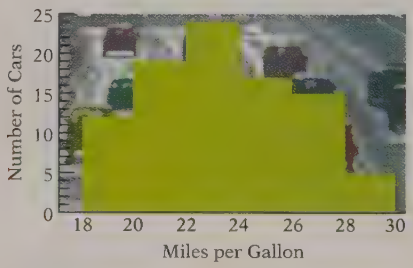


Figure 27

Twenty-four cars get between 22 and 24 miles per gallon.

A precision tool company has 85 employees. Their hourly wages are recorded in the histogram in Figure 28.

The ratio of the number of employees whose hourly wage is between \$14 and \$16 to the total number of employees is $\frac{17 \text{ employees}}{85 \text{ employees}} = \frac{1}{5}$.

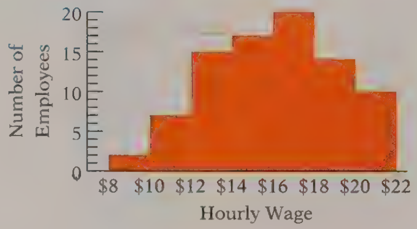


Figure 28

Example 1
Use Figure 28 to find the number of employees whose hourly wage is between \$16 and \$20.

Strategy
To find the number of employees:

- Read the histogram to find the number of employees whose hourly wage is between \$16 and \$18 and the number whose hourly wage is between \$18 and \$20.
- Add the two numbers.

Solution
Number whose wage is between \$16 and \$18: 20; between \$18 and \$20: 14

20 + 14 = 34

34 employees have an hourly wage between \$16 and \$20.

You Try It 1
Use Figure 28 to find the number of employees whose hourly wage is between \$10 and \$14.

Your strategy

Your solution

Solution on p. S19

Objective B

To read a frequency polygon

The speeds of 70 cars on a highway were measured by radar. The results are recorded in the frequency polygon in Figure 29. A **frequency polygon** is a graph that displays information in a manner similar to a histogram. A dot is placed above the center of each class interval at a height corresponding to that class's frequency. The dots are then connected to form a broken-line graph. The center of a class interval is called the **class midpoint.**

Class Inteval (miles per hour)	*Class Midpoint*	*Class Frequency*
30–40	35	7
40–50	45	13
50–60	55	25
60–70	65	21
70–80	75	4

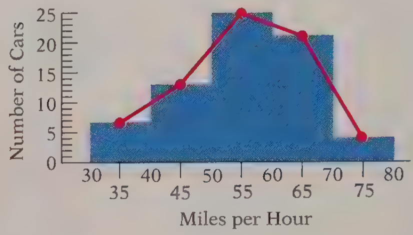


Figure 29

Twenty-five cars were traveling between 50 and 60 miles per hour.

The per capita incomes in a recent year for the 50 states are recorded in the frequency polygon in Figure 30.

The number of states with a per capita income between $22,000 and $26,000 is 20.

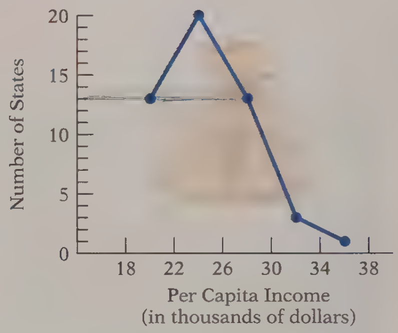


Figure 30

Source: Department of Commerce

Example 2

According to Figure 30, what percent of the states have a per capita income between $22,000 and $26,000?

Strategy

To find the percent, solve the basic percent equation for percent. The base is 50. The amount is 20.

Solution

$$\begin{aligned} \text{Percent} \times \text{base} &= \text{amount} \\ n \times 50 &= 20 \\ n &= 20 \div 50 \\ n &= 0.4 \end{aligned}$$

40% of the states have a per capita income between $22,000 and $26,000.

You Try It 2

Use Figure 30 to find the ratio of the number of states with a per capita income between $18,000 and $22,000 to the number with a per capita income between $26,000 and $30,000.

Your strategy

Your solution

Solution on p. S20

7.3 Exercises

Objective A

The annual tuition for undergraduate college students attending four-year institutions varies depending on the college. The histogram in Figure 31 shows the tuition amounts for a representative sample of 120 students from various parts of the United States.

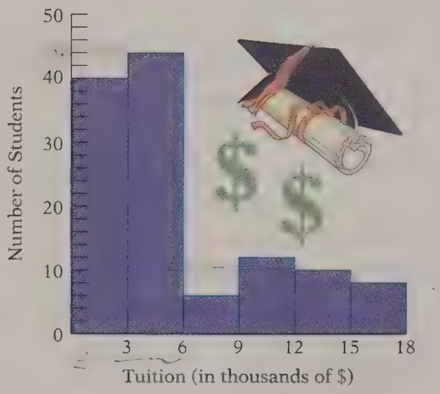


Figure 31
Source: Educational Testing Service

1. How many students have a tuition that is between $3000 and $6000 per year?

2. What is the ratio of the number of students whose tuition is between $9000 per year and $12,000 per year to the total number of students represented?

3. How many students pay more than $12,000 annually for tuition?

4. What percent of the total number of students spend less than $6000 annually?

The histogram in Figure 32 is based on data from the American Automobile Manufacturers Association. It shows the ages of a sample of 1000 cars in a typical city in the United States.

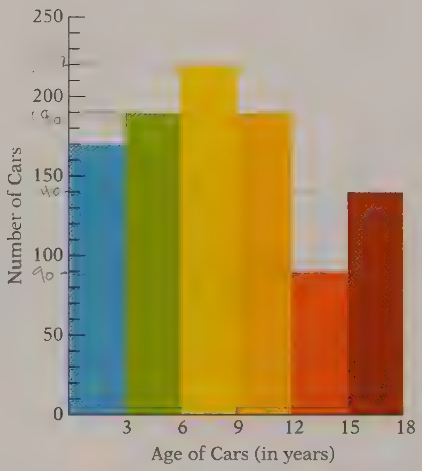


Figure 32
Source: American Automobile Manufacturers Association

5. How many cars are between 6 and 12 years old?

6. Find the ratio of the number of cars between 12 and 15 years old to the total number of cars.

7. Find the number of cars more than 12 years old.

8. Find the percent of cars that are less than 9 years old.

According to a Maritz AmeriPoll, the average U.S. adult goes to a shopping mall about two times a month. The histogram shows the average time 100 adults spend in the mall per trip.

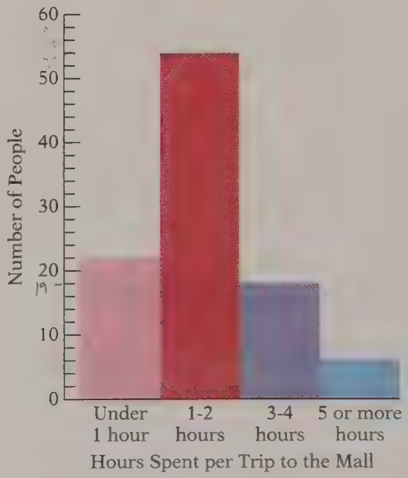


Figure 33
Source: Maritz AmeriPoll

9. Find the number of adults who spend between 1 and 2 hours at the mall.

10. Find the number of adults who spend between 3 and 4 hours at the mall.

11. What percent of the adults spend less than 1 hour at the mall?

12. What percent of the adults spend 5 or more hours at the mall?

Objective B

The frequency polygon in Figure 34 shows the finishing times of the entrants in the University & College Women's Distance Medley Relay Finals at the Drake Relays in April 2000.

13. How many teams competed in the relay finals?

14. How many teams completed the relay in more than 12 minutes?

15. What percent of the teams completed the relay in less than 12 minutes? Round to the nearest tenth of a percent.

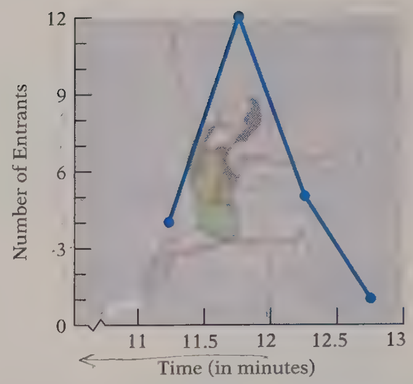


Figure 34

The frequency polygon in Figure 35 is based on data from a Gallup poll survey of 74 people who purchased lottery tickets.

16. How many people purchased between 1 and 10 tickets?

17. What percent of the people purchased between 20 and 30 tickets each month? Round to the nearest tenth of a percent.

18. What percent of the people purchased more than 10 tickets each month? Round to the nearest tenth of a percent.

19. Is it possible to determine from the graph how many people purchased 15 lottery tickets? Explain.

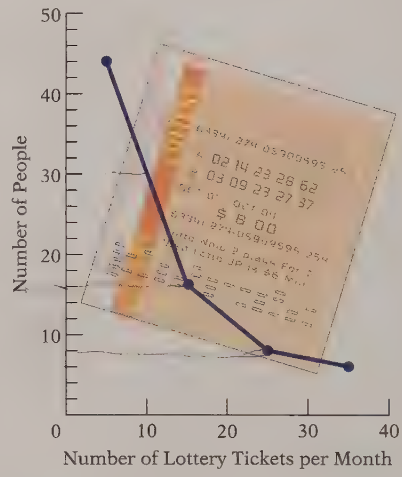


Figure 35

The frequency polygon in Figure 36 shows the distribution of scores of the approximately 1,080,000 students who took an SAT exam.

20. How many students scored between 1200 and 1400 on the exam?

21. What percent of the number of students who took the exam scored between 800 and 1000? Round to the nearest tenth of a percent.

22. How many students scored below 1000?

23. How many students scored above 800?

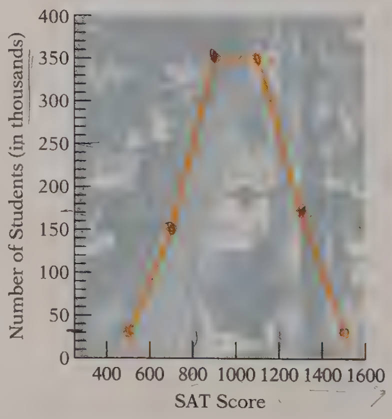


Figure 36

Source: Educational Testing Service

APPLYING THE CONCEPTS

24. Write a paragraph explaining the difference between a histogram and a bar graph.

25. In your own words, describe a frequency table.

7.4

Statistical Measures

Objective A

To find the mean, median, and mode of a distribution

The average score on the math portion of the SAT was 432. The EPA estimates that a 2001 Ford Escort averages 35 miles per gallon on the highway. The average rainfall for portions of Kauai is 350 inches per year. Each of these statements uses one number to describe an entire collection of numbers. Such a number is called an *average.*

In statistics there are various ways to calculate an average. Three of the most common—*mean, median,* and *mode*—are discussed here.

An automotive engineer tests the miles-per-gallon ratings of 15 cars and records the results as follows:

Miles-per-Gallon Ratings of 15 Cars														
25	22	21	27	25	35	29	31	25	26	21	39	34	32	28

The **mean** of the data is the sum of the measurements divided by the number of measurements. The symbol for the mean is $\bar{x}$.

Formula for the Mean

$$\bar{x} = \frac{\text{sum of the data values}}{\text{number of data values}}$$

To find the mean for the data above, add the numbers and then divide by 15.

$$\bar{x} = \frac{25 + 22 + 21 + 27 + 25 + 35 + 29 + 31 + 25 + 26 + 21 + 39 + 34 + 32 + 28}{15}$$

$$= \frac{420}{15} = 28$$

The mean number of miles per gallon for the 15 cars tested was 28 miles per gallon.

The mean is one of the most frequently computed averages. It is the one that is commonly used to calculate a student's performance in a class.

➡ The test scores for a student taking American history were 78, 82, 91, 87, and 93. What was the mean score for this student?

Strategy
To find the mean, divide the sum of the test scores by 5, the number of scores.

Solution

$$\bar{x} = \frac{78 + 82 + 91 + 87 + 93}{5} = \frac{431}{5} = 86.2$$

The mean score for the history student was 86.2.

The **median** of data is the number that separates the data into two equal parts when the numbers are arranged from smallest to largest (or from largest to smallest). There is an equal number of values above the median and below the median.

To find the median of a set of numbers, first arrange the numbers from smallest to largest. The median is the number in the middle.

The result of arranging the miles-per-gallon ratings given on the previous page from smallest to largest is shown below.

$$\underbrace{21\quad 21\quad 22\quad 25\quad 25\quad 25\quad 26}_{\substack{\text{7 values below}\\ \text{the median}}}\quad \underset{\substack{\uparrow\\ \text{Middle number}\\ \textbf{Median}}}{27}\quad \underbrace{28\quad 29\quad 31\quad 32\quad 34\quad 35\quad 39}_{\substack{\text{7 values above}\\ \text{the median}}}$$

The median is 27 miles per gallon.

If data contain an *even* number of values, the median is the mean of the two middle numbers.

➡ The selling prices of the last six homes sold by a real estate agent were \$175,000, \$150,000, \$250,000, \$130,000, \$245,000, and \$190,000. Find the median selling price of these homes.

Strategy
To find the median, arrange the numbers from smallest to largest. Because there is an even number of values, the median is the mean of the two middle numbers.

Solution

$$130{,}000\quad 150{,}000\quad \underbrace{175{,}000\quad 190{,}000}_{\text{Middle 2 numbers}}\quad 245{,}000\quad 250{,}000$$

$$\text{Median} = \frac{175{,}000 + 190{,}000}{2} = 182{,}500$$

The median selling price was \$182,500.

The **mode** of a set of numbers is the value that occurs most frequently. If a set of numbers has no number occurring more than once, then the data have no mode.

Here again are the data for the gasoline mileage ratings of 15 cars.

Miles-per-Gallon Ratings of 15 Cars

25	22	21	27	25	35	29	31	25	26	21	39	34	32	28

25 is the number that occurs most frequently.

The mode is 25 miles per gallon.

Note from the miles-per-gallon example that the mean, median, and mode may be different.

Example 1
Twenty students were asked the number of units in which they were enrolled. The responses were as follows:

15	12	13	15	17	18	13	20	9	16
14	10	15	12	17	16	6	14	15	12

Find the mean number of units taken by these students.

Strategy
To find the mean number of units:

- Find the sum of the 20 numbers.
- Divide the sum by 20.

Solution

$$15 + 12 + 13 + 15 + 17 + 18 + 13 + 20 + 9 + 16 + 14 + 10 + 15 + 12 + 17 + 16 + 6 + 14 + 15 + 12 = 279$$

$$\bar{x} = \frac{279}{20} = 13.95$$

The mean is 13.95 units.

You Try It 1
The amounts spent by 12 customers at a McDonald's restaurant were as follows:

6.26	8.23	5.09	8.11	7.50	6.69
5.66	4.89	5.25	9.36	6.75	7.05

Find the mean amount spent by these customers. Round to the nearest cent.

Your strategy

Your solution

Example 2
The starting hourly wages for an apprentice electrician for six different work locations are \$10.90, \$11.25, \$10.10, \$11.08, \$11.56, and \$10.55. Find the median starting hourly wage.

Strategy
To find the median starting hourly wage:

- Arrange the numbers from smallest to largest.
- Because there is an even number of values, the median is the mean of the two middle numbers.

Solution
10.10, 10.55, 10.90, 11.08, 11.25, 11.56

$$\text{Median} = \frac{10.90 + 11.08}{2} = 10.99$$

The median starting hourly wage is \$10.99.

You Try It 2
The amounts of weight lost, in pounds, by 10 participants in a 6-month weight-reduction program were 22, 16, 31, 14, 27, 16, 29, 31, 40, and 10. Find the median weight loss for these participants.

Your strategy

Your solution

Solutions on p. S20

Objective 6

To draw a box-and-whiskers plot

Recall from the last objective that an average is one number that helps to describe all the numbers in a set of data. For example, we know from the statement

The average annual snowfall in Erie, Pennsylvania, is 85 inches.

that Erie gets a lot of snow each winter.

Now look at these two statements.

The average annual temperature in San Francisco, California, is 57°.
The average annual temperature in St. Louis, Missouri, is 57°.

The average annual temperature in both cities is the same. However, we do not expect the climate in St. Louis to be like San Francisco's climate. Although both cities have the same average annual temperature, their temperature ranges differ. In fact, the difference between the average monthly high temperatures in July and January in San Francisco is 14°, whereas the difference between the average monthly high temperatures in July and January in St. Louis is 50°.

Note that for this example, a single number (the average annual temperature) does not provide us with a very complete picture of the climate of either of these two cities.

One method used to picture an entire set of data is a box-and-whiskers plot. To prepare a box-and-whiskers plot, we begin by separating a set of data into four parts, called quartiles. We will illustrate this by using the average monthly high temperatures for St. Louis, in degrees Fahrenheit. These are listed below from January through December.

39	47	58	72	81	88	89	89	85	76	49	47

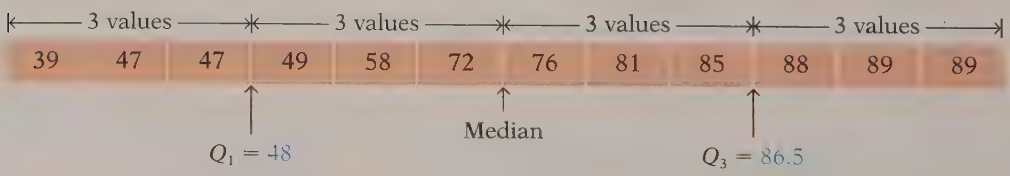
Source: The Weather Channel

First list the numbers in order from smallest to largest and determine the median.

39	47	47	49	58	72	76	81	85	88	89	89

↑
Median = 74

Now find the median of the data values below the median. The median of the data values below the median is called the **first quartile,** symbolized by Q_1. Also find the median of the data values above the median. The median of the data values above the median is called the **third quartile,** symbolized by Q_3.



The first quartile, Q_1, is the number that one-quarter of the data lie below. This means that 25% of the data lie below the first quartile. The third quartile, Q_3, is the number that one-quarter of the data lie above. This means that 25% of the data lie above the third quartile.

The **range** of a set of numbers is the difference between the largest number and the smallest number in the set. The range describes the spread of the data. For the data above,

$$\text{Range} = \text{largest value} - \text{smallest value} = 89 - 39 = 50$$

TAKE NOTE

50% of the data in a distribution lie in the interquartile range.

The **interquartile range** is the difference between the third quartile, Q_3, and the first quartile, Q_1. For the data above,

$$\text{Interquartile range} = Q_3 - Q_1 = 86.5 - 48 = 38.5$$

The interquartile range is the distance that spans the "middle" 50% of the data values. Because it excludes the bottom fourth of the data values and the top fourth of the data values, it excludes any extremes in the numbers in the set.

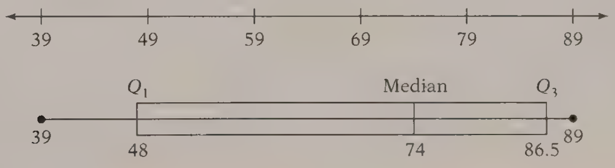
A **box-and-whiskers plot,** or **boxplot,** is a graph that shows five numbers: the smallest value, the first quartile, the median, the third quartile, and the greatest value. Here are these five values for the data on St. Louis temperatures.

The smallest number	39
The first quartile, Q_1	48
The median	74
The third quartile, Q_3	86.5
The largest number	89

Think of a number line that includes the five values listed above. With this in mind, mark off the five values. Draw a box that spans the distance from Q_1 to Q_3. Draw a vertical line the height of the box at the median.

TAKE NOTE

The number line at the right is shown as a reference only. It is not a part of the boxplot. It shows that the numbers labeled on the boxplot are plotted according to the distances on the number line above it.



Listed below are the average monthly high temperatures for San Francisco.

57	60	61	64	68	71	71	73	74	73	60	59

Source: The Weather Channel

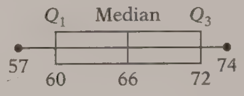
We can perform the same calculations on these data to determine the five values needed for the box-and-whiskers plot.

The smallest number	57
The first quartile, Q_1	60
The median	66
The third quartile, Q_3	72
The largest number	74

TAKE NOTE

It is the "whiskers" on the box-and-whiskers plot that show the range of the data. The "box" on the box-and-whiskers plot shows the interquartile range of the data.

The box-and-whiskers plot is shown at the right with the same scale used for the data on the St. Louis temperatures.



Note that by comparing the two boxplots, we can see that the range of temperatures in St. Louis is greater than the range of temperatures in San Francisco. For the St. Louis temperatures, there is a greater spread of the data below the median than above the median, whereas the spreads of the data above and below the median of the San Francisco boxplot are nearly equal.

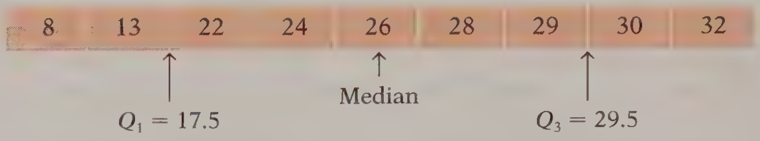
The numbers of avalanche deaths in the United States during each of nine consecutive winters were 8, 24, 29, 13, 28, 30, 22, 26, and 32. (Source: Colorado Avalanche Information Center) Draw a box-and-whiskers plot of the data and determine the interquartile range.

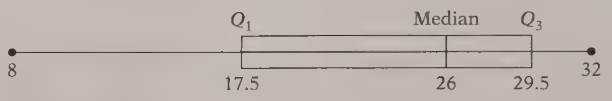
TAKE NOTE
Note that the left whisker in this box-and-whiskers plot is quite long. This indicates a set of data in which the median is closer to the largest data value. If a boxplot has a long whisker on the right, the median is closer to the smallest data value. If the two whiskers are approximately the same length, then the smallest and largest values are about the same distance from the median. See Example 3.

Strategy
To draw the box-and-whiskers plot, arrange the data from smallest to largest. Then find the median, Q_1, and Q_3. Use the smallest value, Q_1, the median, Q_3, and the largest value to draw the box-and-whiskers plot.

To find the interquartile range, find the difference between Q_3 and Q_1.

Solution





Interquartile range $= Q_3 - Q_1 = 29.5 - 17.5 = 12$

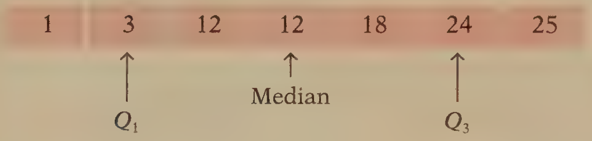
The interquartile range is 12 deaths.

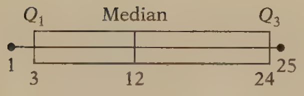
Example 3
The average monthly snowfall, in inches, in Buffalo, New York, from October through April is 1, 12, 24, 25, 18, 12, and 3. (Source: The Weather Channel) Draw a box-and-whiskers plot of the data.

Strategy
To draw the box-and-whiskers plot:

- Arrange the data from smallest to largest.
- Find the median, Q_1, and Q_3.
- Use the smallest value, Q_1, the median, Q_3, and the largest value to draw the box-and-whiskers plot.

Solution





You Try It 3
The average monthly snowfall, in inches, in Denver, Colorado, from October through April is 4, 7, 7, 8, 8, 9, and 13. (Source: The Weather Channel) Draw a box-and-whiskers plot of the data. How does the spread of the data within the interquartile range compare with that in Example 3?

Your strategy

Your solution

Solution on p. S20

7.4 Exercises

Objective A

1. State whether the mean, median, or mode is being used.
   - **a.** Half of the houses in the new development are priced under \$125,000.
   - **b.** The average bill for lunch at the college union is \$7.95.
   - **c.** The college bookstore sells more green college sweatshirts than any other color.
   - **d.** In a recent year, there were as many people age 26 and younger in the world as there were people age 26 and older.
   - **e.** The majority of full-time students carry a load of 12 credit hours per semester.
   - **f.** The average annual return on this investment is 6.5%.

2. The number of big-screen televisions sold each month for one year was recorded by an electronics store. The results were 15, 12, 20, 20, 19, 17, 22, 24, 17, 20, 15, and 27. Calculate the mean, the median, and the mode of the number of televisions sold per month.

3. The number of seats occupied on a jet for 16 transatlantic flights was recorded. The numbers were 309, 422, 389, 412, 401, 352, 367, 319, 410, 391, 330, 408, 399, 387, 411, and 398. Calculate the mean, the median, and the mode of the number of seats occupied per flight.

4. The times, in seconds, for a 100-meter dash at a college track meet were 10.45, 10.23, 10.57, 11.01, 10.26, 10.90, 10.74, 10.64, 10.52, and 10.78. Calculate the mean and the median times for the 100-meter dash.

5. A consumer research group purchased identical items in eight grocery stores. The costs for the purchased items were \$45.89, \$52.12, \$41.43, \$40.67, \$48.73, \$42.45, \$47.81, and \$45.82. Calculate the mean and the median costs of the purchased items.

6. One measure of a computer's hard-drive speed is called access time; this is measured in milliseconds (thousandths of a second). Find the mean and median for 11 hard drives whose access times were 9, 8.5, 8, 8.5, 9, 9.5, 10, 9.5, 7, 8.5, and 8.5. Round to the nearest tenth.

7. Eight health maintenance organizations (HMOs) presented group health insurance plans to a company. The monthly rates per employee were \$423, \$390, \$405, \$396, \$426, \$355, \$404, and \$430. Calculate the mean and the median monthly rates for these eight companies.

8. The lengths of the terms, in years, of all the former Supreme Court Chief Justices are given in the table below. Find the mean and median length of term for the Chief Justices.

5	0	4	34	28	8	14	21
10	8	11	4	7	15	17	

9. The life expectancies, in years, in 10 selected Central and South American countries are given at the right. Find the mean and median life expectancies in these countries.

Brazil	62
Chile	75
Costa Rica	78
Ecuador	70
Guatemala	64
Panama	75
Peru	66
Trinidad and Tobago	71
Uruguay	74
Venezuela	73

10. Your scores on six history tests were 78, 92, 95, 77, 94, and 88. If an "average score" of 90 receives an A for the course, which average, the mean or the median, would you prefer that the instructor use?

11. One student received scores of 85, 92, 86, and 89. A second student received scores of 90, 97, 91, and 94 (exactly 5 points more on each exam). Are the means of the two students the same? If not, what is the relationship between the means of the two students?

12. The table below shows the defense expenditures, in billions of dollars, by the federal government for 1965 through 1973, years during which the United States was actively involved in the Vietnam War.

a. Calculate the mean annual defense expenditure. Round to the nearest tenth of a billion.

b. Find the median annual defense expenditure.

c. If the year 1965 were eliminated from the data, how would that affect the mean? The median?

Year	*1965*	*1966*	*1967*	*1968*	*1969*	*1970*	*1971*	*1972*	*1973*
Expenditures	\$49.6	\$56.8	\$70.1	\$80.5	\$81.2	\$80.3	\$77.7	\$78.3	\$76.0

Source: Statistical Abstract of the United States

Objective B

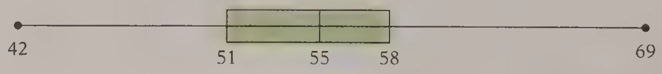
13. **a.** What percent of the data in a set of numbers lie above Q_3?

b. What percent of the data in a set of numbers lie above Q_1?

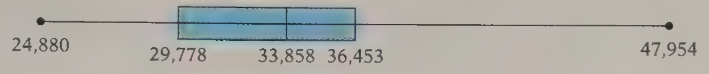
c. What percent of the data in a set of numbers lie below Q_3?

d. What percent of the data in a set of numbers lie below Q_1?

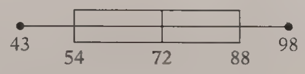
14. The box-and-whiskers plot below shows the distribution of the ages of presidents of the United States at the time of their inauguration. What is the youngest age in the set of data? The oldest age? The first quartile? The third quartile? The median? Find the range and the interquartile range.



15. The box-and-whiskers plot below shows the distribution of median incomes for 50 states and the District of Columbia. What is the lowest value in the set of data? The highest value? The first quartile? The third quartile? The median? Find the range and the interquartile range.



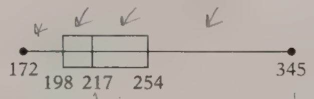
16. An aptitude test was taken by 200 students at the Fairfield Middle School. The box-and-whiskers plot at the right shows the distribution of their scores.



a. How many students scored over 88?
b. How many students scored below 72?
c. How many scores are represented in each quartile?
d. What percent of the students had scores of at least 54?

115

17. The cholesterol levels for 80 adults were recorded and then displayed in the box-and-whiskers plot shown at the right.



a. How many adults had a cholesterol level above 217?
b. How many adults had a cholesterol level below 254?
c. How many cholesterol levels are represented in each quartile?
d. What percent of the adults had a cholesterol level of not more than 198?

18. The gasoline consumption of 19 cars was tested, and the results were recorded in the table below.
a. Find the range, the first quartile, the third quartile, and the interquartile range.
b. Draw a box-and-whiskers plot of the data.
c. Is the data value 21 in the interquartile range?

Miles per Gallon for 19 Cars

33	21	30	32	20	31	25	20	16	24
22	31	30	28	26	19	21	17	26	

19. Carbon dioxide is among the gases that contribute to global warming. The world's biggest emitters of carbon dioxide are listed below. The figures are emissions in millions of metric tons per year.
a. Find the range, the first quartile, the third quartile, and the interquartile range.
b. Draw a box-and-whiskers plot of the data.
c. What data value is responsible for the long "whisker" at the right?

Carbon Dioxide Emissions (in millions of metric tons per year)

Canada	0.41	Japan	1.06
China	2.60	Russian Federation	2.10
Germany	0.87	Ukraine	0.61
India	0.76	United Kingdom	0.56
Italy	0.41	United States	4.80

Source: U.S. State Department

20. The average monthly amounts of rainfall, in inches, from January through December for Seattle, Washington, and Houston, Texas, are listed below.

a. Is the difference between the means greater than 1 inch?

b. What is the difference between the medians?

c. Draw a box-and-whiskers plot of each set of data. Use the same scale.

d. Describe the difference between the distributions of the data for Seattle and Houston.

Seattle	6.0	4.2	3.6	2.4	1.6	1.4	0.7	1.3	2.0	3.4	5.6	6.3
Houston	3.2	3.3	2.7	4.2	4.7	4.1	3.3	3.7	4.9	3.7	3.4	3.7

Source: The Weather Channel

21. The average monthly amounts of rainfall, in inches, from January through December for Orlando, Florida, and Portland, Oregon, are listed below.

a. Is the difference between the means greater than 1 inch?

b. What is the difference between the medians?

c. Draw a box-and-whiskers plot of each set of data. Use the same scale.

d. Describe the difference between the distributions of the data for Orlando and Portland.

Orlando	2.1	2.8	3.2	2.2	4.0	7.4	7.8	6.3	5.6	2.8	1.8	1.8
Portland	6.2	3.9	3.6	2.3	2.1	1.5	0.5	1.1	1.6	3.1	5.2	6.4

Source: The Weather Channel

APPLYING THE CONCEPTS

22. Explain how to determine (a) the mean, (b) the median, and (c) the mode of a set of data.

23. Write a set of data with five data values for which the mean, median, and mode are all 55.

24. A set of data has a mean of 16, a median of 15, and a mode of 14. Which of these numbers must be a value in the data? Explain your answer.

25. Explain each notation.

a. Q_1

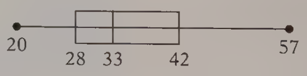
b. Q_3

c. $\bar{x}$

26. What values are shown on a box-and-whiskers plot? Explain how each is displayed.

27. The box in a box-and-whiskers plot represents 50%, or one-half, of the data in a set. Why is the box in Example 3 in this section not one-half of the entire length of the box-and-whiskers plot?

28. Create a set of data containing 25 numbers that would correspond to the box-and-whiskers plot shown at the right.



7.5 Introduction to Probability

Objective A To calculate the probability of simple events

Point of Interest

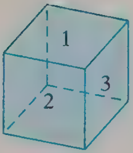
It was dice playing that led Antoine Gombaud, Chevalier de Mere, to ask Blaise Pascal, a French mathematician, to figure out the probability of throwing two sixes. Pascal and Pierre Fermat solved the problem, and their explorations led to the birth of probability theory.

A weather forecaster estimates that there is a 75% chance of rain. A state lottery director claims that there is a $\frac{1}{9}$ chance of winning a prize in a new game offered by the lottery. Each of these statements involves uncertainty to some extent. The degree of uncertainty is called **probability.** For the statements above, the probability of rain is 75%, and the probability of winning a prize in the new lottery game is $\frac{1}{9}$.

A probability is determined from an **experiment,** which is any activity that has an observable outcome. Examples of experiments include

Tossing a coin and observing whether it lands heads or tails
Interviewing voters to determine their preference for a political candidate
Drawing a card from a standard deck of 52 cards

All the possible outcomes of an experiment are called the **sample space** of the experiment. The outcomes are listed between braces. For example,



The number cube shown at the left is rolled once. Any of the numbers from 1 to 6 could show on the top of the cube. The sample space is

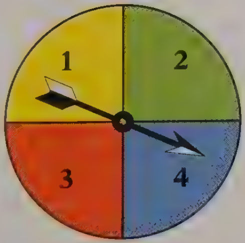
$$\{1, 2, 3, 4, 5, 6\}$$

A fair coin is tossed once. (A fair coin is one for which heads and tails have an equal chance of landing face up.) If H represents "heads up" and T represents "tails up," then the sample space is

$$\{H, T\}$$

An **event** is one or more outcomes of an experiment. For the experiment of rolling the six-sided cube described above, some possible events are

The number is even: $\{2, 4, 6\}$
The number is a multiple of 3: $\{3, 6\}$
The number is less than 10: $\{1, 2, 3, 4, 5, 6\}$
Note that in this case, the event is the entire sample space.

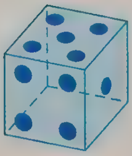


⇨ The spinner at the left is spun once. Assume that the spinner does not come to rest on a line.

a. What is the sample space?
b. List the outcomes in the event that the spinner points to an odd number.

a. The arrow could come to rest on any one of the four sectors. The sample space is $\{1, 2, 3, 4\}$.
b. $\{1, 3\}$

In discussing experiments and events, it is convenient to refer to the **favorable outcomes** of an experiment. These are the outcomes of an experiment that satisfy the requirements of a particular event.

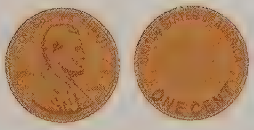


For instance, consider the experiment of rolling a fair die once. The sample space is

$$\{1, 2, 3, 4, 5, 6\}$$

and one possible event would be rolling a number that is divisible by 3. The outcomes of the experiment that are favorable to the event are 3 and 6:

$$\{3, 6\}$$



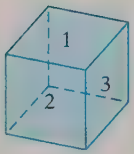
The outcomes of the experiment of tossing a fair coin are *equally likely.* Either one of the outcomes is just as likely as the other. If a fair coin is tossed once, the probability of a head is $\frac{1}{2}$, and the probability of a tail is $\frac{1}{2}$. Both events are equally likely. The theoretical probability formula, given below, applies to experiments for which the outcomes are equally likely.

Theoretical Probability Formula

The theoretical probability of an event is a fraction with the number of favorable outcomes of the experiment in the numerator and the total number of possible outcomes in the denominator.

$$\text{Probability of an event} = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

A probability of an event is a number from 0 to 1 that tells us how likely it is that this outcome will happen.



A probability of 0 means that the event is impossible.
The probability of getting a heads when rolling the die shown at the left is 0.

A probability of 1 means that the event must happen.
The probability of getting either a heads or tails when tossing a coin is 1.

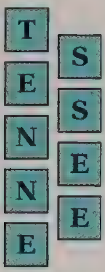
A probability of $\frac{1}{4}$ means that it is expected that the outcome will happen 1 in every 4 times the experiment is performed.

TAKE NOTE
The phrase *at random* means that each card has an equal chance of being drawn.

➡ Each of the letters of the word *TENNESSEE* is written on a card, and the cards are placed in a hat. If one card is drawn at random from the hat, what is the probability that the card has the letter *E* on it?

Count the possible outcomes of the experiment.

There are 9 letters in *TENNESSEE.*
There are 9 possible outcomes of the experiment.



Count the number of outcomes of the experiment that are favorable to the event that a card with the letter *E* on it is drawn.

There are 4 cards with an *E* on them.

Use the probability formula.

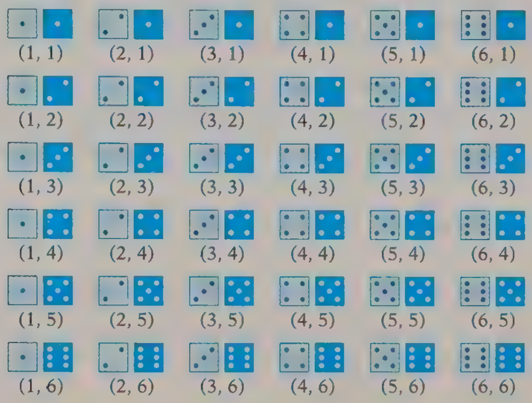
$$\text{Probability of the event} = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}} = \frac{4}{9}$$

The probability of drawing an *E* is $\frac{4}{9}$.

As shown above, calculating the probability of an event requires counting the number of possible outcomes of an experiment and the number of outcomes that are favorable to the event. One way to do this is to list the outcomes of the experiment in a systematic way. Using a table is often helpful.

When two dice are rolled, the sample space for the experiment can be recorded systematically as in the following table.

Possible Outcomes from Rolling Two Dice



Point of Interest

Romans called a die that was marked on four faces a *talus*, which meant "anklebone." The anklebone was considered an ideal die because it is roughly a rectangular solid and it has no marrow, so loose anklebones from sheep were more likely than other bones to be lying around after the wolves had left their prey.

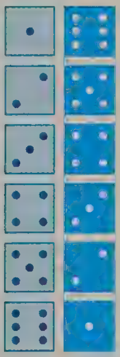
➡ Two dice are rolled once. Calculate the probability that the sum of the numbers on the two dice is 7.

Use the table above to count the number of possible outcomes of the experiment.

There are 36 possible outcomes.

Count the number of outcomes of the experiment that are favorable to the event that a sum of 7 is rolled.

There are 6 favorable outcomes: (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), and (6, 1)



Use the probability formula.

$$\text{Probability of the event} = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}} = \frac{6}{36} = \frac{1}{6}$$

The probability of a sum of 7 is $\frac{1}{6}$.

The probabilities calculated above are **theoretical probabilities** because the calculations are based on theory—for example, that either side of a coin is equally likely to land face up or that each of the six sides of a fair die are equally likely to land face up. Not all probabilities arise from such assumptions.

Empirical probabilities are based on observations of certain events. For instance, a weather forecast of a 75% chance of rain is an empirical probability. From historical records kept by the weather bureau, when a similar weather pattern existed, rain occurred 75% of the time. It is theoretically impossible to predict the weather, and only observations of past weather patterns can be used to predict future weather conditions.

Empirical Probability Formula

The empirical probability of an event is a ratio of the number of observations of the event to the total number of observations.

$$\text{Probability of an event} = \frac{\text{number of observations of the event}}{\text{total number of observations}}$$

For example, suppose the records of an insurance company show that of 2549 claims for theft filed by policy holders, 927 were claims for more than \$5000. The empirical probability that the next claim for theft that this company receives will be a claim for more than \$5000 is the ratio of the number of claims for over \$5000 to the total number of claims. The probability is approximately 0.36.

$$\frac{927}{2549} \approx 0.36$$

Example 1

There are three choices, *a*, *b*, or *c*, for each of the two questions on a multiple-choice quiz. If the instructor randomly chooses which questions will have an answer of *a*, *b*, or *c*, what is the probability that the two correct answers on this quiz will be the same letter?

Strategy

To find the probability:

- List the outcomes of the experiment in a systematic way.
- Count the number of possible outcomes of the experiment.
- Count the number of outcomes of the experiment that are favorable to the event that the two correct answers on the quiz will be the same letter.
- Use the probability formula.

Solution

Possible outcomes: (a, a) (b, a) (c, a)
(a, b) (b, b) (c, b)
(a, c) (b, c) (c, c)

There are 9 possible outcomes.

There are 3 favorable outcomes:
(a, a), (b, b), (c, c)

$$\text{Probability} = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}} = \frac{3}{9} = \frac{1}{3}$$

The probability that the two correct answers will be the same letter is $\frac{1}{3}$.

You Try It 1

A professor writes three true/false questions for a quiz. If the professor randomly chooses which questions will have a true answer and which will have a false answer, what is the probability that the test will have 2 true questions and 1 false question?

Your strategy

Your solution

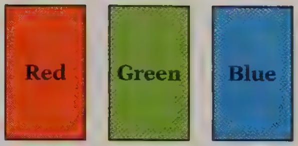
Solution on pp. S20–S21

7.5 Exercises

Objective A

1. A coin is tossed four times. List all the possible outcomes of the experiment as a sample space.

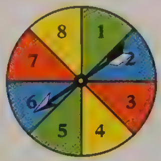
2. Three cards—one red, one green, and one blue—are to be arranged in a stack. Using R for red, G for green, and B for blue, list all the different stacks that can be formed. (Some computer monitors are called RGB monitors for the colors red, green, and blue.)



3. A tetrahedral die is one with four triangular sides. The sides show the numbers from 1 to 4. Say two tetrahedral dice are rolled. List all the possible outcomes of the experiment as a sample space.

4. A coin is tossed and then a die is rolled. List all the possible outcomes of the experiment as a sample space. [To get you started, (H, 1) is one of the possible outcomes.]

5. The spinner at the right is spun once. Assume that the spinner does not come to rest on a line.
   a. What is the sample space?
   b. List the outcomes in the event that the number is less than 4.



6. A coin is tossed four times.
   a. What is the probability that the outcomes of the tosses are exactly in the order HHTT? (See Exercise 1.)
   b. What is the probability that the outcomes of the tosses consist of two heads and two tails?
   c. What is the probability that the outcomes of the tosses consist of one head and three tails?

7. Two dice are rolled.
   a. What is the probability that the sum of the dots on the upward faces is 5?
   b. What is the probability that the sum of the dots on the upward faces is 15?
   c. What is the probability that the sum of the dots on the upward faces is less than 15?
   d. What is the probability that the sum of the dots on the upward faces is 2?

8. A dodecahedral die has 12 sides numbered from 1 to 12. The die is rolled once.
   a. What is the probability that the upward face shows an 11?
   b. What is the probability that the upward face shows a 5?

9. Two tetrahedral dice are rolled (see Exercise 3).
   a. What is the probability that the sum on the upward faces is 4?
   b. What is the probability that the sum on the upward faces is 6?

10. A dodecahedral die has 12 sides numbered from 1 to 12. The die is rolled once.
    a. What is the probability that the upward face shows a number divisible by 4?
    b. What is the probability that the upward face shows a number that is a multiple of 3?

11. Two dice are rolled. Which has the greater probability, throwing a sum of 10 or throwing a sum of 5?

12. Two dice are rolled once. Calculate the probability that the two numbers on the dice are equal.

13. Each of the letters of the word *MISSISSIPPI* is written on a card, and the cards are placed in a hat. One card is drawn at random from the hat.
a. What is the probability that the card has the letter *I* on it?
b. Which is greater, the probability of choosing an *S* or that of choosing a *P*?

14. There are five choices—choices *a* through *e*—for each question on a multiple-choice test. What is the probability of choosing the correct answer for a certain question by just guessing?

15. Three blue marbles, four green marbles, and five red marbles are placed in a bag. One marble is chosen at random.
a. What is the probability that the marble chosen is green?
b. Which is greater, the probability of choosing a blue marble or that of choosing a red marble?

16. Which has the greater probability, drawing a jack, queen, or king from a deck of cards or drawing a spade?

17. In a history class, a set of exams earned the following grades: 4 A's, 8 B's, 22 C's, 10 D's, and 3 F's. If a single student's exam is chosen from this class, what is the probability that it received a B?

18. A survey of 95 people showed that 37 preferred (to using a credit card) a cash discount of 2% if an item was purchased using cash or a check. Judging on the basis of this survey, what is the empirical probability that a person prefers a cash discount? Write the answer as a decimal rounded to the nearest hundredth.

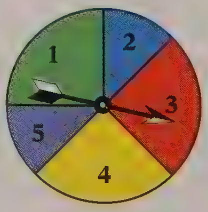
19. A survey of 725 people showed that 587 had a group health insurance plan where they worked. On the basis of this survey, what is the empirical probability that an employee has a group health insurance plan? Write the answer as a decimal rounded to the nearest hundredth.

20. A television cable company surveyed some of its customers and asked them to rate the cable service as excellent, satisfactory, average, unsatisfactory, or poor. The results are recorded in the table at the right. What is the probability that a customer who was surveyed rated the service as satisfactory or excellent?

Quality of Service	*Number Who Voted*
Excellent	98
Satisfactory	87
Average	129
Unsatisfactory	42
Poor	21

APPLYING THE CONCEPTS

21. If the spinner at the right is spun once, is each of the numbers 1 through 5 equally likely? Why or why not?



22. The probability of tossing a coin and having it land heads up is $\frac{1}{2}$. Does this mean that if that coin is tossed 100 times, it will land heads 50 times? Explain your answer.

23. Why can the probability of an event not be $\frac{5}{3}$?

Focus on Problem Solving

Inductive Reasoning

Suppose that, beginning in January, you save \$25 each month. The total amount you have saved at the end of each month can be described by a list of numbers.

25	50	75	100	125	150	175	
Jan	Feb	Mar	Apr	May	June	July	...

The list of numbers that indicates your total savings is an *ordered* list of numbers called a **sequence.** Each of the numbers in a sequence is called a **term** of the sequence. The list is ordered because the position of a number in the list indicates the month in which that total amount has been saved. For example, the 7th term of the sequence (indicating July) is 175. This number means that a total of \$175 has been saved by the end of the 7th month.

Now consider a person who has a different savings plan. The total amount saved by this person for the first seven months is given by the sequence

$$20, 35, 50, 65, 80, 95, 110, \ldots$$

The process you use to discover the next number in the above sequence is *inductive reasoning.* **Inductive reasoning** involves making generalizations from specific examples; in other words, we reach a conclusion by making observations about particular facts or cases. In the case of the above sequence, the person saved \$15 per month after the first month.

Here is another example of inductive reasoning. Find the next two letters of the sequence A, B, E, F, I, J, ...

By trying different patterns, we can determine that a pattern for this sequence is

$$\underline{A, B}, C, D, \underline{E, F}, G, H, \underline{I, J}, \ldots$$

That is, write two letters, skip two letters, write two letters, skip two letters, and so on. The next two letters are M, N.

Use inductive reasoning to solve the following problems.

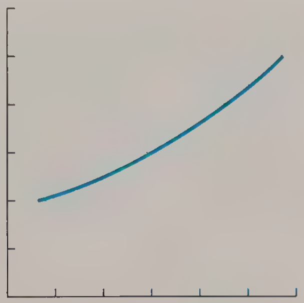
1. What is the next term of the sequence, ban, ben, bin, bon, ... ?
2. Using a calculator, determine the decimal representation of several proper fractions that have a denominator of 99. For instance, you may use $\frac{8}{99}$, $\frac{23}{99}$, and $\frac{75}{99}$. Now use inductive reasoning to explain the pattern, and use your reasoning to find the decimal representation of $\frac{53}{99}$ without a calculator.
3. Find the next number in the sequence 1, 1, 2, 3, 5, 8, 13, 21, ...
4. The decimal representation of a number begins 0.10100100010000100000 ... . What are the next 10 digits in this number?
5. The first seven rows of a triangle of numbers called Pascal's triangle are given below. Find the next row.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

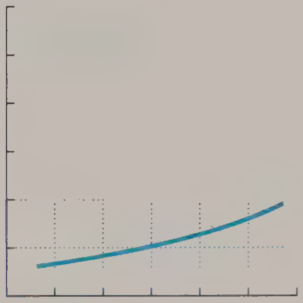
Projects and Group Activities

Deceptive Graphs

A graphical representation of data can sometimes be misleading. Consider the graphs shown below. A financial advisor with an investment firm claims that an investment with the firm grew as shown in the graph on the left, whereas an investment with a competitor grew as shown in the graph on the right. Apparently, one would have accumulated more money by choosing the investment on the left.



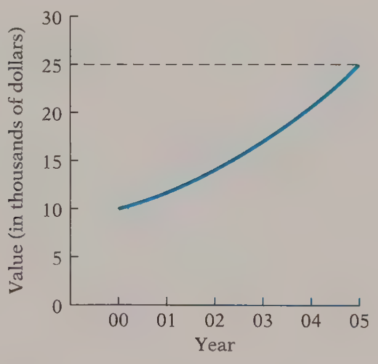
Growth of Investment with Broker A

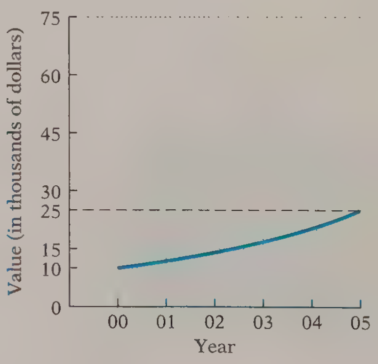


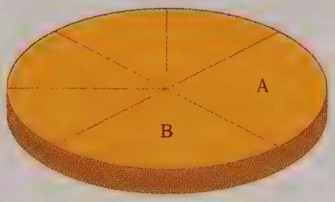
Growth of Investment with Broker B

However, these graphs have a serious flaw. There are no labels on the horizontal and vertical axes. Therefore, it is impossible to tell which investment increased more or over what time interval. When labels are not placed on the axes of a graph, the data represented are meaningless. This is one way in which advertisers use a visual impact to distort the true meaning of data.

The graphs below are the same as those drawn above except that scales have been drawn along each axis. Now it is possible to tell how each investment performed. Note that both turned in exactly the same performance!



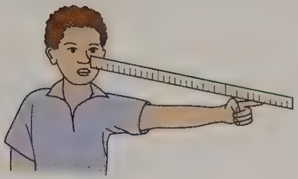




Drawing a circle graph as an oval is another way of distorting data. This is especially true if a three-dimensional representation is given. From the appearance of the circle graph at the left, one would think that region B is larger than region A. However, that isn't true. Measure the angle of each sector to see this for yourself. As you read newspapers and magazines, find examples of graphs that may distort the actual data. Discuss how these graphs should be drawn to be more accurate.

Collecting, Organizing, Displaying, and Analyzing Data

Before standardized units of measurement, measurements were made in terms of the human body. For example, the cubit was the distance from the end of the elbow to the tips of the fingers. The yard was the distance from the tip of the nose to the tip of the fingers on an outstretched arm.



For each student in the class, find the measure from the tip of the nose to the tip of the fingers on an outstretched arm. Round each measure to the nearest centimeter. Record all the measurements on the board.

1. From the data collected, determine each of the following.

Mean ________
Median ________
Mode ________
Range ________
First quartile, Q_1 ________
Third quartile, Q_3 ________
Interquartile range ________

2. Prepare a box-and-whiskers plot of the data.

3. Write a description of the spread of the data.

4. Explain why we need standardized units of measurement.

Chapter Summary

Key Words

Statistics is the branch of mathematics concerned with *data,* or numerical information. [p. 289]

A *graph* is a pictorial representation of data. [p. 289]

A *pictograph* uses symbols to represent information. [p. 289]

A *circle graph* represents data by the size of the sectors. [p. 291]

A *bar graph* represents data by the height of the bars. [p. 297]

A *broken-line graph* represents data by the position of the lines and shows trends and comparisons. [p. 298]

A *histogram* is a special kind of bar graph. In a histogram, the width of each bar corresponds to a range of numbers called a *class interval.* The height of each bar corresponds to the number of occurrences of data in each class interval and is called the *class frequency.* [p. 301]

A *frequency polygon* is a graph that displays information in a manner similar to a histogram. A dot is placed above the center of each class interval at a height corresponding to that class's frequency. The dots are connected to form a broken-line graph. The center of a class interval is called the *class midpoint.* [p. 302]

The *mean* of a set of data is the sum of the numbers in the set divided by the number of items in the set. The symbol for mean is $\bar{x}$. [p. 305]

The *median* of a set of data is the number that separates the data into two equal parts when the numbers are arranged from smallest to largest. There is an equal number of values above the median and below the median. [p. 306]

The *mode* of a set of numbers is the value that occurs most frequently. [p. 306]

The *range* of a set of numbers is the difference between the largest number and the smallest number in the set. The range describes the spread of the data. [p. 309]

The *first quartile,* symbolized by Q_1, is the number that one-quarter of the data lie below. The *third quartile,* symbolized by Q_3, is the number that one-quarter of the data lie above. [p. 308]

The *interquartile range* is the difference between the third quartile, Q_3, and the first quartile, Q_1. [p. 309]

A *box-and-whiskers plot,* or *boxplot,* is a graph that shows five numbers of a data set: the lowest number, the first quartile, the median, the third quartile, and the highest number. [p. 309]

An *experiment* is any activity that has an observable outcome. All the possible outcomes of an experiment are called the *sample space* of the experiment. [p. 315]

An *event* is one or more outcomes of an experiment. *Favorable outcomes* of an experiment are the outcomes that satisfy the requirements of a particular event. [pp. 315–316]

The *probability* of an event is a number from 0 to 1 that tells us how likely it is that a certain outcome will happen. [pp. 315–316]

Essential Rules

To Find the Mean

Divide the sum of the numbers by the number of addends.

$$\bar{x} = \frac{\text{sum of all the data values}}{\text{number of data values}}$$ [p. 305]

To Find the Median

1. Arrange the numbers from smallest to largest.
2. If there is an *odd* number of values in the set of data, the median is the middle number. If there is an *even* number of values in the set of data, the median is the mean of the two middle numbers. [p. 306]

To Find the Mode

Find the number in the set of data that occurs more often than any other number in the set. [p. 306]

To Find the Range of a Set of Data

Subtract the smallest number in the set from the largest number in the set. Range = largest value − smallest value [p. 309]

To Find Q_1

Arrange the numbers from smallest to largest and locate the median. Q_1 is the median of the lower half of the data. [p. 308]

To Find Q_3

Arrange the numbers from smallest to largest and locate the median. Q_3 is the median of the upper half of the data. [p. 308]

To Find the Interquartile Range

Interquartile range $= Q_3 - Q_1$ [p. 309]

Theoretical Probability Formula

$$\text{Probability of an event} = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$ [p. 316]

Empirical Probability Formula

$$\text{Probability of an event} = \frac{\text{number of observations of the event}}{\text{total number of observations}}$$ [p. 318]

Chapter Review

The circle graph in Figure 37 shows the approximate amount of money that government agencies spent on maintaining Internet web sites for a 3-year period.

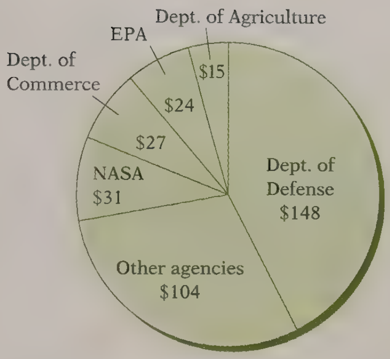


Figure 37 Millions of dollars that federal agencies spent on web sites
Source: General Accounting Office

1. Find the total amount of money that these agencies spent on maintaining web sites.

2. What is the ratio of the amount spent by the Department of Commerce to the amount spent by the EPA?

3. What percent of the total money spent did NASA spend? Round to the nearest tenth of a percent.

The double-line graph in Figure 38 shows the populations of California and Texas.

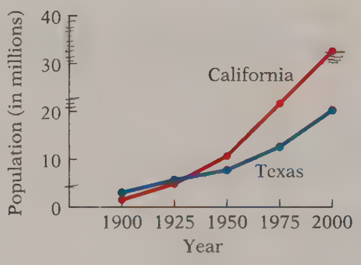


Figure 38 Populations of California and Texas

4. In 1900, which state had the larger population?

5. In 2000, approximately how much greater was the population of California than the population of Texas?

6. During which 25-year period did the population of Texas increase the least?

The frequency polygon in Figure 39 shows the range of scores for the first 80 games of a season for the New York Knicks basketball team.

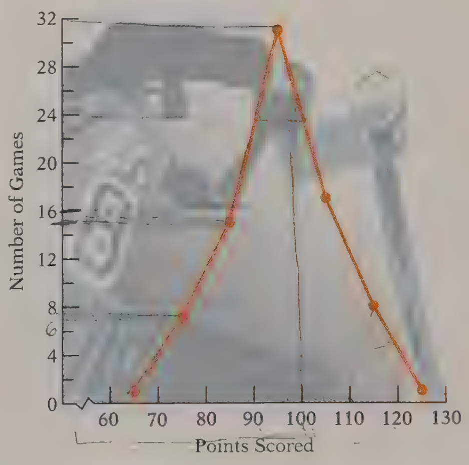


Figure 39
Source: Sports Illustrated web site at **http://CNNSI.com/**

7. Find the number of games in which fewer than 100 points were scored by the Knicks.

8. What is the ratio of the number of games in which 90 to 100 points were scored to the number of games in which 110 to 120 points were scored?

9. In what percent of the games were 110 points or more scored? Round to the nearest tenth of a percent.

The pictograph in Figure 40 shows the numbers of passengers annually passing through the five busiest airports in the United States.

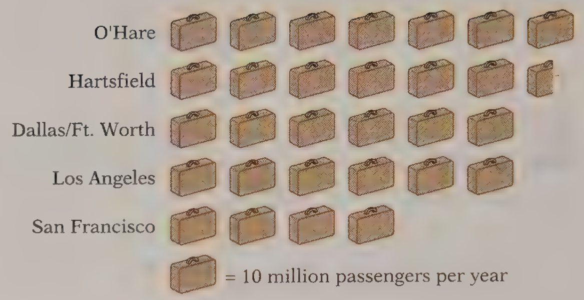


Figure 40 The busiest U.S. airports

Source: Airports Association Council International–North America; Air Transport Association of America

10. How many more passengers pass through O'Hare each year than pass through the Los Angeles airport each year?

11. What is the ratio of the number of passengers passing through the San Francisco airport to the number of passengers passing through the Dallas/Ft. Worth airport each year? Write your answer using a colon.

The double-bar graph in Figure 41 shows the total days open and the days of full operation of ski resorts in different regions of the country.

12. Find the difference between the total days open and the days of full operation for Midwest ski areas.

13. What percent of the total days open were the days of full operation for the Rocky Mountain ski areas?

14. Which region had the lowest number of days of full operation? How many days of full operation did this region have?

15. A coin is tossed four times. What is the probability that the outcomes of the tosses consist of one tail and three heads?

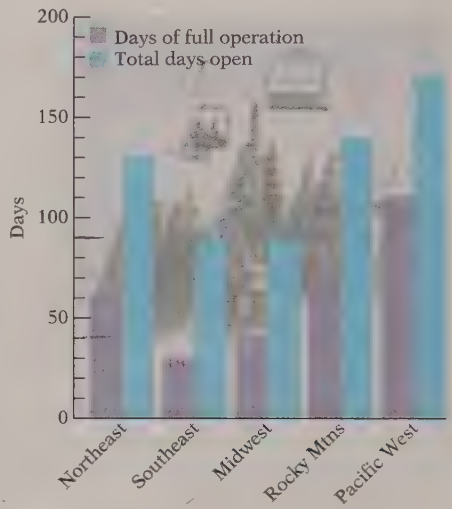


Figure 41

Source: Economic Analysis of United States Ski Areas

Based on a Gallup poll, the numbers of hours that the 46 people surveyed slept during a typical weekday night are shown in the histogram in Figure 42.

16. How many people slept 8 hours or more?

17. What percent of the people surveyed slept 7 hours? Round to the nearest tenth of a percent.

18. The heart rates of 24 women tennis players were measured after each of them had run one-quarter of a mile. The results are listed in the table below. Find the mean, median, and mode for the data. Round to the nearest tenth.

80	82	99	91	93	87	103	94	73	96	86	80
97	94	108	81	100	109	91	84	78	96	96	100

19. For the data in Exercise 18, find the range and the interquartile range.

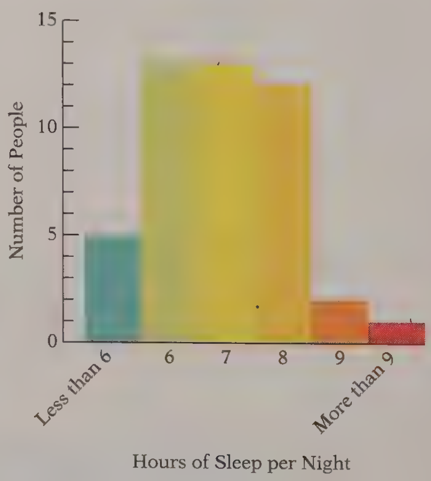


Figure 42

Chapter Test

Forty college students were surveyed to see how much money they spent each week on dining out in restaurants. The results are recorded in the frequency polygon shown in Figure 43.

1. How many students spent between \$15 and \$25 ~~per week?~~

2. Find the ratio of the number of students who spent between \$10 and \$15 to the number who spent between \$15 and \$20.

3. What percent of the students surveyed spent less than \$15 per week?

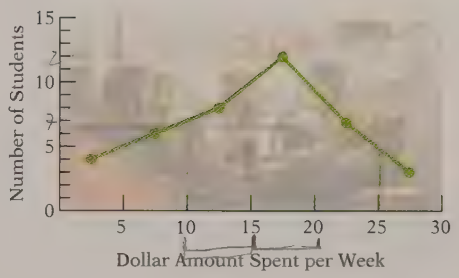


Figure 43

The pictograph in Figure 44 is based on the results of a Gallup poll survey of married couples. Each individual was asked to give a letter grade to the marriage.

4. Find the total number of people who were surveyed.

5. Find the ratio of the number of people who gave their marriage a B to the number who gave it a C.

6. What percent of the total number of people surveyed gave their marriage an A?

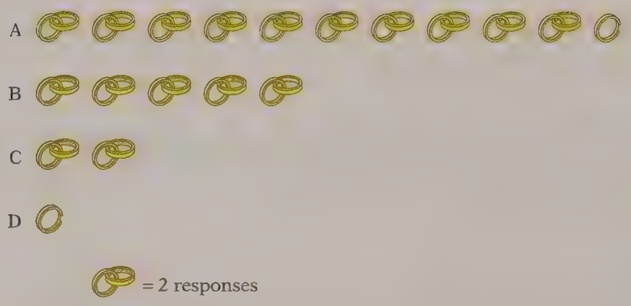


Figure 44 Survey of married couples rating their marriage

The bar graph in Figure 45 shows the number of fatalities that occurred in accidents on amusement rides in the 1990s in the United States.

7. During which two consecutive years were the numbers of fatalities the same?

8. Find the total number of fatalities on amusement rides during 1991 through 1999.

9. How many more such fatalities occurred during the years 1995 through 1998 than occurred during the years 1991 through 1994?

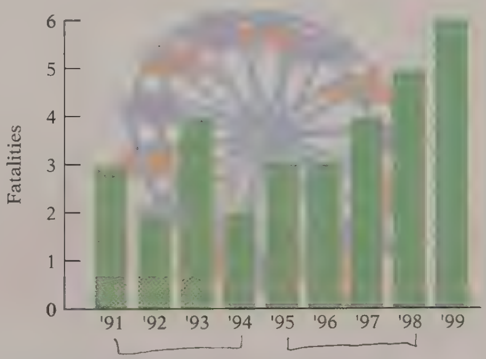


Figure 45 Number of fatalities in accidents on amusement rides

Source: USA Today, April 7, 2000

The circle graph in Figure 46 categorizes the 655 films released during a recent year by their ratings.

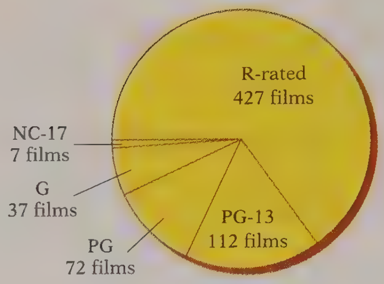


Figure 46 Ratings of films released

Source: MPA Worldwide Market Research

10. How many more R-rated films were released than PG films?

11. How many times more PG-13 films were released than NC-17?

12. What percent of the films released were rated G? Round to the nearest tenth of a percent.

The histogram in Figure 47 gives information about the per capita incomes, by state, in the United States.

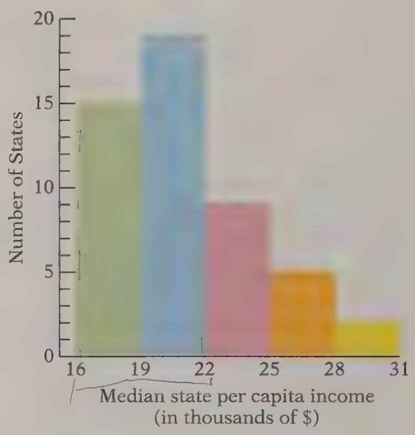


Figure 47

Source: Bureau of Labor Statistics

13. How many states have per capita incomes between \$16,000 and \$22,000?

14. What percent of the states have a per capita income that is between \$19,000 and \$22,000?

15. What percent of the states have a per capita income that is \$25,000 or more?

16. A box contains 50 balls, of which 15 are red. If 1 ball is randomly selected from the box, what is the probability of the ball's being red?

The line graph in Figure 48 shows the number of students enrolled in colleges.

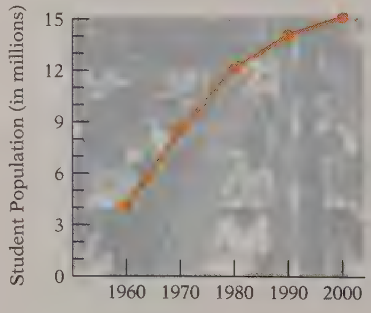


Figure 48 Student enrollment in public and private colleges

Source: National Center for Educational Statistics

17. During which decade did the student population increase the least?

18. Approximate the increase in the college enrollment from 1960 to 2000.

19. The length of time (in days) that various batteries operated a portable CD player continuously are given in the table below. Find the mean and median for the data.

2.9	2.4	3.1	2.5	2.6	2.0	3.0	2.3	2.4	2.7
2.0	2.4	2.6	2.7	2.1	2.9	2.8	2.4	2.0	2.8

20. For the data in Exercise 19, draw a box-and-whiskers plot.

Cumulative Review

1. Simplify: $2^2 \cdot 3^3 \cdot 5$

2. Simplify: $3^2 \cdot (5 - 2) \div 3 + 5$

3. Find the LCM of 24 and 40.

4. Write $\frac{60}{144}$ in simplest form.

5. Find the total of $4\frac{1}{2}$, $2\frac{3}{8}$, and $5\frac{1}{5}$.

6. Subtract: $12\frac{5}{8} - 7\frac{11}{12}$

7. Multiply: $\frac{5}{8} \times 3\frac{1}{5}$

8. Find the quotient of $3\frac{1}{5}$ and $4\frac{1}{4}$.

9. Simplify: $\frac{5}{8} \div \left(\frac{3}{4} - \frac{2}{3}\right) + \frac{3}{4}$

10. Write two hundred nine and three hundred five thousandths in standard form.

11. Find the product of 4.092 and 0.69.

12. Convert $16\frac{2}{3}$ to a decimal. Round to the nearest hundredth.

13. Write "330 miles on 12.5 gallons of gas" as a unit rate.

14. Solve the proportion: $\frac{n}{5} = \frac{16}{25}$

15. Write $\frac{4}{5}$ as a percent.

16. 8 is 10% of what?

17. What is 38% of 43?

18. What percent of 75 is 30?

19. Tanim Kamal, a salesperson at a department store, receives $100 per week plus 2% commission on sales. Find the income for a week in which Tanim had $27,500 in sales.

20. A life insurance policy costs $4.15 for every $1000 of insurance. At this rate, what is the cost for $50,000 of life insurance?

21. A contractor borrowed $125,000 for 6 months at an annual simple interest rate of 11%. Find the interest due on the loan.

22. A compact disc player with a cost of $180 is sold for $279. Find the markup rate.

23. The circle graph in Figure 49 shows how a family's monthly income of $3000 is budgeted. How much is budgeted for food?

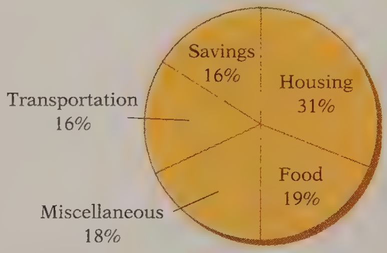


Figure 49 Budget for a monthly income of $3000

24. The double-broken-line graph in Figure 50 shows two students' scores on 5 math tests of 30 problems each. Find the difference between the numbers of problems that the two students answered correctly on Test 1.

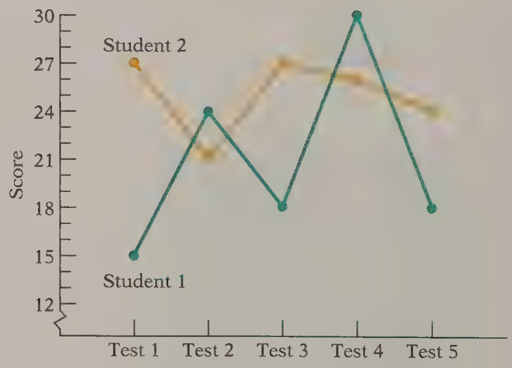


Figure 50

25. The average daily high temperatures, in degrees Fahrenheit, for a week in Newtown were 56°, 72°, 80°, 75°, 68°, 62°, and 74°. Find the mean high temperature for the week. Round to the nearest tenth of a degree.

26. Two dice are rolled. What is the probability that the sum of the dots on the upward faces is 8?

Chapter

8

U.S. Customary Units of Measurement

Police officers investigating the scene of a traffic accident have a number of tools at their disposal. One such tool is a nomograph, such as the one in the **project on page 352.** A nomograph is used to determine the speed of a car given the length of the skid marks left by the car and the road conditions at the time. For example, a car that skids 200 ft on wet concrete was traveling 60 mph.

Objectives

Section 8.1

A To convert measurements of length in the U.S. Customary System

B To perform arithmetic operations with measurements of length

C To solve application problems

Section 8.2

A To convert measurements of weight in the U.S. Customary System

B To perform arithmetic operations with measurements of weight

C To solve application problems

Section 8.3

A To convert measurements of capacity in the U.S. Customary System

B To perform arithmetic operations with measurements of capacity

C To solve application problems

Section 8.4

A To use units of energy in the U.S. Customary System

B To use units of power in the U.S. Customary System

Need help? For on-line student resources, such as section quizzes, visit this textbook's web site at **math.college.hmco.com/students.**

Prep Test

For Exercises 1 to 8, add, subtract, multiply, or divide.

1. $\begin{array}{r} 485 \\ +\ 217 \\ \hline \end{array}$

2. $\begin{array}{r} 145 \\ -\ 87 \\ \hline \end{array}$

3. $36 \times \frac{1}{9}$

4. $\frac{5}{3} \times 6$

5. $400 \times \frac{1}{8} \times \frac{1}{2}$

6. $5\frac{3}{4} \times 8$

7. $3\overline{)714}$

8. $12\overline{)18}$

Go Figure

Mandy walks to work at a constant rate. One-third of the way to work, she passes a bank. Three-fourths of the way to work, she passes a book store. At the bank her watch reads 7:52 A.M. and at the book store it reads 8:02 A.M. At what time is Mandy one-half of the way to work?

8.1

Length

Objective A

To convert measurements of length in the U.S. Customary System

A **measurement** includes a number and a unit.

$$\underbrace{\begin{matrix}3\\7\\12\end{matrix}}_{\text{Number}}\qquad\underbrace{\begin{matrix}\text{feet}\\\text{miles}\\\text{yards}\end{matrix}}_{\text{Unit}}$$

Point of Interest

The ancient Greeks devised the foot measurement, which they usually divided into 16 fingers. It was the Romans who subdivided the foot into 12 units called inches. The word *inch* is derived from the Latin word *uncia*, which means "a twelfth part."

The Romans also used a unit called *pace*, which equaled two steps. One thousand paces equaled one mile. The word *mile* is derived from the Latin word *mille*, which means "1000."

Standard units of measurement have been established to simplify trade and commerce.

The unit of length, or distance, that is called the **yard** was originally defined as the length of a specified bronze bar located in London.

The standard U.S. Customary System units of length are **inch, foot, yard,** and **mile.** Equivalences between units of length in the U.S. Customary System are

$$\begin{aligned}12 \text{ inches (in.)} &= 1 \text{ foot (ft)}\\ 3 \text{ ft} &= 1 \text{ yard (yd)}\\ 36 \text{ in.} &= 1 \text{ yard (yd)}\\ 5280 \text{ ft} &= 1 \text{ mile (mi)}\end{aligned}$$

These equivalences can be used to form conversion rates to change one unit of measurement to another. For example, because 3 ft = 1 yd, the conversion rates $\frac{3 \text{ ft}}{1 \text{ yd}}$ and $\frac{1 \text{ yd}}{3 \text{ ft}}$ are both equivalent to 1.

➡ Convert 27 ft to yards.

$$\begin{aligned}27 \text{ ft} &= 27 \text{ ft} \times \boxed{\frac{1 \text{ yd}}{3 \text{ ft}}}\\ &= 27 \cancel{\text{ft}} \times \frac{1 \text{ yd}}{3 \cancel{\text{ft}}}\\ &= \frac{27 \text{ yd}}{3}\\ &= 9 \text{ yd}\end{aligned}$$

➡ Convert 5 yd to feet.

$$\begin{aligned}5 \text{ yd} &= 5 \text{ yd} \times \boxed{\frac{3 \text{ ft}}{1 \text{ yd}}}\\ &= 5 \cancel{\text{yd}} \times \frac{3 \text{ ft}}{1 \cancel{\text{yd}}}\\ &= \frac{15 \text{ ft}}{1}\\ &= 15 \text{ ft}\end{aligned}$$

Note that in the conversion rate chosen, the unit in the numerator is the same as the unit desired in the answer. The unit in the denominator is the same as the unit in the given measurement.

Example 1 Convert 40 in. to feet.

Solution

$$40 \text{ in.} = 40 \text{ } \cancel{\text{in.}} \times \frac{1 \text{ ft}}{12 \text{ } \cancel{\text{in.}}}$$

$$= \frac{40 \text{ ft}}{12} = 3\frac{1}{3} \text{ ft}$$

You Try It 1 Convert 14 ft to yards.

Your solution

Example 2 Convert $3\frac{1}{4}$ yd to feet.

Solution

$$3\frac{1}{4} \text{ yd} = \frac{13}{4} \text{ yd} = \frac{13}{4} \text{ } \cancel{\text{yd}} \times \frac{3 \text{ ft}}{1 \text{ } \cancel{\text{yd}}}$$

$$= \frac{39 \text{ ft}}{4} = 9\frac{3}{4} \text{ ft}$$

You Try It 2 Convert 9240 ft to miles.

Your solution

Solutions on p. S21

Objective B

To perform arithmetic operations with measurements of length

When performing arithmetic operations with measurements of length, write the answer in simplest form. For example, 1 ft 14 in. should be written as 2 ft 2 in.

⇒ Convert: 50 in. = ____ ft ____ in.

$$\begin{array}{r} 4 \text{ ft } 2 \text{ in.} \\ 12\overline{)\ 50} \\ \underline{-48} \\ 2 \end{array}$$

- **Because 12 in. = 1 ft, divide 50 in. by 12. The whole-number part of the quotient is the number of feet. The remainder is the number of inches.**

50 in. = 4 ft 2 in.

Example 3 Convert: 17 in. = ____ ft ____ in.

Solution

$$\begin{array}{r} 1 \text{ ft } 5 \text{ in.} \\ 12\overline{)\ 17} \\ \underline{-12} \\ 5 \end{array}$$

- **12 in. = 1 ft**

17 in. = 1 ft 5 in.

You Try It 3 Convert:
42 in. = ____ ft ____ in.

Your solution

Example 4 Convert: 31 ft = ____ yd ____ ft

Solution

$$\begin{array}{r} 10 \text{ yd } 1 \text{ ft} \\ 3\overline{)\ 31} \\ \underline{-30} \\ 1 \end{array}$$

- **3 ft = 1 yd**

31 ft = 10 yd 1 ft

You Try It 4 Convert: 14 ft = ____ yd ____ ft

Your solution

Solutions on p. S21

Example 5 Find the sum of 4 ft 4 in. and 1 ft 11 in.

Solution

$$\begin{array}{r} 4\text{ ft}\quad 4\text{ in.} \\ +\ 1\text{ ft}\ 11\text{ in.} \\ \hline 5\text{ ft}\ 15\text{ in.} \end{array}$$

• 15 in. = 1 ft 3 in.

5 ft 15 in. = 6 ft 3 in.

You Try It 5 Find the sum of 3 ft 5 in. and 4 ft 9 in.

Your solution

Example 6 Subtract: 9 ft 6 in. − 3 ft 8 in.

Solution

$$\begin{array}{r} \overset{8\text{ ft}}{\cancel{9\text{ ft}}}\quad \overset{18\text{ in.}}{\cancel{6\text{ in.}}} \\ -\ 3\text{ ft}\quad 8\text{ in.} \\ \hline 5\text{ ft}\ 10\text{ in.} \end{array}$$

• Borrow 1 ft (12 in.) from 9 ft and add to 6 in.

You Try It 6 Subtract: 4 ft 2 in. − 1 ft 8 in.

Your solution

Example 7 Multiply: 3 yd 2 ft × 4

Solution

$$\begin{array}{r} 3\text{ yd}\ 2\text{ ft} \\ \times\qquad 4 \\ \hline 12\text{ yd}\ 8\text{ ft} \end{array}$$

• 8 ft = 2 yd 2 ft

12 yd 8 ft = 14 yd 2 ft

You Try It 7 Multiply: 4 yd 1 ft × 8

Your solution

Example 8 Find the quotient of 4 ft 3 in. and 3.

Solution

$$\begin{array}{r|rr} & 1\text{ ft} & 5\text{ in.} \\ \hline 3 & 4\text{ ft} & 3\text{ in.} \\ & -3\text{ ft} & \\ \cline{2-2} & 1\text{ ft} = & 12\text{ in.} \\ \cline{3-3} & & 15\text{ in.} \\ & & -15\text{ in.} \\ \cline{3-3} & & 0 \end{array}$$

You Try It 8 Find the quotient of 7 yd 1 ft and 2.

Your solution

Example 9 Multiply: $2\frac{3}{4}$ ft × 3

Solution

$$\begin{aligned} 2\frac{3}{4}\text{ ft} \times 3 &= \frac{11}{4}\text{ ft} \times 3 \\ &= \frac{33}{4}\text{ ft} \\ &= 8\frac{1}{4}\text{ ft} \end{aligned}$$

You Try It 9 Subtract: $6\frac{1}{4}$ ft − $3\frac{2}{3}$ ft

Your solution

Solutions on p. S21

Objective C To solve application problems

Example 10

A concrete block is 9 in. high. How many rows of blocks are required for a retaining wall that is 6 ft high?

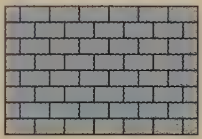
Strategy

To find the number of rows of blocks, convert 9 in. to feet. Then divide the height of the wall (6 ft) by the height of each block.

Solution

$$9\text{ in.} = \frac{9\cancel{\text{ in.}}}{1}\cdot\frac{1\text{ ft}}{12\cancel{\text{ in.}}} = \frac{9\text{ ft}}{12} = 0.75\text{ ft}$$

$$\frac{6\text{ ft}}{0.75\text{ ft}} = 8$$



The wall will have 8 rows of blocks.

You Try It 10

The floor of a storage room is being tiled. Eight tiles, each a 9-inch square, fit across the width of the floor. Find the width, in feet, of the storage room.

Your strategy

Your solution

Example 11

A plumber used 3 ft 9 in., 2 ft 6 in., and 11 in. of copper tubing to install a sink. Find the total length of copper tubing used.

Strategy

To find the total length of copper tubing used, add the three lengths of copper tubing (3 ft 9 in., 2 ft 6 in., and 11 in.).

Solution

$$\begin{array}{lrr} & 3\text{ ft} & 9\text{ in.} \\ & 2\text{ ft} & 6\text{ in.} \\ + & & 11\text{ in.} \\ \hline & 5\text{ ft} & 26\text{ in.} \end{array}$$

• 26 in. = 2 ft 2 in.

5 ft 26 in. = 7 ft 2 in.

The plumber used 7 ft 2 in. of copper tubing.

You Try It 11

A board 9 ft 8 in. is cut into four pieces of equal length. How long is each piece?

Your strategy

Your solution

Solutions on p. S21

8.1 Exercises

Objective A

Convert.

1. 6 ft = ________ in.

2. 9 ft = ________ in.

3. 30 in. = ________ ft

4. 64 in. = ________ ft

5. 13 yd = ________ ft

6. $4\frac{1}{2}$ yd = ________ ft

7. 16 ft = ________ yd

8. $4\frac{1}{2}$ ft = ________ yd

9. $2\frac{1}{3}$ yd = ________ in.

10. 5 yd = ________ in.

11. 120 in. = ________ yd

12. 66 in. = ________ yd

13. 2 mi = ________ ft

14. $1\frac{1}{2}$ mi = ________ ft

15. $7\frac{1}{2}$ in. = ________ ft

Objective B

Perform the arithmetic operation.

16. 100 in. = ____ ft ____ in.

17. 6400 ft = ____ mi ____ ft

18. 15 in. = ____ ft ____ in.

19. $\begin{array}{r} 6\text{ ft } 7\text{ in.} \\ +\ 3\text{ ft } 4\text{ in.} \\ \hline \end{array}$

20. $\begin{array}{r} 9\text{ ft } 11\text{ in.} \\ +\ 3\text{ ft } \ \ 6\text{ in.} \\ \hline \end{array}$

21. $\begin{array}{r} 5\text{ ft } 3\text{ in.} \\ -\ 2\text{ ft } 6\text{ in.} \\ \hline \end{array}$

22. $\begin{array}{r} 9\text{ yd } 1\text{ ft} \\ -\ 3\text{ yd } 2\text{ ft} \\ \hline \end{array}$

23. $\begin{array}{r} 2\text{ ft } 5\text{ in.} \\ \times \qquad 6 \\ \hline \end{array}$

24. $3\frac{2}{3}$ ft × 4

25. $2\overline{)5\text{ ft } 4\text{ in.}}$

26. $12\frac{1}{2}$ in. ÷ 3

27. $4\frac{2}{3}$ ft + $6\frac{1}{2}$ ft

28. $\begin{array}{r} 3\text{ yd } 2\text{ ft} \\ +\ 6\text{ yd } 2\text{ ft} \\ \hline \end{array}$

29. $\begin{array}{r} 1\text{ mi } 4200\text{ ft} \\ +\ 2\text{ mi } 3600\text{ ft} \\ \hline \end{array}$

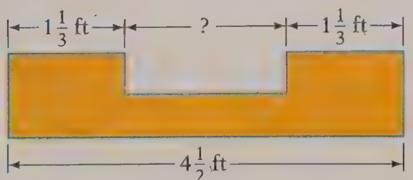
30. $\begin{array}{r} 5\text{ yd } 1\text{ ft} \\ -\ 2\text{ yd } 2\text{ ft} \\ \hline \end{array}$

Objective C Application Problems

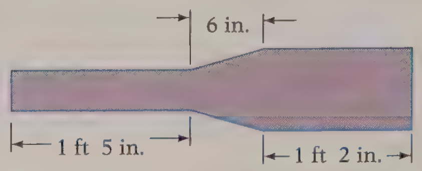
31. A kitchen counter is to be covered with tile that is 4 in. square. How many tiles can be placed along one row of a counter top that is 4 ft 8 in. long?

32. Thirty-two yards of material were used for making pleated draperies. How many feet of material were used?

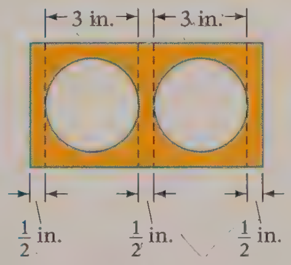
33. Find the missing dimension.



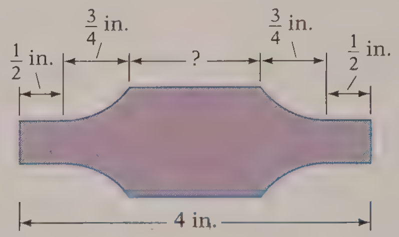
34. Find the total length of the shaft.



35. What length of material is needed to drill two holes 3 in. in diameter and leave $\frac{1}{2}$ in. between the holes and on either side as shown in the diagram?



36. Find the missing dimension in the figure.



37. A board $6\frac{2}{3}$ ft long is cut into four equal pieces. How long is each piece?

38. How long must a board be if four pieces, each 3 ft 4 in. long, are to be cut from it?

39. A picture is 1 ft 9 in. high and 1 ft 6 in. wide. Find the length of framing needed to frame the picture.

40. You bought 32 ft of baseboard to install in the kitchen of your house. How many inches of baseboard did you purchase?

41. Forty-five bricks, each 9 in. long, are laid end to end to make the base for a wall. Find the length of the wall in feet.

42. A roof is constructed with nine rafters, each 8 ft 4 in. long. Find the total number of feet of material needed to build the rafters.

APPLYING THE CONCEPTS

43. There are approximately 200,000,000 adults living in the United States. Assume that the average adult is 19 in. wide from shoulder to shoulder. If all the adults in the United States are standing shoulder to shoulder, could they reach around Earth at the equator, a distance of approximately 25,000 mi?

44. How good are you at estimating lengths or distances? Estimate the length of a pencil, the width of your room, the length of a block, and the distance to the grocery store. Then measure these lengths and compare the results with your estimates.

8.2 Weight

Objective A

To convert measurements of weight in the U.S. Customary System

Weight is a measure of how strongly Earth is pulling on an object. The unit of weight called the **pound** is defined as the weight of a standard solid kept at the Bureau of Standards in Washington, D.C. The U.S. Customary System units of weight are **ounce, pound,** and **ton.**

Equivalences between units of weight in the U.S. Customary System are

$$16 \text{ ounces (oz)} = 1 \text{ pound (lb)}$$
$$2000 \text{ lb} = 1 \text{ ton}$$

These equivalences can be used to form conversion rates to change one unit of measurement to another. For example, because 16 oz = 1 lb, the conversion rates $\frac{16\text{ oz}}{1\text{ lb}}$ and $\frac{1\text{ lb}}{16\text{ oz}}$ are both equivalent to 1.

➡ Convert 62 oz to pounds.

$$\begin{aligned} 62\text{ oz} &= 62\text{ oz} \times \boxed{\frac{1\text{ lb}}{16\text{ oz}}} \\ &= \frac{62\ \cancel{\text{oz}}}{1} \times \frac{1\text{ lb}}{16\ \cancel{\text{oz}}} \\ &= \frac{62\text{ lb}}{16} \\ &= 3\frac{7}{8}\text{ lb} \end{aligned}$$

- The conversion rate must contain lb (the unit desired in the answer) in the numerator and must contain oz (the original unit) in the denominator.

Point of Interest

The Romans used two different systems of weights. In both systems, the smallest unit was the *uncia*, abbreviated to "oz," from which the term *ounce* is derived. In one system, there were 16 ounces to 1 pound. In the second system, a pound, which was called the libra, equaled 12 unciae. The abbreviation "lb" for pound comes from the word *libra*.

The avoirdupois system of measurement and the troy system of measurement have their heritage in the two Roman systems.

Example 1 Convert $3\frac{1}{2}$ tons to pounds.

Solution

$$\begin{aligned} 3\frac{1}{2}\text{ tons} &= \frac{7}{2}\ \cancel{\text{tons}} \times \frac{2000\text{ lb}}{1\ \cancel{\text{ton}}} \\ &= \frac{14{,}000\text{ lb}}{2} = 7000\text{ lb} \end{aligned}$$

You Try It 1 Convert 3 lb to ounces.

Your solution

Example 2 Convert 42 oz to pounds.

Solution

$$\begin{aligned} 42\text{ oz} &= 42\ \cancel{\text{oz}} \times \frac{1\text{ lb}}{16\ \cancel{\text{oz}}} \\ &= \frac{42\text{ lb}}{16} = 2\frac{5}{8}\text{ lb} \end{aligned}$$

You Try It 2 Convert 4200 lb to tons.

Your solution

Solutions on p. S22

Objective B To perform arithmetic operations with measurements of weight

When performing arithmetic operations with measurements of weight, write the answer in simplest form. For example, 1 lb 22 oz should be written 2 lb 6 oz.

Example 3 Find the difference between 14 lb 5 oz and 8 lb 14 oz.

Solution

$$\begin{array}{rr} \overset{13\text{ lb}}{\cancel{14\text{ lb}}} & \overset{21\text{ oz}}{\cancel{5\text{ oz}}} \\ -\ 8\text{ lb} & 14\text{ oz} \\ \hline 5\text{ lb} & 7\text{ oz} \end{array}$$

- Borrow 1 lb (16 oz) from 14 lb and add it to 5 oz.

You Try It 3 Find the difference between 7 lb 1 oz and 3 lb 4 oz.

Your solution

Example 4 Divide: 7 lb 14 oz ÷ 3

Solution

$$\begin{array}{rrr} & 2\text{ lb} & 10\text{ oz} \\ \cline{2-3} 3) & 7\text{ lb} & 14\text{ oz} \\ & -6\text{ lb} & \\ \cline{2-2} & 1\text{ lb} = & 16\text{ oz} \\ \cline{3-3} & & 30\text{ oz} \\ & & -30\text{ oz} \\ \cline{3-3} & & 0 \end{array}$$

You Try It 4 Multiply: 3 lb 6 oz × 4

Your solution

Solutions on p. S22

Objective C To solve application problems

Example 5

Sirina Jasper purchased 4 lb 8 oz of oat bran and 2 lb 11 oz of wheat bran. She plans to blend the two brans and then repackage the mixture in 3-ounce packages for a diet supplement. How many 3-ounce packages can she make?

Strategy

To find the number of 3-ounce packages:

- Add the amount of oat bran (4 lb 8 oz) to the amount of wheat bran (2 lb 11 oz).
- Convert the sum to ounces.
- Divide the total ounces by the weight of each package (3 oz).

Solution

$$\begin{array}{rr} 4\text{ lb} & 8\text{ oz} \\ +\ 2\text{ lb} & 11\text{ oz} \\ \hline 6\text{ lb} & 19\text{ oz} \end{array} = 7\text{ lb } 3\text{ oz} = 115\text{ oz}$$

$$\frac{115\text{ oz}}{3\text{ oz}} \approx 38.3$$

She can make 38 packages.

You Try It 5

Find the weight in pounds of 12 bars of soap. Each bar weighs 7 oz.

Your strategy

Your solution

Solution on p. S22

8.2 Exercises

Objective A

Convert.

1. 64 oz = ______ lb

2. 36 oz = ______ lb

3. 8 lb = ______ oz

4. 7 lb = ______ oz

5. 3200 lb = ______ tons

6. 7000 lb = ______ tons

7. 6 tons = ______ lb

8. $1\frac{1}{4}$ tons = ______ lb

9. 66 oz = ______ lb

10. 90 oz = ______ lb

11. $1\frac{1}{2}$ lb = ______ oz

12. $2\frac{5}{8}$ lb = ______ oz

13. $1\frac{3}{10}$ tons = ______ lb

14. $\frac{4}{5}$ ton = ______ lb

15. 500 lb = ______ ton

16. 5000 lb = ______ tons

17. 180 oz = ______ lb

18. 12 oz = ______ lb

Objective B

Perform the arithmetic operation.

19. 9000 lb = ______ tons ______ lb

20. 85 oz = ______ lb ______ oz

21. 40 oz = ______ lb ______ oz

22. $\begin{array}{r} 4\text{ lb } \ \ 7\text{ oz} \\ +\ 3\text{ lb } 12\text{ oz} \\ \hline \end{array}$

23. $\begin{array}{r} 1\text{ ton } \ \ 800\text{ lb} \\ +\ 3\text{ tons } 1600\text{ lb} \\ \hline \end{array}$

24. $\begin{array}{r} 7\text{ lb } 5\text{ oz} \\ -\ 3\text{ lb } 8\text{ oz} \\ \hline \end{array}$

25. $\begin{array}{r} 3\text{ tons } 500\text{ lb} \\ -\ 1\text{ ton } \ \ 800\text{ lb} \\ \hline \end{array}$

26. $\begin{array}{r} 3\text{ lb } 6\text{ oz} \\ \times \qquad 4 \\ \hline \end{array}$

27. $5\frac{1}{2}$ lb × 6

28. $2\overline{)3\text{ lb } 8\text{ oz}}$

29. $4\frac{2}{3}$ lb × 3

30. $\begin{array}{r} 7\text{ lb } 7\text{ oz} \\ +\ 6\text{ lb } 9\text{ oz} \\ \hline \end{array}$

31. $\begin{array}{r} 6\frac{1}{2}\text{ oz} \\ +\ 2\frac{1}{2}\text{ oz} \\ \hline \end{array}$

32. $\begin{array}{r} 6\frac{3}{8}\text{ lb} \\ -\ 2\frac{5}{6}\text{ lb} \\ \hline \end{array}$

33. 5 lb 12 oz ÷ 4

Objective C Application Problems

34. A machinist has 25 iron rods to mill. Each rod weighs 20 oz. Find the total weight of the rods in pounds.

35. A fireplace brick weighs $2\frac{1}{2}$ lb. What is the weight of a load of 800 bricks?

36. A college bookstore received 1200 textbooks, each weighing 9 oz. Find the total weight of the 1200 textbooks in pounds.

37. A 4 × 4-inch tile weighs 7 oz. Find the weight, in pounds, of a package of 144 tiles.

38. A farmer ordered 20 tons of feed for 100 cattle. After 15 days, the farmer has 5 tons of feed left. How many pounds of food has each cow eaten per day?

39. A case of soft drink contains 24 cans, each weighing 6 oz. Find the weight, in pounds, of the case of soft drink.

40. A baby weighed 7 lb 8 oz at birth. At 6 months of age, the baby weighed 15 lb 13 oz. Find the baby's increase in weight during the 6 months.

41. Shampoo weighing 5 lb 4 oz is divided equally and poured into four containers. How much shampoo is in each container?

42. A steel rod weighing 16 lb 11 oz is cut into three pieces. Find the weight of each piece of steel rod.

43. Find the cost of a ham roast that weighs 5 lb 10 oz if the price per pound is $3.20.

44. A candy store buys candy weighing 12 lb for $14.40. The candy is repackaged and sold in 6-ounce packages for $1.15 each. Find the markup on the 12 lb of candy.

45. A manuscript weighing 2 lb 3 oz is mailed at the parcel post rate of $.25 per ounce. Find the cost of mailing the manuscript.

APPLYING THE CONCEPTS

46. Write a paragraph describing the growing need for precision in our measurements as civilization progressed. Include a discussion of the need for precision in the space industry.

47. Estimate the weight of a nickel, a textbook, a friend, and a car. Then find the actual weights and compare them with your estimates.

8.3 Capacity

Objective A

To convert measurements of capacity in the U.S. Customary System

Point of Interest

The word *quart* has its root in the Medieval Latin word *quartus*, which means "fourth." Thus a quart is $\frac{1}{4}$ of a gallon. The same Latin word is the source of such other English words as *quarter, quartile, quadrilateral,* and *quartet.*

Liquid substances are measured in units of **capacity.** The standard U.S. Customary units of capacity are the **fluid ounce, cup, pint, quart,** and **gallon.** Equivalences between units of capacity in the U.S. Customary System are

$$\begin{aligned} 8\text{ fluid ounces (fl oz)} &= 1\text{ cup (c)} \\ 2\text{ c} &= 1\text{ pint (pt)} \\ 2\text{ pt} &= 1\text{ quart (qt)} \\ 4\text{ qt} &= 1\text{ gallon (gal)} \end{aligned}$$

These equivalences can be used to form conversion rates to change one unit of measurement to another. For example, because 8 fl oz = 1 c, the conversion rates $\frac{8\text{ fl oz}}{1\text{ c}}$ and $\frac{1\text{ c}}{8\text{ fl oz}}$ are both equivalent to 1.

➡ Convert 36 fl oz to cups.

$$\begin{aligned} 36\text{ fl oz} &= 36\text{ fl oz} \times \boxed{\frac{1\text{ c}}{8\text{ fl oz}}} \\ &= \frac{36\ \cancel{\text{fl oz}}}{1} \times \frac{1\text{ c}}{8\ \cancel{\text{fl oz}}} \\ &= \frac{36\text{ c}}{8} \\ &= 4\frac{1}{2}\text{ c} \end{aligned}$$

- The conversion rate must contain c in the numerator and fl oz in the denominator.

➡ Convert 3 qt to cups.

$$\begin{aligned} 3\text{ qt} &= 3\text{ qt} \times \boxed{\frac{2\text{ pt}}{1\text{ qt}}} \times \boxed{\frac{2\text{ c}}{1\text{ pt}}} \\ &= \frac{3\ \cancel{\text{qt}}}{1} \times \frac{2\ \cancel{\text{pt}}}{1\ \cancel{\text{qt}}} \times \frac{2\text{ c}}{1\ \cancel{\text{pt}}} \\ &= \frac{12\text{ c}}{1} \\ &= 12\text{ c} \end{aligned}$$

- The direct equivalence is not given above. Use two conversion rates. First convert quarts to pints, and then convert pints to cups. The unit in the denominator of the second conversion rate and the unit in the numerator of the first conversion rate must be the same in order to cancel.

Example 1 Convert 42 c to quarts.

Solution

$$\begin{aligned} 42\text{ c} &= 42\ \cancel{\text{c}} \times \frac{1\ \cancel{\text{pt}}}{2\ \cancel{\text{c}}} \times \frac{1\text{ qt}}{2\ \cancel{\text{pt}}} \\ &= \frac{42\text{ qt}}{4} = 10\frac{1}{2}\text{ qt} \end{aligned}$$

You Try It 1 Convert 18 pt to gallons.

Your solution

Solution on p. S22

Objective B

To perform arithmetic operations with measurements of capacity

When performing arithmetic operations with measurements of capacity, write the answer in simplest form. For example, 1 c 12 fl oz should be written as 2 c 4 fl oz.

Example 2 What is 4 gal 1 qt decreased by 2 gal 3 qt?

Solution

$$\begin{array}{r} \overset{3\text{ gal}}{\cancel{4\text{ gal}}}\ \overset{5\text{ qt}}{\cancel{1\text{ qt}}} \\ -\ 2\text{ gal}\ 3\text{ qt} \\ \hline 1\text{ gal}\ 2\text{ qt} \end{array}$$

- **Borrow 1 gal (4 qt) from 4 gal and add to 1 qt.**

You Try It 2 Find the quotient of 4 gal 2 qt and 3.

Your solution

Solution on p. S22

Objective C

To solve application problems

Example 3

A can of apple juice contains 25 fl oz. Find the number of quarts of apple juice in a case of 24 cans.

Strategy

To find the number of quarts of apple juice in one case:

- Multiply the number of cans (24) by the number of fluid ounces per can (25) to find the total number of fluid ounces in the case.
- Convert the number of fluid ounces in the case to quarts.

Solution

$24 \times 25\text{ fl oz} = 600\text{ fl oz}$

$$600\text{ fl oz} = \frac{600\ \cancel{\text{fl oz}}}{1} \cdot \frac{1\ \cancel{\text{c}}}{8\ \cancel{\text{fl oz}}} \cdot \frac{1\ \cancel{\text{pt}}}{2\ \cancel{\text{c}}} \cdot \frac{1\text{ qt}}{2\ \cancel{\text{pt}}}$$

$$= \frac{600\text{ qt}}{32} = 18\frac{3}{4}\text{ qt}$$

One case of apple juice contains $18\frac{3}{4}$ qt.

You Try It 3

Five students are going backpacking in the desert. Each student requires 3 qt of water per day. How many gallons of water should they take for a 3-day trip?

Your strategy

Your solution

Solution on p. S22

8.3 Exercises

Objective A

Convert.

1. 60 fl oz = ______ c

2. 48 fl oz = ______ c

3. 3 c = ______ fl oz

4. $2\frac{1}{2}$ c = ______ fl oz

5. 8 c = ______ pt

6. 5 c = ______ pt

7. $3\frac{1}{2}$ pt = ______ c

8. 12 pt = ______ qt

9. 22 qt = ______ gal

10. 10 qt = ______ gal

11. $2\frac{1}{4}$ gal = ______ qt

12. 7 gal = ______ qt

13. $7\frac{1}{2}$ pt = ______ qt

14. $3\frac{1}{2}$ qt = ______ pt

15. 20 fl oz = ______ pt

16. $1\frac{1}{2}$ pt = ______ fl oz

17. 17 c = ______ qt

18. $1\frac{1}{2}$ qt = ______ c

Objective B

Perform the arithmetic operation.

19. 14 qt =
______ gal ______ qt

20. 9 pt =
______ qt ______ pt

21. 5 pt =
______ qt ______ pt

22. $\begin{array}{r} 3\text{ gal }2\text{ qt} \\ +\ 4\text{ gal }3\text{ qt} \\ \hline \end{array}$

23. $\begin{array}{r} 4\text{ qt }1\text{ pt} \\ +\ 2\text{ qt }1\text{ pt} \\ \hline \end{array}$

24. $\begin{array}{r} 3\text{ gal }1\text{ qt} \\ -\ 1\text{ gal }2\text{ qt} \\ \hline \end{array}$

25. $\begin{array}{r} 3\text{ c }3\text{ fl oz} \\ -\ 2\text{ c }5\text{ fl oz} \\ \hline \end{array}$

26. $\begin{array}{r} 2\text{ qt }1\text{ pt} \\ \times\quad 5 \\ \hline \end{array}$

27. $3\frac{1}{2}$ pt × 5

28. $5\overline{)6\text{ gal }1\text{ qt}}$

29. $3\frac{1}{2}$ gal ÷ 4

30. $\begin{array}{r} 5\text{ c }3\text{ fl oz} \\ +\ 3\text{ c }6\text{ fl oz} \\ \hline \end{array}$

31. $\begin{array}{r} 3\text{ gal }3\text{ qt} \\ +\ 1\text{ gal }2\text{ qt} \\ \hline \end{array}$

32. $\begin{array}{r} 4\text{ c }6\text{ fl oz} \\ -\ 2\text{ c }7\text{ fl oz} \\ \hline \end{array}$

33. $\begin{array}{r} 3\text{ gal} \\ -\ 1\text{ gal }2\text{ qt} \\ \hline \end{array}$

34. $1\frac{1}{2}\text{ pt} + 2\frac{2}{3}\text{ pt}$

35. $4\frac{1}{2}\text{ gal} - 1\frac{3}{4}\text{ gal}$

36. $2\overline{)3\text{ gal }2\text{ qt}}$

Objective C Application Problems

37. Sixty adults will attend a church social. Each adult will drink 2 c of coffee. How many gallons of coffee should be prepared?

38. The Bayside Playhouse serves punch during intermission. Assume that 200 people will each drink 1 c of punch. How many gallons of punch should be ordered?

39. A solution needed for a chemistry class required 72 fl oz of water, 16 fl oz of one solution, and 48 fl oz of another solution. Find the number of quarts of the final solution.

40. A cafeteria sold 124 cartons of milk in 1 day. Each carton contained 1 c of milk. How many quarts of milk were sold that day?

41. A farmer changed the oil in a tractor seven times during the year. Each oil change required 5 qt of oil. How many gallons of oil did the farmer use in the seven oil changes?

42. There are 24 cans in a case of tomato juice. Each can contains 10 fl oz of tomato juice. Find the number of 1-cup servings in the case of tomato juice.

43. One brand of orange juice costs \$1.59 for 1 qt. Another brand costs \$1.25 for 24 fl oz. Which is the more economical purchase?

44. Mandy carried 6 qt of water for 3 days of desert camping. Water weighs $8\frac{1}{3}$ lb per gallon. Find the weight of water that she carried.

45. A department store bought hand lotion in 5-quart containers and then repackaged the lotion in 8-fluid-ounce bottles. The lotion and bottles cost \$41.50, and each 8-fluid-ounce bottle was sold for \$4.25. How much profit was made on each 5-quart package of lotion?

46. Orlando bought oil in 50-gallon containers for changing the oil in his customers' cars. He paid \$240 for the 50 gal of oil and charged customers \$2.10 per quart. Find the profit Orlando made on one 50-gallon container of oil.

APPLYING THE CONCEPTS

47. Define the following units: grain, dram, furlong, and rod. Give an example where each would be used.

48. Assume that you wanted to invent a new measuring system. Discuss some of the features that would have to be incorporated into the system.

8.4 Energy and Power

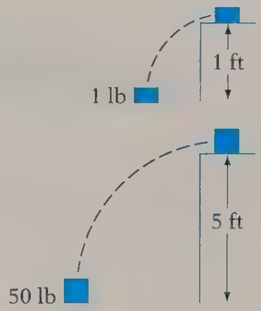
Objective A

To use units of energy in the U.S. Customary System

Energy can be defined as the ability to do work. Energy is stored in coal, in gasoline, in water behind a dam, and in one's own body.

One **foot-pound** (ft · lb) of energy is the amount of energy necessary to lift 1 pound a distance of 1 foot.

To lift 50 lb a distance of 5 ft requires
50 × 5 = 250 ft · lb of energy.



Consumer items that use energy, such as furnaces, stoves, and air conditioners, are rated in **British thermal units** (Btu). For example, a furnace might have a rating of 35,000 Btu per hour, which means that it releases 35,000 Btu of energy in one hour (1 h).

Because 1 Btu = 778 ft · lb, the following conversion rate, equivalent to 1, can be written:

$$\frac{778 \text{ ft} \cdot \text{lb}}{1 \text{ Btu}} = 1$$

Example 1
Convert 250 Btu to foot-pounds.

Solution

$$250 \text{ Btu} = 250 \cancel{\text{Btu}} \times \frac{778 \text{ ft} \cdot \text{lb}}{1 \cancel{\text{Btu}}}$$
$$= 194{,}500 \text{ ft} \cdot \text{lb}$$

You Try It 1
Convert 4.5 Btu to foot-pounds.

Your solution

Example 2
Find the energy required for a 125-pound person to climb a mile-high mountain.

Solution
In climbing the mountain, the person is lifting 125 lb a distance of 5280 ft.

$$\text{Energy} = 125 \text{ lb} \times 5280 \text{ ft}$$
$$= 660{,}000 \text{ ft} \cdot \text{lb}$$

You Try It 2
Find the energy required for a motor to lift 800 lb a distance of 16 ft.

Your solution

Solutions on p. S22

Example 3
A furnace is rated at 80,000 Btu per hour. How many foot-pounds of energy are released in 1 h?

Solution

$$80{,}000 \text{ Btu} = 80{,}000 \cancel{\text{Btu}} \times \frac{778 \text{ ft} \cdot \text{lb}}{1 \cancel{\text{Btu}}}$$
$$= 62{,}240{,}000 \text{ ft} \cdot \text{lb}$$

You Try It 3
A furnace is rated at 56,000 Btu per hour. How many foot-pounds of energy are released in 1 h?

Your solution

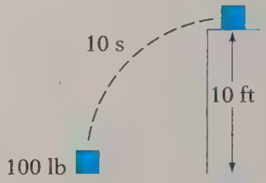
Solution on p. S22

Objective B To use units of power in the U.S. Customary System

Power is the rate at which work is done, or the rate at which energy is released. Power is measured in **foot-pounds per second** $\left(\frac{\text{ft} \cdot \text{lb}}{\text{s}}\right)$. In each of the following examples, the amount of energy released is the same, but the time taken to release the energy is different; thus the power is different.

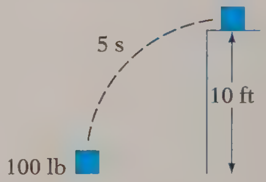
100 lb is lifted 10 ft in 10 s.

$$\text{Power} = \frac{10 \text{ ft} \times 100 \text{ lb}}{10 \text{ s}} = 100 \frac{\text{ft} \cdot \text{lb}}{\text{s}}$$



100 lb is lifted 10 ft in 5 s.

$$\text{Power} = \frac{10 \text{ ft} \times 100 \text{ lb}}{5 \text{ s}} = 200 \frac{\text{ft} \cdot \text{lb}}{\text{s}}$$



The U.S. Customary unit of power is the **horsepower.** A horse doing average work can pull 550 lb a distance of 1 ft in 1 s and can continue this work all day.

$$1 \text{ horsepower (hp)} = 550 \frac{\text{ft} \cdot \text{lb}}{\text{s}}$$

Example 4
Find the power needed to raise 300 lb a distance of 30 ft in 15 s.

Solution $\text{Power} = \frac{30 \text{ ft} \times 300 \text{ lb}}{15 \text{ s}}$

$$= 600 \frac{\text{ft} \cdot \text{lb}}{\text{s}}$$

You Try It 4
Find the power needed to raise 1200 lb a distance of 90 ft in 24 s.

Your solution

Example 5
A motor has a power of 2750 $\frac{\text{ft} \cdot \text{lb}}{\text{s}}$. Find the horsepower of the motor.

Solution $\frac{2750}{550} = 5 \text{ hp}$

You Try It 5
A motor has a power of 3300 $\frac{\text{ft} \cdot \text{lb}}{\text{s}}$. Find the horsepower of the motor.

Your solution

Solutions on p. S22

8.4 Exercises

Objective A

1. Convert 25 Btu to foot-pounds.

2. Convert 6000 Btu to foot-pounds.

3. Convert 25,000 Btu to foot-pounds.

4. Convert 40,000 Btu to foot-pounds.

5. Find the energy required to lift 150 lb a distance of 10 ft.

6. Find the energy required to lift 300 lb a distance of 16 ft.

7. Find the energy required to lift a 3300-pound car a distance of 9 ft.

8. Find the energy required to lift a 3680-pound elevator a distance of 325 ft.

9. Three tons are lifted 5 ft. Find the energy required in foot-pounds.

10. Seven tons are lifted 12 ft. Find the energy required in foot-pounds.

11. A construction worker carries 3-pound blocks up a 10-foot flight of stairs. How many foot-pounds of energy are required to carry 850 blocks up the stairs?

12. A crane lifts an 1800-pound steel beam to the roof of a building 36 ft high. Find the amount of energy the crane requires in lifting the beam.

13. A furnace is rated at 45,000 Btu per hour. How many foot-pounds of energy are released by the furnace in 1 h?

14. A furnace is rated at 22,500 Btu per hour. How many foot-pounds of energy does the furnace release in 1 h?

15. Find the amount of energy in foot-pounds given off when 1 lb of coal is burned. One pound of coal gives off 12,000 Btu of energy when burned.

16. Find the amount of energy in foot-pounds given off when 1 lb of gasoline is burned. One pound of gasoline gives off 21,000 Btu of energy when burned.

Objective B

17. Convert 1100 $\frac{\text{ft} \cdot \text{lb}}{\text{s}}$ to horsepower.

18. Convert 6050 $\frac{\text{ft} \cdot \text{lb}}{\text{s}}$ to horsepower.

19. Convert 4400 $\frac{\text{ft} \cdot \text{lb}}{\text{s}}$ to horsepower.

20. Convert 1650 $\frac{\text{ft} \cdot \text{lb}}{\text{s}}$ to horsepower.

21. Convert 5 hp to foot-pounds per second.

22. Convert 4 hp to foot-pounds per second.

23. Convert 7 hp to foot-pounds per second.

24. Convert 2 hp to foot-pounds per second.

25. Find the power in foot-pounds per second needed to raise 125 lb a distance of 12 ft in 3 s.

26. Find the power in foot-pounds per second needed to raise 500 lb a distance of 60 ft in 8 s.

27. Find the power in foot-pounds per second needed to raise 3000 lb a distance of 40 ft in 25 s.

28. Find the power in foot-pounds per second of an engine that can raise 12,000 lb to a height of 40 ft in 60 s.

29. Find the power in foot-pounds per second of an engine that can raise 180 lb to a height of 40 ft in 5 s.

30. Find the power in foot-pounds per second of an engine that can raise 1200 lb to a height of 18 ft in 30 s.

31. A motor has a power of 4950 $\frac{\text{ft} \cdot \text{lb}}{\text{s}}$. Find the horsepower of the motor.

32. A motor has a power of 16,500 $\frac{\text{ft} \cdot \text{lb}}{\text{s}}$. Find the horsepower of the motor.

33. A motor has a power of 6600 $\frac{\text{ft} \cdot \text{lb}}{\text{s}}$. Find the horsepower of the motor.

APPLYING THE CONCEPTS

34. Pick a source of energy and write an article about it. Include the source, possible pollution problems, and future prospects associated with this form of energy.

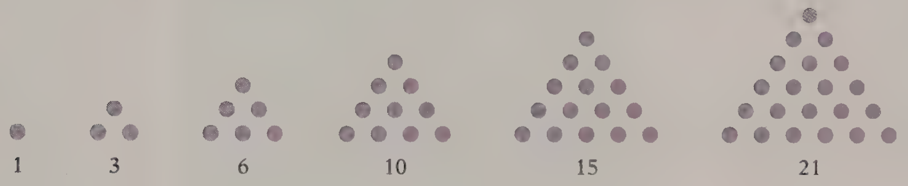
Focus on Problem Solving

Applying Solutions to Other Problems

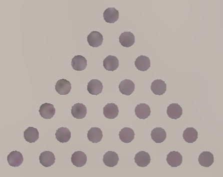
Problem solving in the previous chapters concentrated on solving specific problems. After a problem is solved, however, there is an important question to be asked: "Does the solution to this problem apply to other types of problems?"

To illustrate this extension of problem solving, we will consider *triangular numbers,* which were studied by ancient Greek mathematicians. The numbers 1, 3, 6, 10, 15, and 21 are the first six triangular numbers. What is the next triangular number?

To answer this question, note in the diagram below that a triangle can be formed using the number of dots that correspond to a triangular number.



Observe that the number of dots in each row is one more than the number of dots in the row above. The total number of dots can be found by addition.

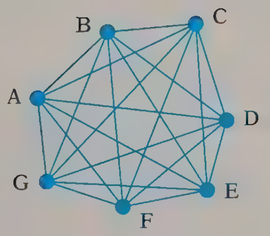


The pattern suggests that the next triangular number (the seventh one) is the sum of the first 7 natural numbers. The seventh triangular number is 28. The diagram at the right shows the seventh triangular number.

Using the pattern for triangular numbers, it is easy to determine that the tenth triangular number is

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55$$

Now consider a situation that may seem to be totally unrelated to triangular numbers. Suppose you are in charge of scheduling softball games for a league. There are seven teams in the league, and each team must play every other team once. How many games must be scheduled?



We label the teams A, B, C, D, E, F, and G. (See the figure at the left.) A line between two teams indicates that the two teams play each other. Beginning with A, there are 6 lines for the 6 teams that A must play. There are 6 teams that B must play, but the line between A and B has already been drawn, so there are only 5 remaining games to schedule for B. Now move on to C. The lines between C and A and between C and B have already been drawn, so there are only 4 additional lines to be drawn to represent the teams C will play. Moving on to D, we see that the lines between D and A, D and B, and D and C have already been drawn, so there are 3 more lines to be drawn to represent the teams D will play.

Note that each time we move from team to team, one fewer line needs to be drawn. When we reach F, there is only one line to be drawn, the one between F and G. The total number of lines drawn is $6 + 5 + 4 + 3 + 2 + 1 = 21$, the sixth triangular number. For a league with 7 teams, the number of games that must be scheduled so that each team plays every other team once is the sixth triangular number. If there were 10 teams in the league, the number of games that must be scheduled would be the ninth triangular number, which is 45.

A college chess team wants to schedule a match so that each of its 15 members plays each other member of the team once. How many matches must be scheduled?

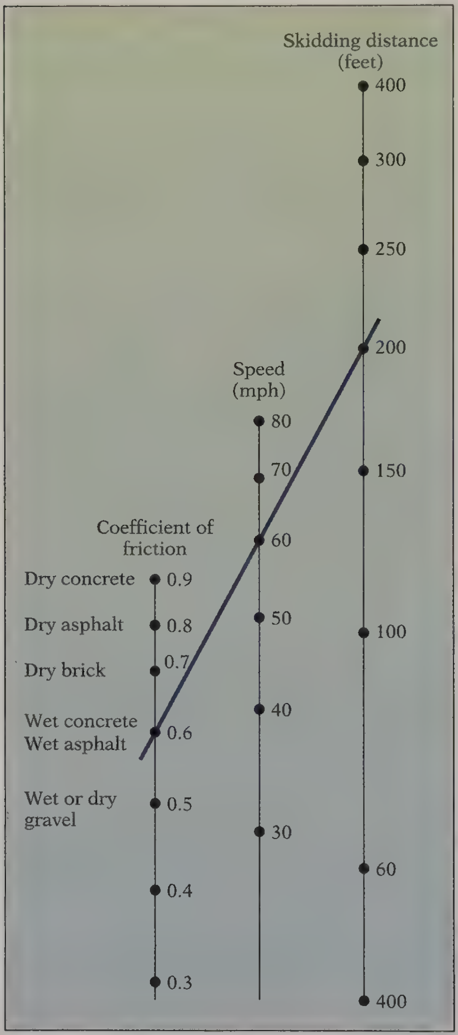
Projects and Group Activities

Astronomical Distances

Although one mile may seem like a long distance, for astronomers this measurement is inadequate for the vastness of space. Other measurements of distance are more useful. An **astronomical unit** is the average distance between Earth and the sun. A **light year** is the distance a ray of light travels in one year. To measure even greater distances within the universe, a **parsec** is used.

Use a reference to find the distance of one astronomical unit, one light year, and one parsec. Then write equivalences between these three units.

Nomographs



A chart is another tool that is used in problem solving. The chart at the left is a nomograph. A **nomograph** is a chart that represents numerical relationships among variables.

One of the details a traffic accident investigator checks when looking into a car accident is the length of the skid marks of the car. This length can help the investigator determine the speed of the car when the brakes were applied.

The nomograph at the left can be used to determine the speed of a car under given conditions. It shows the relationship among the speed of the car, the skidding distance, and the *coefficient of friction*.

The coefficient of friction is an experimentally obtained value that reflects how easy or hard it is to drag one object over another. For instance, it is easier to drag a box across ice than to drag it across a carpet. The coefficient of friction is smaller for the box and ice than it is for the box and carpet.

To use the nomograph at the left, an investigator would draw a line from the coefficient of friction to the skidding distance. The point at which the line crosses the speed line shows how fast the car was going when the brakes were applied. The line from 0.6 to 200 intersects the speed line at 60. This indicates that a car that skidded 200 feet on wet asphalt or wet concrete was traveling at 60 mph.

1. Use the nomograph to determine the speed of a car when the brakes were applied for a car traveling on gravel and for skid marks of 100 ft.

2. Use the nomograph to determine the speed of a car when the brakes were applied for a car traveling on dry concrete and for skid marks of 150 ft.

3. Suppose a car is traveling 80 mi/h when the brakes are applied. Find the difference in skidding distance if the car is traveling on wet concrete rather than dry concrete.

Averages

If two towns are 150 mi apart and you drive between the two towns in 3 h, then your

$$\textit{Average speed} = \frac{\text{total distance}}{\text{total time}} = \frac{150}{3} = 50\text{ mi/h}$$

It is highly unlikely that your speed was *exactly* 50 mi/h the entire time of the trip. Sometimes you will have traveled faster than 50 mi/h, and other times you will have traveled slower than 50 mi/h. Dividing the total distance you traveled by the total time it took to go that distance is an example of calculating an average.

There are many other averages that may be calculated. For instance, the Environmental Protection Agency calculates an estimated miles per gallon (mpg) for new cars. Miles per gallon is an average calculated from the formula

$$\frac{\text{Miles traveled}}{\text{Gallons of gasoline}}$$

For instance, the miles per gallon for a car that travels 308 mi on 11 gal of gas is $\frac{308}{11} = 28$ mpg.

A pilot would not use miles per gallon as a measure of fuel efficiency. Rather, pilots use gallons per hour. A plane that travels 5 h and uses 400 gal of fuel has an average that is calculated as

$$\frac{\text{Gallons of fuel}}{\text{Hours flown}} = \frac{400}{5} = 80\text{ gal/h}$$

Using the examples above, calculate the following averages.

1. Determine the average speed of a car that travels 355 mi in 6 h. Round to the nearest tenth.
2. Determine the miles per gallon of a car that can travel 405 mi on 12 gal of gasoline. Round to the nearest tenth.
3. If a plane flew 2000 mi in 5 h and used 1000 gal of fuel, determine the average number of gallons per hour that the plane used.

Another type of average is grade-point average (GPA). It is calculated by multiplying the units for each class by the grade point for that class, adding the results, and dividing by the total number of units taken. Here is an example using the grading scale A = 4, B = 3, C = 2, D = 1, and F = 0.

Class	*Units*	*Grade*
Math	4	B (= 3)
English	3	A (= 4)
French	5	C (= 2)
Biology	3	B (= 3)

$$\text{GPA} = \frac{4 \cdot 3 + 3 \cdot 4 + 5 \cdot 2 + 3 \cdot 3}{4 + 3 + 5 + 3} = \frac{43}{15} \approx 2.87$$

4. A grading scale that provides for plus or minus grades uses $A = 4$, $A- = 3.7$, $B+ = 3.3$, $B = 3$, $B- = 2.7$, $C+ = 2.3$, $C = 2$, $C- = 1.7$, $D+ = 1.3$, $D = 1$, $D- = 0.7$, and $F = 0$. Calculate the GPA of the student whose grades are given below.

Class	*Units*	*Grade*
Math	5	B +
English	3	C +
Spanish	5	A –
Physical Science	3	B –

Chapter Summary

Key Words

A *measurement* includes a number and a unit. [p. 333]

The U.S. Customary units of length are *inch, foot, yard,* and *mile.* [p. 333]

Weight is a measure of how strongly Earth is pulling on an object. The U.S. Customary units of weight are *ounce, pound,* and *ton.* [p. 339]

Liquid substances are measured in units of *capacity.* The U.S. Customary units of capacity are *fluid ounce, cup, pint, quart,* and *gallon.* [p. 343]

Energy is the ability to do work. One *foot-pound* of energy is the amount of energy required to lift 1 pound a distance of 1 foot. [p. 347]

Furnaces, stoves, and air conditioners are rated in energy units called *British thermal units* (Btu). [p. 347]

Power is the rate at which work is done. The U.S. Customary unit of power is the *horsepower* (hp). [p. 348]

Essential Rules

Equivalences Between Units of Length

12 in. = 1 ft
3 ft = 1 yd
36 in. = 1 yd
5280 ft = 1 mi [p. 333]

Equivalences Between Units of Weight

16 oz = 1 lb
2000 lb = 1 ton [p. 339]

Equivalences Between Units of Capacity

8 fl oz = 1 c
2 c = 1 pt
2 pt = 1 qt
4 qt = 1 gal [p. 343]

Equivalences Between Units of Energy

1 Btu = 778 ft · lb [p. 347]

Equivalences Between Units of Power

$1\text{ hp} = 550\,\dfrac{\text{ft}\cdot\text{lb}}{\text{s}}$ [p. 348]

A conversion rate is used to convert one unit of measurement to another. For example, the conversion rate $\dfrac{12\text{ in.}}{1\text{ ft}}$ is used to convert feet to inches. [p. 333]

Chapter Review

1. Convert 4 ft to inches.

2. What is 7 ft 6 in. divided by 3?

3. Find the energy needed to lift 200 lb a distance of 8 ft.

4. Convert $2\frac{1}{2}$ pt to fluid ounces.

5. Convert 14 ft to yards.

6. Convert 2400 lb to tons.

7. Find the quotient of 7 lb 5 oz and 3.

8. Convert $3\frac{3}{8}$ lb to ounces.

9. Add:

$$\begin{array}{r} 3\text{ ft } 9\text{ in.} \\ +\ 5\text{ ft } 6\text{ in.} \\ \hline \end{array}$$

10. Subtract:

$$\begin{array}{r} 3\text{ tons } \ 500\text{ lb} \\ -\ 1\text{ ton } \ 1500\text{ lb} \\ \hline \end{array}$$

11. Add:

$$\begin{array}{r} 5\text{ lb } 11\text{ oz} \\ +\ 3\text{ lb } \ 8\text{ oz} \\ \hline \end{array}$$

12. Subtract:

$$\begin{array}{r} 5\text{ yd } 1\text{ ft} \\ -\ 3\text{ yd } 2\text{ ft} \\ \hline \end{array}$$

13. Convert 12 c to quarts.

14. Find the product of 2 ft 8 in. and 5.

15. Convert 2.5 hp to foot-pounds per second. $\left(1 \text{ hp} = 550 \frac{\text{ft} \cdot \text{lb}}{\text{s}}.\right)$

16. Multiply:

$$\begin{array}{r} 5 \text{ lb } 8 \text{ oz} \\ \times \quad 8 \\ \hline \end{array}$$

17. Convert 50 Btu to foot-pounds. (1 Btu = 778 ft · lb.)

18. Convert 3850 $\frac{\text{ft} \cdot \text{lb}}{\text{s}}$ to horsepower. $\left(1 \text{ hp} = 550 \frac{\text{ft} \cdot \text{lb}}{\text{s}}.\right)$

19. A board 6 ft 11 in. long is cut from a board 10 ft 5 in. long. Find the length of the remaining piece of board.

20. A book weighing 2 lb 3 oz is mailed at the parcel post rate of $.18 per ounce. Find the cost of mailing the book.

21. A can of pineapple juice contains 18 fl oz. Find the number of quarts in a case of 24 cans.

22. A cafeteria sold 256 cartons of milk in one school day. Each carton contains 1 c of milk. How many gallons of milk were sold that day?

23. A furnace is rated at 35,000 Btu per hour. How many foot-pounds of energy does the furnace release in 1 h? (1 Btu = 778 ft · lb.)

24. Find the power in foot-pounds per second of an engine that can raise 800 lb to a height of 15 ft in 25 s.

Chapter Test

1. Convert $2\frac{1}{2}$ ft to inches.

2. Subtract: 4 ft 2 in. − 1 ft 9 in.

3. A board $6\frac{2}{3}$ ft long is cut into five equal pieces. How long is each piece?

4. Seventy-two bricks, each 8 in. long, are laid end to end to make the base for a wall. Find the length of the wall in feet.

5. Convert $2\frac{7}{8}$ lb to ounces.

6. Convert: 40 oz = ___ lb ___ oz

7. Find the sum of 9 lb 6 oz and 7 lb 11 oz.

8. Divide: 6 lb 12 oz ÷ 4

9. A college bookstore received 1000 workbooks, each weighing 12 oz. Find the total weight of the 1000 workbooks in pounds.

10. An elementary school class gathered 800 aluminum cans for recycling. Four aluminum cans weigh 3 oz. Find the amount the class received if the rate of pay was $.75 per pound for the aluminum cans. Round to the nearest cent.

11. Convert 13 qt to gallons.

12. Convert $3\frac{1}{2}$ gal to pints.

13. What is $1\frac{3}{4}$ gal times 7?

14. Add: 5 gal 2 qt + 2 gal 3 qt

15. A can of grapefruit juice contains 20 fl oz. Find the number of cups of grapefruit juice in a case of 24 cans.

16. Nick, a mechanic, bought oil in 40-gallon containers for changing the oil in customers' cars. He paid $200 for the 40 gallons of oil and charged customers $2.15 per quart. Find the profit Nick made on one 40-gallon container of oil.

17. Find the energy required to lift 250 lb a distance of 15 ft.

18. A furnace is rated at 40,000 Btu per hour. How many foot-pounds of energy are released by the furnace in 1 h? (1 Btu = 778 ft · lb.)

19. Find the power needed to lift 200 lb a distance of 20 ft in 25 s.

20. A motor has a power of 2200 $\frac{\text{ft} \cdot \text{lb}}{\text{s}}$. Find the motor's horsepower. $\left(1 \text{ hp} = 550 \frac{\text{ft} \cdot \text{lb}}{\text{s}}.\right)$

Cumulative Review

1. Find the LCM of 9, 12, and 15.

2. Write $\frac{43}{8}$ as a mixed number.

3. Subtract: $5\frac{7}{8} - 2\frac{7}{12}$

4. What is $5\frac{1}{3}$ divided by $2\frac{2}{3}$?

5. Simplify: $\frac{5}{8} \div \left(\frac{3}{8} - \frac{1}{4}\right) - \frac{5}{8}$

6. Round 2.0972 to the nearest hundredth.

7. Multiply: $\begin{array}{r} 0.0792 \\ \times \quad 0.49 \\ \hline \end{array}$

8. Solve the proportion: $\frac{n}{12} = \frac{44}{60}$

9. Find $2\frac{1}{2}\%$ of 50.

10. 18 is 42% of what? Round to the nearest hundredth.

11. A 7.2-pound roast costs $29.88. Find the unit cost.

12. Add: $3\frac{2}{5}$ in. + $5\frac{1}{3}$ in.

13. Convert: 24 oz = ___ lb ___ oz

14. Multiply: 3 lb 8 oz × 9

15. Subtract: $4\frac{1}{3}$ qt − $1\frac{5}{6}$ qt

16. Find 2 lb 10 oz less than 4 lb 6 oz.

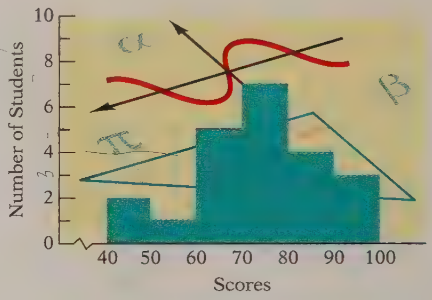
17. An investor receives a dividend of $56 from 40 shares of stock. At the same rate, find the dividend that 200 shares of stock would yield.

18. Anna had a balance of $578.56 in her checkbook. She wrote checks of $216.98 and $34.12 and made a deposit of $315.33. What is her new checking account balance?

19. An account executive receives a salary of $800 per month plus a commission of 2% on all sales over $25,000. Find the total monthly income of an account executive who has monthly sales of $140,000.

20. A health inspector found that 3% of a 2500-pound shipment of carrots were spoiled and could not be sold. Find the amount of carrots that could be sold.

21. The scores on the final exam of a trigonometry class are recorded in the histogram in the accompanying figure. What percent of the class received a score between 80% and 90%? Round to the nearest percent.



22. Hayes Department Store uses a markup rate of 40% on all merchandise. What is the selling price of a compact disk player that cost the store $220?

23. A construction firm received a loan of $200,000 for 8 months at a simple interest rate of 11%. Find the interest paid on the loan.

24. Six college students spent several weeks panning for gold during their summer vacation. The students obtained 1 lb 3 oz of gold, which they sold for $400 per ounce. How much money did each student receive if they shared the money equally? Round to the nearest dollar.

25. Four books were mailed at the postal rate of $.28 per ounce. The books weighed 1 lb 3 oz, 13 oz, 1 lb 8 oz, and 1 lb. Find the cost of mailing the books.

26. One brand of yogurt costs $.79 for 8 oz, and 36 oz of another brand can be bought for $2.98. Which purchase is the better buy?

27. Two dice are rolled. What is the probability that the sum of the dots on the upward faces is 9?

28. Find the energy required to lift 400 lb a distance of 8 ft.

29. Find the power, in foot-pounds per second, needed to raise 600 lb a distance of 8 ft in 12 s.

Chapter 9

The Metric System of Measurement

We have all heard the rules for good health. Eating right, watching our weight, and exercising regularly are vital components of physical well being. Understanding the relationship between the number of Calories we eat and the number of Calories we burn during exercise can help us maintain our ideal weight. **Exercises 1 to 10 on page 377** focus on these topics.

Objectives

Section 9.1

- **A** To convert units of length in the metric system of measurement
- **B** To solve application problems

Section 9.2

- **A** To convert units of mass in the metric system of measurement
- **B** To solve application problems

Section 9.3

- **A** To convert units of capacity in the metric system of measurement
- **B** To solve application problems

Section 9.4

- **A** To use units of energy in the metric system of measurement

Section 9.5

- **A** To convert U.S. Customary units to metric units
- **B** To convert metric units to U.S. Customary units

Need help? For on-line student resources, such as section quizzes, visit this textbook's web site at **math.college.hmco.com/students.**

Prep Test

For Exercises 1 to 10, add, subtract, multiply, or divide.

1. $3.732 \times 10{,}000$

2. 65.9×10^4

3. $41.07 \div 1000$

4. $28{,}496 \div 10^3$

5. $6 - 0.875$

6. $5 + 0.96$

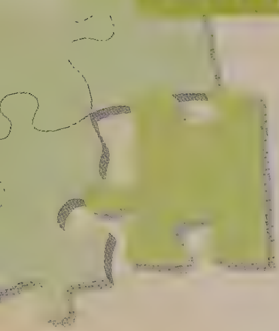
7. 3.25×0.04

8. $35 \times \frac{1.61}{1}$

9. $1.67 \times \frac{1}{3.34}$

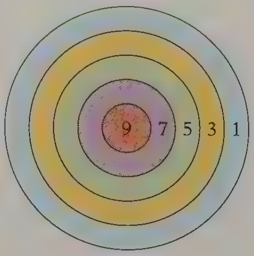
10. $4\frac{1}{2} \times 150$

Go Figure



Suppose you threw six darts and all six hit the target shown. Which of the following could be your score?

4, 15, 58, 28, 29, 31

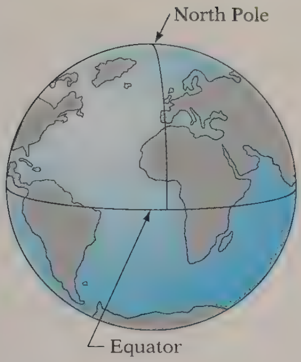


9.1 Length

Objective A

To convert units of length in the metric system of measurement

In 1789, an attempt was made to standardize units of measurement internationally in order to simplify trade and commerce between nations. A commission in France developed a system of measurement known as the **metric system.** The basic unit of length in the metric system is the **meter.** One meter is approximately the distance from a doorknob to the floor.



All units of length in the metric system are derived from the meter. Prefixes to the basic unit denote the length of each unit. For example, the prefix "centi-" means one-hundredth, so 1 centimeter is 1 one-hundredth of a meter.

kilo- = 1000	1 kilometer (km) = 1000 meters (m)
hecto- = 100	1 hectometer (hm) = 100 m
deca- = 10	1 decameter (dam) = 10 m
	1 meter (m) = 1 m
deci- = 0.1	1 decimeter (dm) = 0.1 m
centi- = 0.01	1 centimeter (cm) = 0.01 m
milli- = 0.001	1 millimeter (mm) = 0.001 m

Point of Interest

Originally the meter (spelled *metre* in some countries) was defined as $\frac{1}{10,000,000}$ of the distance from the equator to the north pole. Modern scientists have redefined the meter as 1,650,763.73 wavelengths of the orange-red light given off by the element krypton.

Conversion between units of length in the metric system involves moving the decimal point to the right or to the left. Listing the units in order from largest to smallest will indicate how many places to move the decimal point and in which direction.

To convert 4200 cm to meters, write the units in order from largest to smallest.

km hm dam m dm cm mm

2 positions

Converting cm to m requires moving 2 positions to the left.

4200 cm = 42.00 m

2 places

Move the decimal point the same number of places and in the same direction.

A metric measurement that involves two units is customarily written in terms of one unit. Convert the smaller unit to the larger unit and then add.

To convert 8 km 32 m to kilometers, first convert 32 m to kilometers.

km hm dam m dm cm mm

- Converting m to km requires moving 3 positions to the left.

32 m = 0.032 km

- Move the decimal point the same number of places and in the same direction.

8 km 32 m = 8 km + 0.032 km
= 8.032 km

- Add the result to 8 km.

Example 1 Convert 0.38 m to millimeters.

Solution 0.38 m = 380 mm

You Try It 1 Convert 3.07 m to centimeters.

Your solution

Example 2 Convert 4 m 62 cm to meters.

Solution 62 cm = 0.62 m

4 m 62 cm = 4 m + 0.62 m
= 4.62 m

You Try It 2 Convert 3 km 750 m to kilometers.

Your solution

Solutions on p. S23

Objective B To solve application problems

TAKE NOTE
Although in this text we will always change units to the larger unit, it is possible to perform the calculation by changing to the smaller unit.

2 m − 85 cm
= 200 cm − 85 cm
= 115 cm

Note that 115 cm = 1.15 m.

In the application problems in this section, we perform arithmetic operations with the measurements of length in the metric system. It is important to remember that before measurements can be added or subtracted, they must be expressed in terms of the same unit. In this textbook, unless otherwise stated, the units should be changed to the larger unit before the arithmetic operation is performed.

To subtract 85 cm from 2 m, convert 85 cm to meters.

2 m − 85 cm = 2 m − 0.85 m
= 1.15 m

Example 3
A piece measuring 142 cm is cut from a board 4.20 m long. Find the length of the remaining piece.

Strategy
To find the length of the remaining piece:

- Convert the length of the piece cut (142 cm) to meters.
- Subtract the length of the piece cut from the original length.

Solution
142 cm = 1.42 m

4.20 m − 142 cm = 4.20 m − 1.42 m
= 2.78 m

The length of the remaining piece is 2.78 m.

You Try It 3
A bookcase 175 cm long has four shelves. Find the cost of the shelves when the price of the lumber is $15.75 per meter.

Your strategy

Your solution

Solution on p. S23

9.1 Exercises

Objective A

Convert.

1. 42 cm = _____ mm

2. 62 cm = _____ mm

3. 81 mm = _____ cm

4. 68.2 mm = _____ cm

5. 6804 m = _____ km

6. 3750 m = _____ km

7. 2.109 km = _____ m

8. 32.5 km = _____ m

9. 432 cm = _____ m

10. 61.7 cm = _____ m

11. 0.88 m = _____ cm

12. 3.21 m = _____ cm

13. 7038 m = _____ km

14. 2589 m = _____ km

15. 3.5 km = _____ m

16. 9.75 km = _____ m

17. 260 cm = _____ m

18. 705 cm = _____ m

19. 1.685 m = _____ cm

20. 0.975 m = _____ cm

21. 14.8 cm = _____ mm

22. 6 m 42 cm = _____ m

23. 62 m 7 cm = _____ m

24. 42 cm 6 mm = _____ cm

25. 31 cm 9 mm = _____ cm

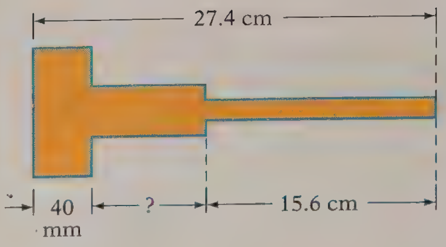
26. 62 km 482 m = _____ km

27. 8 km 75 m = _____ km

Objective B *Application Problems*

28. How many shelves, each 140 cm long, can be cut from a board that is 4.20 m in length? Find the length of the board remaining after the shelves are cut.

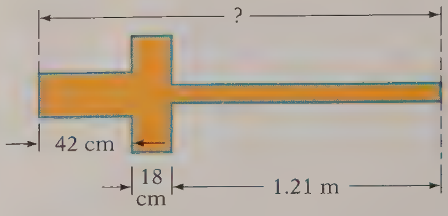
29. Find the missing dimension, in centimeters, in the diagram at the right.



30. A walk-a-thon had two checkpoints. One checkpoint was 1400 m from the starting point. The second checkpoint was 1200 m from the first checkpoint. The second checkpoint was 1800 m from the finish line. How long was the walk? Express the answer in kilometers.

31. Twenty rivets are used to fasten two steel plates together. The plates are 3.4 m long, and the rivets are equally spaced, with a rivet at each end. Find the distance between the rivets. Round to the nearest tenth of a centimeter.

32. Find the total length, in centimeters, of the shaft in the diagram at the right.



33. You purchase a 50-meter roll of fencing, at a cost of $14.95 per meter, in order to build a dog run that is 340 cm wide and 1380 cm long. After you cut the four pieces of fencing from the roll, how much of the fencing is left on the roll?

34. Carmine is a member of a group that has adopted 10 km of highway. During this week, Carmine cleaned up litter along the highway for 2500 m, 1500 m, 1200 m, 1300 m, and 1400 m. Find the average number of kilometers Carmine cleaned up on each of the 5 days this week.

35. The distance between Earth and the sun is 150,000,000 km. Light travels 300,000,000 m in 1 s. How long does it take for light to reach Earth from the sun?

36. The circumference of Earth is 40,000 km. How long would it take to travel the circumference of Earth at a speed of 85 km per hour? Round to the nearest tenth.

37. Light travels 300,000 km in 1 s. How far does light travel in 1 day?

APPLYING THE CONCEPTS

38. Other prefixes in the metric system are becoming more commonly used as a result of technological advances. Find the meaning of the following prefixes: tera-, giga-, mega-, micro-, nano-, and pico.

39. Write a short history of the metric system.

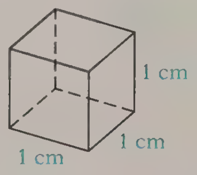
9.2 Mass

Objective A

To convert units of mass in the metric system of measurement

Mass and weight are closely related. Weight is a measure of how strongly Earth is pulling on an object. Therefore, an object's weight is less in space than on Earth's surface. However, the amount of material in the object, its **mass,** remains the same. On the surface of Earth, mass and weight can be used interchangeably.

The basic unit of mass in the metric system is the **gram.** If a box that is 1 cm long on each side is filled with water, then the mass of that water is 1 gram.



1 gram = the mass of water in the box

Point of Interest

An average snowflake weighs about $\frac{1}{300}$ g and contains approximately 100,000,000,000,000,000,000 water molecules. You may have heard the expression "No two snowflakes are the same." It is this large number of water molecules, and the number of their possible arrangements, that led to that statement.

The gram is a very small unit of mass. A paper clip weighs about 1 gram. The kilogram (1000 grams) is a more useful unit of mass in consumer applications. This textbook weighs about 1 kilogram.

The units of mass in the metric system have the same prefixes as the units of length:

1 kilogram (kg) = 1000 grams (g)
1 hectogram (hg) = 100 g
1 decagram (dag) = 10 g
1 gram (g) = 1 g
1 decigram (dg) = 0.1 g
1 centigram (cg) = 0.01 g
1 milligram (mg) = 0.001 g

Conversion between units of mass in the metric system involves moving the decimal point to the right or to the left. Listing the units in order from largest to smallest will indicate how many places to move the decimal point and in which direction.

To convert 324 g to kilograms, first write the units in order from largest to smallest.

$$\underbrace{\text{kg} \quad \text{hg} \quad \text{dag} \quad \text{g}}_{3\text{ positions}} \quad \text{dg} \quad \text{cg} \quad \text{mg}$$

Converting g to kg requires moving 3 positions to the left.

$$324\text{ g} = \underbrace{0.324}_{3\text{ places}}\text{ kg}$$

Move the decimal point the same number of places and in the same direction.

Example 1 Convert 4.23 g to milligrams.

Solution 4.23 g = 4230 mg

You Try It 1 Convert 42.3 mg to grams.

Your solution

Solution on p. S23

Example 2 Convert 2 kg 564 g to kilograms.

Solution 564 g = 0.564 kg

2 kg 564 g = 2 kg + 0.564 kg
= 2.564 kg

You Try It 2 Convert 3 g 54 mg to grams.

Your solution

Solution on p. S23

Objective B To solve application problems

TAKE NOTE
Although in this text we will always change units to the larger unit, it is possible to perform the calculation by changing to the smaller unit.

3 kg − 750 g
= 3000 g − 750 g
= 2250 g

Note that

2250 g = 2.250 kg.

In the application problems in this section, we perform arithmetic operations with the measurements of mass in the metric system. Remember that before measurements can be added or subtracted, they must be expressed in terms of the same unit. In this textbook, unless otherwise stated, the units should be changed to the larger unit before the arithmetic operation is performed.

To subtract 750 g from 3 kg, convert 750 g to kilograms.

3 kg − 750 g = 3 kg − 0.750 kg
= 2.250 kg

Example 3
Find the cost of three packages of ground meat weighing 540 g, 670 g, and 890 g if the price per kilogram is $9.89. Round to the nearest cent.

Strategy
To find the cost of the meat:

- Find the total weight of the three packages.
- Convert the total weight to kilograms.
- Multiply the weight by the cost per kilogram ($9.89).

Solution
540 g + 670 g + 890 g = 2100 g

2100 g = 2.1 kg

2.1 × 9.89 = 20.769

The cost of the meat is $20.77.

You Try It 3
How many kilograms of fertilizer are required to fertilize 400 trees in an apple orchard if 300 g of fertilizer are used for each tree?

Your strategy

Your solution

Solution on p. S23

9.2 Exercises

Objective A

Convert.

1. 420 g = _____ kg

2. 7421 g = _____ kg

3. 127 mg = _____ g

4. 43 mg = _____ g

5. 4.2 kg = _____ g

6. 0.027 kg = _____ g

7. 0.45 g = _____ mg

8. 325 g = _____ mg

9. 1856 g = _____ kg

10. 8900 g = _____ kg

11. 4057 mg = _____ g

12. 1970 mg = _____ g

13. 1.37 kg = _____ g

14. 5.1 kg = _____ g

15. 0.0456 g = _____ mg

16. 0.2 g = _____ mg

17. 18,000 g = _____ kg

18. 0.87 kg = _____ g

19. 3 kg 922 g = _____ kg

20. 1 kg 47 g = _____ kg

21. 7 g 891 mg = _____ g

22. 209 g 42 mg = _____ g

23. 4 kg 63 g = _____ kg

24. 18 g 5 mg = _____ g

Objective B *Application Problems*

25. A 1.19-kilogram container of Quaker Oats contains 30 servings. Find the number of grams in one serving of the oatmeal. Round to the nearest gram.

26. A patient is advised to supplement her diet with 2 g of calcium per day. The calcium tablets she purchases contain 500 mg of calcium per tablet. How many tablets per day should the patient take?

27. **a.** One egg contains 274 mg of cholesterol. How many grams of cholesterol are in a dozen eggs?
b. One glass of milk contains 33 mg of cholesterol. How many grams of cholesterol are in four glasses of milk?

28. A carat is a unit of weight equal to 200 mg. Find the weight in grams of a 10-carat precious stone.

29. The nutrition label for a corn bread mix is shown at the right.
a. How many kilograms of mix are in the package?
b. How many grams of sodium are contained in two servings of the corn bread?

Nutrition Facts

Serving Size 1/6 pkg. (31g mix)
Servings Per Container 6

Amount Per Serving	Mix	Prepared
Calories	110	160
Calories from Fat	10	50
	% Daily Value*	
Total Fat 1g	1%	9%
Saturated Fat 0g	0%	7%
Cholesterol 0mg	0%	12%
Sodium 210mg	9%	11%
Total Carbohydrate 24g	8%	8%
Sugars 6g		
Protein 2g		

30. Find the cost of three packages of ground meat weighing 470 g, 680 g, and 590 g if the price per kilogram is \$8.40.

31. Eighty grams of grass seed are used for every 100 m^2 of lawn. How many kilograms of grass seed are needed to cover 2000 m^2?

32. A commuter flight charges \$9.95 for each kilogram or part of a kilogram over 15 kg of luggage weight. How much extra must be paid for three pieces of luggage weighing 6450 g, 5850 g, and 7500 g?

33. A health food store buys nuts in 10-kilogram containers and repackages the nuts for resale. The store packages the nuts in 200-gram bags, costing \$.04 each, and sells them for \$2.89 per bag. Find the profit on a 10-kilogram container of nuts costing \$75.

34. A train car is loaded with 15 automobiles weighing 1405 kg each. Find the total weight of the automobiles.

35. During one year the United States exported 37,141 million kg of wheat, 2,680 million kg of rice, and 40,365 million kg of corn. What percent of the total of these grain exports was corn? Round to the nearest tenth of a percent.

APPLYING THE CONCEPTS

36. Define a metric ton. Convert the weights in Exercise 35 to metric tons.

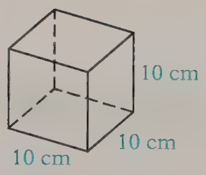
37. Discuss the advantages and disadvantages of the U.S. Customary System and the metric system of measurement.

9.3 Capacity

Objective A

To convert units of capacity in the metric system of measurement

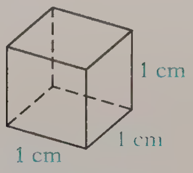
The basic unit of capacity in the metric system is the liter. One **liter** is defined as the capacity of a box that is 10 cm long on each side.



The units of capacity in the metric system have the same prefixes as the units of length:

1 kiloliter (kl) = 1000 L
1 hectoliter (hl) = 100 L
1 decaliter (dal) = 10 L
1 liter (L) = 1 L
1 deciliter (dl) = 0.1 L
1 centiliter (cl) = 0.01 L
1 milliliter (ml) = 0.001 L

The milliliter is equal to 1 **cubic centimeter** (cm^3).



1 ml = 1 cm^3

Conversion between units of capacity in the metric system involves moving the decimal point to the right or to the left. Listing the units in order from largest to smallest will indicate how many places to move the decimal point and in which direction.

To convert 824 ml to liters, first write the units in order from largest to smallest.

$\text{kl} \quad \text{hl} \quad \text{dal} \quad \underbrace{\text{L} \quad \text{dl} \quad \text{cl} \quad \text{ml}}_{3 \text{ positions}}$

Converting ml to L requires moving 3 positions to the left.

$824 \text{ ml} = 0.\underbrace{824}_{3 \text{ places}} \text{ L}$

Move the decimal point the same number of places and in the same direction.

Example 1 Convert 4 L 32 ml to liters.

Solution 32 ml = 0.032 L

4 L 32 ml = 4 L + 0.032 L
= 4.032 L

You Try It 1 Convert 2 kl 167 L to liters.

Your solution

Solution on p. S23

Example 2 Convert 1.23 L to cubic centimeters.

Solution 1.23 L = 1230 ml = 1230 cm^3

You Try It 2 Convert 325 cm^3 to liters.

Your solution

Solution on p. S23

Objective B To solve application problems

TAKE NOTE
Although in this text we will always change units to the larger unit, it is possible to perform the calculation by changing to the smaller unit.

$$\begin{aligned}&2.5 \text{ kl} + 875 \text{ L}\\&\quad = 2500 \text{ L} + 875 \text{ L}\\&\quad = 3375 \text{ L}\end{aligned}$$

Note that

3375 L = 3.375 kl.

In the application problems in this section, we perform arithmetic operations with the measurements of capacity in the metric system. Remember that before measurements can be added or subtracted, they must be expressed in terms of the same unit. In this textbook, unless otherwise stated, the units should be changed to the larger unit before the arithmetic operation is performed.

To add 2.5 kl and 875 L, convert 875 L to kiloliters.

$$\begin{aligned}2.5 \text{ kl} + 875 \text{ L} &= 2.5 \text{ kl} + 0.875 \text{ kl}\\&= 3.375 \text{ kl}\end{aligned}$$

Example 3
A laboratory assistant is in charge of ordering acid for three chemistry classes of 30 students each. Each student requires 80 ml of acid. How many liters of acid should be ordered? (The assistant must order by the whole liter.)

Strategy
To find the number of liters to be ordered:

- Find the number of milliliters of acid needed by multiplying the number of classes (3) by the number of students per class (30) by the number of milliliters of acid required by each student (80).
- Convert milliliters to liters.
- Round up to the nearest whole number.

Solution
3(30)(80) = 7200 ml

7200 ml = 7.2 L

7.2 rounded up to the nearest whole number is 8.

The assistant should order 8 L of acid.

You Try It 3
For $299.50, a cosmetician buys 5 L of moisturizer and repackages it in 125-milliliter jars. Each jar costs the cosmetician $.85. Each jar of moisturizer is sold for $19.95. Find the profit on the 5 L of moisturizer.

Your strategy

Your solution

Solution on p. S23

9.3 Exercises

Objective A

Convert.

1. 4200 ml = ______ L

2. 7.5 ml = ______ L

3. 3.42 L = ______ ml

4. 0.037 L = ______ ml

5. 423 ml = ______ cm^3

6. 0.32 ml = ______ cm^3

7. 642 cm^3 = ______ ml

8. 0.083 cm^3 = ______ ml

9. 42 cm^3 = ______ L

10. 3075 cm^3 = ______ L

11. 0.435 L = ______ cm^3

12. 2.57 L = ______ cm^3

13. 4.62 kl = ______ L

14. 0.035 kl = ______ L

15. 1423 L = ______ kl

16. 897 L = ______ kl

17. 1.267 L = ______ cm^3

18. 4.105 L = ______ cm^3

19. 3 L 42 ml = ______ L

20. 1 L 127 ml = ______ L

21. 3 kl 4 L = ______ kl

22. 6 kl 32 L = ______ kl

23. 8 L 200 ml = ______ L

24. 9 kl 505 L = ______ kl

Objective B *Application Problems*

25. The air in Earth's atmosphere is 78% nitrogen and 21% oxygen.

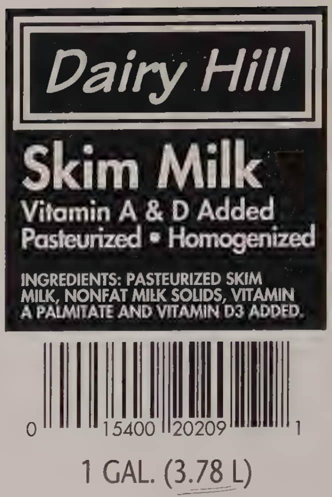
a. Without calculating, determine whether the amount of oxygen in 50 L of air is more or less than 25 L.

b. Find the amount of oxygen in 50 L of air.

26. A can of tomato juice contains 1.36 L. How many 170-milliliter servings are in one can of tomato juice?

27. An athletic club uses 800 ml of chlorine each day for its swimming pool. How many liters of chlorine are used in a month of 30 days?

28. The printed label from a container of milk is shown at the right. How many 230-milliliter servings are in the container? Round to the nearest whole number.



Dairy Hill

Skim Milk

Vitamin A & D Added

Pasteurized • Homogenized

INGREDIENTS: PASTEURIZED SKIM MILK, NONFAT MILK SOLIDS, VITAMIN A PALMITATE AND VITAMIN D3 ADDED.

0 15400 20209 1

1 GAL. (3.78 L)

29. A flu vaccine is being given for the coming winter season. A medical corporation buys 12 L of flu vaccine. How many patients can be immunized if each person receives 3 cm^3 of the vaccine?

30. A chemistry experiment requires 12 ml of an acid solution. How many liters of acid should be ordered when 4 classes of 90 students each are going to perform the experiment? (The acid must be ordered by the whole liter.)

31. A case of 12 one-liter bottles of apple juice costs $19.80. A case of 24 cans, each can containing 340 ml of apple juice, costs $14.50. Which case of apple juice is the better buy?

32. For $195, a pharmacist purchases 5 L of cough syrup and repackages it in 250-milliliter bottles. Each bottle costs the pharmacist $.55. Each bottle of cough syrup is sold for $17.89. Find the profit on the 5 L of cough syrup.

33. A service station operator bought 85 kl of gasoline for $23,750. The gasoline was sold for $.379 per liter. Find the profit on the 85 kl of gasoline.

34. A wholesale distributor purchased 32 kl of cooking oil for $44,480. The wholesaler repackaged the cooking oil in 1.25-liter bottles. The bottles cost $.21 each. Each bottle of cooking oil was sold for $2.97. Find the distributor's profit on the 32 kl of cooking oil.

APPLYING THE CONCEPTS

35. After a 280-milliliter serving is taken from a 3-liter bottle of water, how much water remains in the container? Write the answer in three different ways.

36. Write an essay describing problems in trade and manufacturing that arise between the United States and Europe because they use different systems of measurement.

9.4 Energy

Objective A To use units of energy in the metric system of measurement

Two commonly used units of energy in the metric system are the calorie and the watt-hour.

Heat is generally measured in units called calories or in larger units called Calories (with a capital C). A **Calorie** is 1000 calories and should be called a kilocalorie, but it is common practice in nutritional references and food labeling to simply call it a Calorie. A Calorie is the amount of heat required to raise the temperature of 1 kg of water 1 degree Celsius. One Calorie is also the energy required to lift 1 kg a distance of 427 m.

⇒ Swimming uses 480 Calories per hour. How many Calories are used by swimming $\frac{1}{2}$ h each day for 30 days?

Strategy To find the number of Calories used:

- Find the number of hours spent swimming.
- Multiply the number of hours spent swimming by the Calories used per hour.

Solution $\frac{1}{2} \times 30 = 15$ hours

$15(480) = 7200$

7200 Calories are used by swimming $\frac{1}{2}$ h each day for 30 days.

The **watt-hour** is used for measuring electrical energy. One watt-hour is the amount of energy required to lift 1 kg a distance of 370 m. A light bulb rated at 100 watts (W) will emit 100 watt-hours (Wh) of energy each hour.

1000 watt-hours (Wh) = 1 kilowatt-hour (kWh)

⇒ A 150-watt bulb is on for 8 h. At 8¢ per kilowatt-hour, find the cost of the energy used.

Strategy To find the cost:

- Find the number of watt-hours used.
- Convert to kilowatt-hours.
- Multiply the number of kilowatt-hours used by the cost per kilowatt-hour.

Solution $\overset{w}{150} \times \overset{h}{8} = 1200$ wh

1200 Wh = 1.2 kWh

$1.2 \times 0.08 = 0.096$

The cost of the energy used is \$.096.

Example 1

Walking uses 180 Calories per hour. How many Calories will you burn off by walking $5\frac{1}{4}$ h during one week?

Strategy

To find the number of Calories, multiply the number of hours spent walking by the Calories used per hour.

Solution

$$5\frac{1}{4} \times 180 = \frac{21}{4} \times 180 = 945$$

You will burn off 945 Calories.

You Try It 1

Housework requires 240 Calories per hour. How many Calories are burned off by doing $4\frac{1}{2}$ h of housework?

Your strategy

Your solution

Example 2

A clothes iron is rated at 1200 W. If the iron is used for 1.5 h, how much energy, in kilowatt-hours, is used?

Strategy

To find the energy used:

- Find the number of watt-hours used.
- Convert watt-hours to kilowatt-hours.

Solution

$1200 \times 1.5 = 1800$

1800 Wh = 1.8 kWh

1.8 kWh of energy are used.

You Try It 2

Find the number of kilowatt-hours of energy used when a 150-watt light bulb burns for 200 h.

Your strategy

Your solution

Example 3

A TV set rated at 1800 W is on for an average of 3.5 h per day. At 7.2¢ per kilowatt-hour, find the cost of operating the set for 1 week.

Strategy

To find the cost:

- Multiply to find the total number of hours the set is used.
- Multiply the product by the number of watts to find the watt-hours.
- Convert watt-hours to kilowatt-hours.
- Multiply the number of kilowatt-hours by the cost per kilowatt-hour.

Solution

$3.5 \times 7 = 24.5$

$24.5 \times 1800 = 44{,}100$

44,100 Wh = 44.1 kWh

$44.1 \times 0.072 = 3.1752$

The cost is $3.1752.

You Try It 3

A microwave oven rated at 500 W is used an average of 20 min per day. At 8.7¢ per kilowatt-hour, find the cost of operating the oven for 30 days.

Your strategy

Your solution

Solutions on p. S24

9.4 Exercises

Objective A Application Problems

1. How many Calories can you eliminate from your diet by omitting 1 slice of bread per day for 30 days? One slice of bread contains 110 Calories.

2. How many Calories can you eliminate from your diet in 2 weeks by omitting 400 Calories per day?

3. A nutrition label from a package of crisp bread is shown at the right.

   a. How many Calories are in $1\frac{1}{2}$ servings?

   b. How many Calories from fat are in 6 slices of the bread?

Nutrition Facts
Serving Size 2 Slices (18g)
Servings Per Container about 15

Amount Per Serving
Calories 60 Calories from Fat 10

	% Daily Value*
Total Fat 1g	2%
Saturated Fat 0g	0%
Polyunsaturated Fat 0.5g	
Monounsaturated Fat 0.5g	
Cholesterol 0mg	0%
Sodium 60mg	3%
Total Carbohydrate 10g	3%
Dietary Fiber 3g	10%
Sugars 1g	
Protein 2g	

Vitamin A 0% • Vitamin C 0%
Calcium 0% • Iron 4%

* Percent Daily Values are based on a 2,000 calorie diet. Your daily values may be higher or lower depending on your calorie needs

	Calories:	2,000	2,500
Total Fat	Less than	65g	80g
Saturated Fat	Less than	20g	25g
Cholesterol	Less than	300mg	300mg
Sodium	Less than	2,400mg	2,400mg
Total Carbohydrate		300g	375g
Dietary Fiber		25g	30g

Calories per gram:
Fat 9 • Carbohydrate 4 • Protein 4

4. Moderately active people need 20 Calories per pound of body weight to maintain their weight. How many Calories should a 150-pound, moderately active person consume per day to maintain that weight?

5. People whose daily activity level would be described as light need 15 Calories per pound of body weight to maintain their weight. How many Calories should a 135-pound, lightly active person consume per day to maintain that weight?

6. For a healthy diet, it is recommended that 55% of the daily intake of Calories come from carbohydrates. Find the daily intake of Calories from carbohydrates that is appropriate if you want to limit your Calorie intake to 1600 Calories.

7. Playing singles tennis requires 450 Calories per hour. How many Calories do you burn in 30 days playing 45 min per day?

8. After playing golf for 3 h, Ruben had a banana split containing 550 Calories. Playing golf uses 320 Calories per hour.

   a. Without doing the calculations, did the banana split contain more or fewer Calories than Ruben burned off playing golf?

   b. Find the number of Calories Ruben gained or lost from these two activities.

9. Hiking requires approximately 315 Calories per hour. How many hours would you have to hike to burn off the Calories in a 375-Calorie sandwich, a 150-Calorie soda, and a 280-Calorie ice cream cone? Round to the nearest tenth.

10. Riding a bicycle requires 265 Calories per hour. How many hours would Shawna have to ride a bicycle to burn off the Calories in a 320-Calorie milkshake, a 310-Calorie cheeseburger, and a 150-Calorie apple? Round to the nearest tenth.

11. An oven uses 500 W of energy. How many watt-hours of energy are used to cook a 5-kilogram roast for $2\frac{1}{2}$ h?

12. A 21-inch color TV set is rated at 90 W. The TV is used an average of $3\frac{1}{2}$ h each day for a week. How many kilowatt-hours of energy are used during the week?

13. A fax machine is rated at 9 W when the machine is in standby mode and at 36 W when in operation. How many kilowatt-hours of energy are used during a week in which the fax machine is in standby mode for 39 h and in operation for 6 h?

14. A 120-watt CD player is on an average of 2 h a day. Find the cost of listening to the CD player for 2 weeks at a cost of 9.4¢ per kilowatt-hour. Round to the nearest cent.

15. How much does it cost to run a 2200-watt air conditioner for 8 h at 9¢ per kilowatt-hour? Round to the nearest cent.

16. A space heater is used for 3 h. The heater uses 1400 W per hour. Electricity costs 11.1¢ per kilowatt-hour. Find the cost of using the electric heater. Round to the nearest cent.

17. A 60-watt Sylvania Long Life Soft White Bulb has a light output of 835 lumens and an average life of 1250 h. A 34-watt Sylvania Energy Saver Bulb has a light output of 400 lumens and an average life of 1500 h.

a. Is the light output of the Energy Saver Bulb more or less than half that of the Long Life Soft White Bulb?

b. If electricity costs 10.8¢ per kilowatt-hour, what is the difference in cost between using the Long Life Soft White Bulb for 150 h and using the Energy Saver Bulb for 150 h? Round to the nearest cent.

18. A house is insulated to save energy. The house used 265 kWh of electrical energy per month before insulation and saves 45 kWh of energy per month after insulation. What percent decrease does this amount represent? Round to the nearest tenth.

19. A welder uses 6.5 kW of energy each hour. Find the cost of using the welder for 6 h a day for 30 days. The cost is 9.4¢ per kilowatt-hour.

APPLYING THE CONCEPTS

20. Write an essay on how to improve the energy efficiency of a home.

21. A maintenance intake of Calories allows a person to neither gain nor lose weight. Consult a book on nutrition in order to make a table of weights and the corresponding maintenance intake of Calories. Then for each weight, add another column indicating the appropriate Calorie intake when an individual at that weight wants to lose 1 lb per week.

9.5

Conversion Between the U.S. Customary and the Metric Systems of Measurement

Objective A

To convert U.S. Customary units to metric units

More than 90% of the world's population use the metric system of measurement. Therefore, converting U.S. Customary units to metric units is essential in trade and commerce—for example, in importing foreign goods and exporting domestic goods. Approximate equivalences between the two systems follow.

Point of Interest

The definition of 1 inch has been changed as a consequence of the wide acceptance of the metric system. One inch is now exactly 25.4 mm.

Units of Length	*Units of Weight*	*Units of Capacity*
1 in. = 2.54 cm	1 oz ≈ 28.35 g	1 L ≈ 1.06 qt
1 m ≈ 3.28 ft	1 lb ≈ 454 g	1 gal ≈ 3.79 L
1 m ≈ 1.09 yd	1 kg ≈ 2.2 lb	
1 mi ≈ 1.61 km		

These equivalences can be used to form conversion rates to change one unit of measurement to another. For example, because 1 mi ≈ 1.61 km, the conversion rates $\frac{1\text{ mi}}{1.61\text{ km}}$ and $\frac{1.61\text{ km}}{1\text{ mi}}$ are both approximately equal to 1.

Point of Interest

Back in September of 1999, CNN reported that "NASA lost a $125 million Mars orbiter because a Lockheed Martin engineering team used English units of measurement while the agency's team used the . . . metric system for a key spacecraft operation." One team used miles, the other kilometers. This resulted in the orbiter breaking up when it hit the Martian atmosphere because the approach was too near the surface.

➡ Convert 55 mi to kilometers.

$$
\begin{aligned}
55\text{ mi} &\approx 55\text{ mi} \times \boxed{\frac{1.61\text{ km}}{1\text{ mi}}} \\
&= \frac{55\ \cancel{\text{mi}}}{1} \times \frac{1.61\text{ km}}{1\ \cancel{\text{mi}}} \\
&= \frac{88.55\text{ km}}{1} \\
55\text{ mi} &\approx 88.55\text{ km}
\end{aligned}
$$

- **The conversion rate must contain km in the numerator and mi in the denominator.**

Example 1

Convert 45 mi/h to kilometers per hour.

Solution

$$
\begin{aligned}
\frac{45\text{ mi}}{\text{h}} &\approx \frac{45\ \cancel{\text{mi}}}{\text{h}} \times \frac{1.61\text{ km}}{1\ \cancel{\text{mi}}} \\
&\approx \frac{72.45\text{ km}}{1\text{ h}} \\
&\approx 72.45\text{ km/h}
\end{aligned}
$$

You Try It 1

Convert 60 ft/s to meters per second. Round to the nearest hundredth.

Your solution

Solution on p. S24

Example 2

The price of gasoline is $1.59/gal. Find the cost per liter. Round to the nearest tenth of a cent.

Solution

$$\frac{\$1.59}{\text{gal}} \approx \frac{\$1.59}{\cancel{\text{gal}}} \times \frac{1\ \cancel{\text{gal}}}{3.79\ \text{L}} = \frac{\$1.59}{3.79\ \text{L}}$$

$$\approx \$.420/\text{L}$$

You Try It 2

The price of milk is $2.69/gal. Find the cost per liter. Round to the nearest cent.

Your solution

Solution on p. S24

Objective B To convert metric units to U.S. Customary units

Metric units are used in the United States. Cereal is sold by the gram, 35-mm film is available, and soda is sold by the liter. The same conversion rates used in Objective A are used for converting metric units to U.S. Customary units.

Example 3

Convert 200 m to feet.

Solution

$$200\ \text{m} \approx 200\ \cancel{\text{m}} \times \frac{3.28\ \text{ft}}{1\ \cancel{\text{m}}}$$

$$\approx 656\ \text{ft}$$

You Try It 3

Convert 45 cm to inches. Round to the nearest hundredth.

Your solution

Example 4

Convert 90 km/h to miles per hour. Round to the nearest hundredth.

Solution

$$\frac{90\ \text{km}}{\text{h}} \approx \frac{90\ \cancel{\text{km}}}{\text{h}} \times \frac{1\ \text{mi}}{1.61\ \cancel{\text{km}}} = \frac{90\ \text{mi}}{1.61\ \text{h}}$$

$$\approx 55.90\ \text{mi/h}$$

You Try It 4

Express 75 km/h in miles per hour. Round to the nearest hundredth.

Your solution

Example 5

The price of gasoline is $.392/L. Find the cost per gallon. Round to the nearest cent.

Solution

$$\frac{\$.392}{1\ \text{L}} \approx \frac{\$.392}{1\ \cancel{\text{L}}} \times \frac{3.79\ \cancel{\text{L}}}{1\ \text{gal}} \approx \frac{\$1.49}{1\ \text{gal}}$$

$$\approx \$1.49/\text{gal}$$

You Try It 5

The price of ice cream is $1.75/L. Find the cost per gallon. Round to the nearest cent.

Your solution

Solutions on p. S24

9.5 Exercises

Objective A

Convert. Round to the nearest hundredth.

1. Convert the 100-yard dash to meters.

2. Find the weight in kilograms of a 145-pound person.

3. Find the height in meters of a person 5 ft 8 in. tall.

4. Find the number of liters in 2 c of soda.

5. How many kilograms does a 15-pound turkey weigh?

6. Find the number of liters in 14.3 gal of gasoline.

7. Find the number of milliliters in 1 c.

8. The winning long jump at a track meet was 29 ft 2 in. Convert this distance to meters.

9. Express 65 mi/h in kilometers per hour.

10. Express 30 mi/h in kilometers per hour.

11. Fat-free hot dogs cost $3.49/lb. Find the cost per kilogram.

12. Seedless watermelon costs $.59/lb. Find the cost per kilogram.

13. The cost of gasoline is $1.47/gal. Find the cost per liter.

14. Deck stain costs $24.99/gal. Find the cost per liter.

15. The distance around Earth is 24,887 mi. Convert this distance to kilometers.

16. The distance from Earth to the sun is 93,000,000 mi. Convert this distance to kilometers.

Objective B

Convert. Round to the nearest hundredth.

17. Convert the 100-meter dash to feet.

18. Find the weight in pounds of an 86-kilogram person.

19. Find the number of gallons in 6 L of antifreeze.

20. Find the height in inches of a person 1.85 m tall.

21. Find the distance of the 1500-meter race in feet.

22. Find the weight in ounces of 327 g of cereal.

23. How many gallons of water does a 24-liter aquarium hold?

24. Find the width in inches of 35-mm film.

25. Express 80 km/h in miles per hour.

26. Express 30 m/s in feet per second.

27. Gasoline costs $.385/L. Find the cost per gallon.

28. A 5-kilogram ham costs $10/kg. Find the cost per pound.

29. A backpack tent weighs 2.1 kg. Find its weight in pounds.

30. A 2.5-kilogram bag of grass seed costs $7.89. Find the cost per pound.

31. Gary is planning a 5-day backpacking trip and decides to hike an average of 5 h each day. Hiking requires an extra 320 Calories per hour. How many pounds will Gary lose during the trip if he consumes an extra 900 Calories each day? (3500 Calories is equivalent to 1 lb.)

32. Swimming requires 550 Calories per hour. How many pounds could be lost by swimming $1\frac{1}{2}$ h each day for 5 days if no extra calories were consumed? (3500 Calories is equivalent to 1 lb.)

33. The speed of light is 300,000 km/s. Convert this speed to miles per second.

APPLYING THE CONCEPTS

34. For the following U.S. Customary units, make an estimate of the metric equivalent. Then perform the conversion and see how close you came to the actual measurement.

60 mi/h ≈
120 lb ≈
6 ft ≈
1 mi ≈
1 gal ≈
1 quarter-mile ≈

35. Determine whether the statement is true or false.

a. A liter is more than a gallon.
b. A meter is less than a yard.
c. 30 mi/h is less than 60 km/h.
d. A kilogram is greater than a pound.
e. An ounce is less than a gram.

36. Should the United States keep the U.S. Customary System or convert to the metric system? Justify your position.

Focus on Problem Solving

Working Backward

Sometimes the solution to a problem can be found by *working backward.* This problem-solving technique can be used to find a winning strategy for a game called Nim.

There are many variations of this game. For our game, there are two players, Player A and Player B, who alternately place 1, 2, or 3 matchsticks in a pile. The object of the game is to place the 32nd matchstick in the pile. Is there a strategy that Player A can use to guarantee winning the game?

Working backward, if there are 29, 30, or 31 matchsticks in the pile when it is A's turn to play, A can win by placing 3 matchsticks ($29 + 3 = 32$), 2 matchsticks, ($30 + 2 = 32$), or 1 matchstick ($31 + 1 = 32$) on the pile. If there are to be 29, 30, or 31 matchsticks in the pile when it is A's turn, there must be 28 matchsticks in the pile when it is B's turn.

Working backward from 28, if there are to be 28 matches in the pile at B's turn, there must be 25, 26, or 27 at A's turn. Player A can then add 3 matchsticks, 2 matchsticks, or 1 matchstick to the pile to bring the number to 28. For there to be 25, 26, or 27 matchsticks in the pile at A's turn, there must be 24 matchsticks at B's turn.

Now working backward from 24, if there are to be 24 matches in the pile at B's turn, there must be 21, 22, or 23 at A's turn. Player A can then add 3 matchsticks, 2 matchsticks, or 1 matchstick to the pile to bring the number to 24. For there to be 21, 22, or 23 matchsticks in the pile at A's turn, there must be 20 matchsticks at B's turn.

So far, we have found that for Player A to win, there must be 28, 24, or 20 matchsticks in the pile when it is B's turn to play. Note that each time, the number is decreasing by 4. Continuing this pattern, Player A will win if there are 16, 12, 8, or 4 matchsticks in the pile when it is B's turn.

Player A can guarantee winning by making sure that the number of matchsticks in the pile is a multiple of 4. To ensure this, Player A allows Player B to go first and then adds exactly enough matchsticks to the pile to bring the total to a multiple of 4.

For example, suppose B places 3 matchsticks in the pile; then A places 1 matchstick ($3 + 1 = 4$). Now B places 2 matchsticks in the pile. The total is now 6 matchsticks. Player A then places 2 matchsticks in the pile to bring the total to 8, a multiple of 4. If play continues in this way, Player A will win.

Here are some variations of Nim. See whether you can develop a winning strategy for Player A.

1. Suppose the goal is to place the last matchstick in a pile of 30 matches.

2. Suppose the players make two piles of matchsticks, with the final number of matchsticks in each pile to be 20.

3. In this variation of Nim, there are 40 matchsticks in a pile. Each player alternately removes 1, 2, or 3 matches from the pile. The player who removes the last match wins.

Projects and Group Activities

Name That Metric Unit

What unit in the metric system would be used to measure each of the following? If you are working in a group, be sure that each member agrees on the unit to be used and understands why that unit is used before going on to the next item.

1. The distance from Los Angeles to New York
2. The weight of a truck
3. A person's waist
4. The amount of coffee in a mug
5. The weight of a thumbtack
6. The amount of water in a swimming pool
7. The distance a baseball player hits a baseball
8. A person's hat size
9. The amount of protein needed daily
10. A person's weight
11. The amount of maple syrup served with pancakes
12. The amount of water in a water cooler
13. The amount of medication in an aspirin
14. The distance to the grocery store
15. The width of a hair
16. A person's height
17. The weight of a lawn mower
18. The amount of water a family uses monthly
19. The contents of a bottle of salad dressing
20. The newspapers collected at a recycling center

Deductive Reasoning

Suppose that during the last week of your math class, your instructor tells you that if you receive an A on the final exam, you will earn an A in the course. When the final exam grades are posted, you learn that you received an A on the final exam. You can then assume that you will earn an A in the course.

The process used to determine your grade in the math course is deductive reasoning. **Deductive reasoning** involves drawing a conclusion that is based on given facts. The problems below require deductive reasoning.

1. Given that ΔΔΔ = ◇◇◇◇ and ◇◇◇◇ = ÓÓ, then ΔΔΔΔΔΔ = how many Ós?

2. Given that ‡‡ = ••• and ••• = Λ, then ‡‡‡‡ = how many Λs?

3. Given that ÓÓÓ = ΩΩ and ¤ = ΩΩ, then ¤¤ = how many Ós?

4. Given that ∫∫∫∫∫∫ = ∂∂ and ∂∂∂∂ = ¥¥¥, then ¥¥¥¥¥¥ = how many ∫s?

5. Given that ÔÔÔÔÔ = □□□ and □□□□□□ = §§§§, then §§§§§§ = how many Ôs?

6. Chris, Dana, Leslie, and Pat are neighbors. Each drives a different type of vehicle: a compact car, a sedan, a sports car, or a station wagon. From the following statements, determine which type of vehicle each of the neighbors drives. It may be helpful to use the chart provided below.

- **a.** Although the vehicle owned by Chris has more mileage on it than does either the sedan or the sports car, it does not have the highest mileage of all four cars.
- **b.** Pat and the owner of the sports car live on one side of the street, and Leslie and the owner of the compact car live on the other side of the street.
- **c.** Leslie owns the vehicle with the most mileage on it.

TAKE NOTE

To use the chart to solve this problem, write an X in a box to indicate that a possibility has been eliminated. Write a √ to show that a match has been found. When a row or column has 3 X's, a √ is written in the remaining open box in that row or column of the chart.

	Compact	*Sedan*	*Sports Car*	*Wagon*
Chris				
Dana				
Leslie				
Pat				

7. Four neighbors, Anna, Kay, Michelle, and Nicole, each plant a different vegetable (beans, cucumbers, squash, or tomatoes) in their garden. From the following statements, determine which vegetable each neighbor plants.

- **a.** Nicole's garden is bigger than the one that has tomatoes but smaller than the one that has cucumbers.
- **b.** Anna, who planted the largest garden, didn't plant the beans.
- **c.** The person who planted the beans has a garden the same size as Nicole's.
- **d.** Kay and the person who planted the tomatoes also have flower gardens.

8. The Ontkeans, Kedrovas, McIvers, and Levinsons are neighbors. Each of the four families specializes in a different national cuisine (Chinese, French, Italian, or Mexican). From the following statements, determine which cuisine each family specializes in.

- **a.** The Ontkeans invited the family that specializes in Chinese cuisine and the family that specializes in Mexican cuisine for dinner last night.
- **b.** The McIvers live between the family that specializes in Italian cuisine and the Ontkeans. The Levinsons live between the Kedrovas and the family that specializes in Chinese cuisine.
- **c.** The Kedrovas and the family that specializes in Italian cuisine both subscribe to the same culinary magazine.

Search the World Wide Web

Use the Internet to find information regarding the metric system. You might enter *metric* as the word to search for.

Find professional organizations whose members are in careers related to your major. For example, we found information regarding the metric system and architecture, traffic safety services, cooking, surveying, and transportation.

The process of changing our system of measurement to the metric system is called **metrication** or **metric transition.** Find at least two associations involved in metrication.

Describe one way in which a country that has changed to the metric system helped its citizens in the transition to the metric system. For example, one web site shows metric-conversion postal stamps printed in Australia.

Chapter Summary

Key Words

The *metric system of measurement* is a system of measurement based on the decimal system. [p. 363]

The basic unit of measurement of length in the metric system is the *meter.* [p. 363]

The basic unit of mass in the metric system is the *gram.* [p. 367]

The basic unit of capacity in the metric system is the *liter.* [p. 371]

Heat is commonly measured in units called *Calories.* [p. 375]

The *watt-hour* is used in the metric system for measuring electrical energy. [p. 375]

Essential Rules

Prefixes to the basic unit denote the magnitude of each unit in the metric system.

kilo-	1000
hecto-	100
deca-	10
deci-	0.1
centi-	0.01
milli-	0.001 [p. 363]

Conversion between units in the metric system involves moving the decimal point to the right or to the left. Listing the units in order from largest to smallest will indicate how many places to move the decimal point.

1. When converting from a larger unit to a smaller unit, move the decimal point to the **right.**
2. When converting from a smaller unit to a larger unit, move the decimal point to the **left.** [p. 363]

Approximate equivalences between units in the metric system and the U.S. Customary System are used to form conversion rates to change one unit of measurement to another. [p. 379]

Chapter Review

1. Convert 1.25 km to meters.

2. Convert 0.450 g to milligrams.

3. Convert 0.0056 L to milliliters.

4. Convert the 1000-meter run to yards. (1 m ≈ 1.09 yd)

5. Convert 79 mm to centimeters.

6. Convert 5 m 34 cm to meters.

7. Convert 990 g to kilograms.

8. Convert 2550 ml to liters.

9. Convert 4870 m to kilometers.

10. Convert 0.37 cm to millimeters.

11. Convert 6 g 829 mg to grams.

12. Convert 1.2 L to cubic centimeters.

13. Convert 4.050 kg to grams.

14. Convert 8.7 m to centimeters.

15. Convert 192 ml to cubic centimeters.

16. Convert 356 mg to grams.

17. Convert 372 cm to meters.

18. Convert 8.3 kl to liters.

19. Convert 2 L 89 ml to liters.

20. Convert 5410 cm^3 to liters.

21. Convert 3792 L to kiloliters.

22. Convert 468 cm^3 to milliliters.

23. Three pieces of wire are cut from a 50-meter roll. The three pieces measure 240 cm, 560 cm, and 480 cm. How much wire is left on the roll after the three pieces are cut?

24. Find the total cost of three packages of chicken weighing 790 g, 830 g, and 655 g if the cost is $5.59 per kilogram.

25. Cheese costs $3.40 per pound. Find the cost per kilogram. (1 kg ≈ 2.2 lb)

26. One hundred twenty-five guests are expected to attend a reception. Assuming that each person drinks 400 ml of coffee, how many liters of coffee should be prepared?

27. A large egg contains approximately 90 Calories. How many Calories can be eliminated from your diet in a 30-day month by eliminating one large egg per day from your usual breakfast?

28. A TV uses 240 W of energy. The set is on an average of 5 h a day in a 30-day month. At a cost of 9.5¢ per kilowatt-hour, how much does it cost to run the set for 30 days?

29. A backpack weighs 1.90 kg. Find the weight in pounds. Round to the nearest hundredth. (1 kg ≈ 2.2 lb)

30. Cycling burns up approximately 400 Calories per hour. How many hours of cycling are necessary to lose 1 lb? (3500 Calories is equivalent to 1 lb.)

31. Six liters of liquid soap were bought for $11.40 per liter. The soap was repackaged in 150-milliliter plastic containers. The cost of each container was $.26. Each container of soap sold for $3.29 per bottle. Find the profit on the 6 L of liquid soap.

32. A color TV is rated at 80 W. The TV is used an average of 2 h each day for a week. How many kilowatt-hours of energy are used during the week?

33. How many kilograms of fertilizer are necessary to fertilize 500 trees in an orchard if 250 g of fertilizer is used for each tree?

Chapter Test

1. Convert 2.96 km to meters.

2. Convert 0.378 g to milligrams.

3. Convert 0.046 L to milliliters.

4. Convert 919 cm^3 to milliliters.

5. Convert 42.6 mm to centimeters.

6. Convert 7 m 96 cm to meters.

7. Convert 847 g to kilograms.

8. Convert 3920 ml to liters.

9. Convert 5885 m to kilometers.

10. Convert 1.5 cm to millimeters.

11. Convert 3 g 89 mg to grams.

12. Convert 1.6 L to cubic centimeters.

13. Convert 3.29 kg to grams.

14. Convert 4.2 m to centimeters.

15. Convert 96 ml to cubic centimeters.

16. Convert 1375 mg to grams.

17. Convert 402 cm to meters.

18. Convert 8.92 kl to liters.

19. Convert 5 km 38 m to meters.

20. Convert 6020 L to kiloliters.

21. A carpenter needs 30 rafters, each 380 cm long. Find the total length of the rafters in meters.

$100\ cm^2$

22. A tile measuring 20 × 20 cm weighs 250 g. Find the weight, in kilograms, of a box of 144 tiles.

144

0.25 kg

23. The community health clinic is giving flu shots for the coming flu season. Each flu shot contains 2 cm^3 of vaccine. How many liters of vaccine are needed to inoculate 2600 people?

24. Convert 35 mi/h to kilometers per hour. Round to the nearest tenth. (1 mi ≈ 1.61 km)

25 2 steel plates

25. Twenty-five rivets are used to fasten two steel plates together. The plates are 4.20 m long, and the rivets are equally spaced, with a rivet at each end. Find the distance, in centimeters, between the rivets.

200 g

26. Two hundred grams of fertilizer are used for each tree in an orchard containing 1200 trees. At $2.75 per kilogram of fertilizer, how much does it cost to fertilize the orchard?

27. An air conditioner rated at 1600 W is operated an average of 4 h per day. Electrical energy costs 8.5¢ per kilowatt-hour. How much does it cost to operate the air conditioner for 30 days?

3 × 40

28. A laboratory assistant is in charge of ordering acid for three chemistry classes of 40 students each. Each student requires 90 ml of acid. How many liters of acid should be ordered? (The assistant must order by the whole liter.)

90 ml x ______ L

29. The longest ski jump in Olympic history in the men's competition was 137 m. Convert this distance to feet. Round to the nearest hundredth. (1 m ≈ 3.28 ft)

30. At present, the rules in ski jumping allow skis that are up to 31.5 in. longer than the jumper's height, with a maximum length of 110 in. Convert the maximum length of the skis to centimeters. (1 in. = 2.54 cm)

Cumulative Review

1. Simplify: $12 - 8 \div (6 - 4)^2 \cdot 3$

2. Find the total of $5\frac{3}{4}$, $1\frac{5}{6}$, and $4\frac{7}{9}$.

3. Subtract: $4\frac{2}{9} - 3\frac{5}{12}$

4. Divide: $5\frac{3}{8} \div 1\frac{3}{4}$

5. Simplify: $\left(\frac{2}{3}\right)^4 \cdot \left(\frac{9}{4}\right)^2$

6. Subtract: $12.0072 - 9.937$

7. Solve the proportion $\frac{5}{8} = \frac{n}{50}$. Round to the nearest tenth.

8. Write $1\frac{3}{4}$ as a percent.

9. 6.09 is 4.2% of what number?

10. Convert 18 pt to gallons.

11. Convert 875 cm to meters.

12. Convert 3420 m to kilometers.

13. Convert 5.05 kg to grams.

14. Convert 3 g 672 mg to grams.

15. Convert 6 L to milliliters.

16. Convert 2.4 kl to liters.

28

17. The Guerrero family has a monthly income of \$3244 per month. The family spends one-fourth of its monthly income on rent. How much money is left after the rent is paid?

18. The state income tax on a business is $620 plus 0.08 times the profit the business makes. The business made a profit of $82,340.00 last year. Find the amount of state income tax the business paid.

19. The property tax on a $45,000 home is $900. At the same rate, what is the property tax on a home worth $75,000?

20. A car dealer offers new-car buyers a 12% rebate on some models. What rebate would a new-car buyer receive on a car that cost $23,500?

21. Rob Akullian received a dividend of $533 on an investment of $8200. What percent of the investment is the dividend?

22. You received grades of 78, 92, 45, 80, and 85 on five English exams. Find your mean grade.

23. Karla Perella, a ski instructor, receives a salary of $22,500. Her salary will increase by 12% next year. What will her salary be next year?

24. A sporting goods store has regularly priced $80 fishing rods on sale for $62.40. What is the discount rate?

25. Forty-eight blocks, each 9 in. long, are laid end to end to make the base for a wall. Find the length of the wall in feet.

26. A bottle of apple juice contains 24 oz. Find the number of quarts of apple juice in a case of 16 bottles.

27. A garage mechanic bought oil in a 40-gallon container. The mechanic bought the oil for $4.88 per gallon and sold the oil for $1.99 per quart. Find the profit on the 40-gallon container of oil.

28. A school swimming pool uses 1200 ml of chlorine each school day. How many liters of chlorine are used for 20 days during the month?

29. A 1200-watt hair dryer is used an average of 30 min a day. At a cost of 10.5¢ per kilowatt-hour, how much does it cost to operate the hair dryer for 30 days?

30. Convert 60 mi/h to kilometers per hour. Round to the nearest tenth. (1.61 km $\approx$ 1 mi)

Final Examination

1. Subtract: 100,914 − 97,655

2. Find 34,821 divided by 657.

3. Simplify: $3^2 \cdot (5 - 3)^2 \div 3 + 4$

4. Find the LCM of 9, 12, and 16.

5. Add: $\frac{3}{8} + \frac{5}{6} + \frac{1}{5}$

6. Subtract: $7\frac{5}{12} - 3\frac{13}{16}$

7. Find the product of $3\frac{5}{8}$ and $1\frac{5}{7}$.

8. Divide: $1\frac{2}{3} \div 3\frac{3}{4}$

9. Simplify: $\left(\frac{2}{3}\right)^3 \cdot \left(\frac{3}{4}\right)^2$

10. Simplify: $\left(\frac{2}{3}\right)^2 \div \left(\frac{3}{4} + \frac{1}{3}\right) - \frac{1}{3}$

11. Add:
$$\begin{array}{r} 4.972 \\ 28.6 \\ 1.88 \\ +\ 128.725 \\ \hline \end{array}$$

12. Find 90,001 decreased by 29,796.

13. Multiply:
$$\begin{array}{r} 2.97 \\ \times\ 0.0094 \\ \hline \end{array}$$

14. Divide: $0.062\overline{)0.0426}$
Round to the nearest hundredth.

15. Convert 0.45 to a fraction in simplest form.

16. Write "323.4 miles on 13.2 gallons of gas" as a unit rate.

17. Solve the proportion $\frac{12}{35} = \frac{n}{160}$.
Round to the nearest tenth.

18. Write $22\frac{1}{2}\%$ as a fraction.

19. Write 1.35 as a percent.

20. Write $\frac{5}{4}$ as a percent.

21. Find 120% of 30.

22. 12 is what percent of 9?

23. 42 is 60% of what number?

24. Convert $1\frac{2}{3}$ ft to inches.

25. Subtract: 3 ft 2 in. − 1 ft 10 in.

26. Convert 40 oz to pounds.

27. Find the sum of 3 lb 12 oz and 2 lb 10 oz.

28. Convert 18 pt to gallons.

29. Divide: $3\overline{)5\text{ gal }1\text{ qt}}$

30. Convert 2.48 m to centimeters.

31. Convert 4 m 62 cm to meters.

32. Convert 1 kg 614 g to kilograms.

33. Convert 2 L 67 ml to milliliters.

34. Convert 55 mi to kilometers. Round to the nearest hundredth. (1.61 km ≈ 1 mi)

35. How much does it cost to run a 2400-watt air conditioner for 6 h at 8¢ per kilowatt-hour? Round to the nearest cent.

36. Write 0.0000000679 in scientific notation.

37. Find the perimeter of a rectangle with a length of 1.2 m and a width of 0.75 m.

38. Find the area of a rectangle with a length of 9 in. and a width of 5 in.

39. Find the volume of a box with a length of 20 cm, a width of 12 cm, and a height of 5 cm.

40. Add: $-2 + 8 + (-10)$

41. Subtract: $-30 - (-15)$

42. Multiply: $2\frac{1}{2} \times \left(-\frac{1}{5}\right)$

43. Find the quotient of $-1\frac{3}{8}$ and $5\frac{1}{2}$.

44. Simplify: $(-4)^2 \div (1 - 3)^2 - (-2)$

45. Simplify: $2x - 3(x - 4) + 5$

46. Solve: $\frac{2}{3}x = -12$

47. Solve: $3x - 5 = 10$

48. Solve: $8 - 3x = x + 4$

49. You have $872.48 in your checking account. You write checks of $321.88 and $34.23 and then make a deposit of $443.56. Find your new checking account balance.

50. On the basis of a pre-election survey, it is estimated that 5 out of 8 eligible voters will vote in an election. How many people will vote in an election with 102,000 eligible voters?

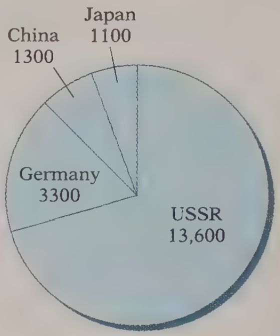
51. This month a company is paying its stockholders a dividend of $1.60 per share. This is 80% of what the dividend per share was one year ago. What was the dividend per share one year ago?

52. A sales executive received commissions of $4320, $3572, $2864, and $4420 during a 4-month period. Find the average income from commissions for the 4 months.

53. A contractor borrows $120,000 for 9 months at an annual interest rate of 10%. What is the simple interest due on the loan?

54. If two dice are tossed, what is the probability that the sum of the dots on the upward faces is divisible by 3?

55. The top four highest death counts, by country, in World War II are shown in the circle graph. Find what percent the death count of Chinese is of the total death count in all four countries. Round to the nearest tenth of a percent.



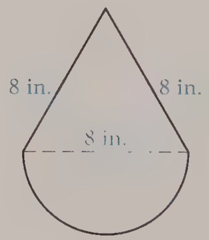
Top Four Highest Death Counts in World War II (in thousands)

Source: U.S. Department of Defense

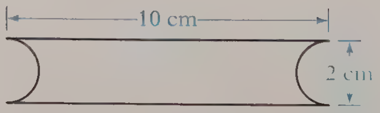
56. A compact disc player that regularly sells for $314.00 is on sale for $226.08. What is the discount rate?

57. A square tile measuring 8 in. by 8 in. weighs 9 oz. Find the weight, in pounds, of a box containing 144 tiles.

58. Find the perimeter of the composite figure. Use 3.14 for π.



59. Find the area of the composite figure. Use 3.14 for π.



60. Five less than the quotient of a number and two is equal to three. Find the number.

Appendix

Compound Interest Table

Compounded Annually

	4%	5%	6%	7%	8%	9%	10%
1 year	1.04000	1.05000	1.06000	1.07000	1.08000	1.09000	1.10000
5 years	1.21665	1.27628	1.33823	1.40255	1.46933	1.53862	1.61051
10 years	1.48024	1.62890	1.79085	1.96715	2.15893	2.36736	2.59374
15 years	1.80094	2.07893	2.39656	2.75903	3.17217	3.64248	4.17725
20 years	2.19112	2.65330	3.20714	3.86968	4.66095	5.60441	6.72750

Compounded Semiannually

	4%	5%	6%	7%	8%	9%	10%
1 year	1.04040	1.05062	1.06090	1.07123	1.08160	1.09203	1.10250
5 years	1.21899	1.28008	1.34392	1.41060	1.48024	1.55297	1.62890
10 years	1.48595	1.63862	1.80611	1.98979	2.19112	2.41171	2.65330
15 years	1.81136	2.09757	2.42726	2.80679	3.24340	3.74531	4.32194
20 years	2.20804	2.68506	3.26204	3.95926	4.80102	5.81634	7.03999

Compounded Quarterly

	4%	5%	6%	7%	8%	9%	10%
1 year	1.04060	1.05094	1.06136	1.07186	1.08243	1.09308	1.10381
5 years	1.22019	1.28204	1.34686	1.41478	1.48595	1.56051	1.63862
10 years	1.48886	1.64362	1.81402	2.00160	2.20804	2.43519	2.68506
15 years	1.81670	2.10718	2.44322	2.83182	3.28103	3.80013	4.39979
20 years	2.21672	2.70148	3.29066	4.00639	4.87544	5.93015	7.20957

Compounded Monthly

	4%	5%	6%	7%	8%	9%	10%
1 year	1.04074	1.051162	1.061678	1.072290	1.083000	1.093807	1.104713
5 years	1.220997	1.283359	1.348850	1.417625	1.489846	1.565681	1.645309
10 years	1.490833	1.647009	1.819397	2.009661	2.219640	2.451357	2.707041
15 years	1.820302	2.113704	2.454094	2.848947	3.306921	3.838043	4.453920
20 years	2.222582	2.712640	3.310204	4.038739	4.926803	6.009152	7.328074

Compounded Daily

	4%	5%	6%	7%	8%	9%	10%
1 year	1.04080	1.05127	1.06183	1.07250	1.08328	1.09416	1.10516
5 years	1.22139	1.28400	1.34983	1.41902	1.49176	1.56823	1.64861
10 years	1.49179	1.64866	1.82203	2.01362	2.22535	2.45933	2.71791
15 years	1.82206	2.11689	2.45942	2.85736	3.31968	3.85678	4.48077
20 years	2.22544	2.71810	3.31979	4.05466	4.95217	6.04830	7.38703

To use this table:

1. Locate the section which gives the desired compounding period.
2. Locate the interest rate in the top row of that section.
3. Locate the number of years in the left-hand column of that section.
4. Locate the number where the interest-rate column and the number-of-years row meet. This is the compound interest factor.

Example An investment yields an annual interest rate of 10% compounded quarterly for 5 years.
The compounding period is "compounded quarterly."
The interest rate is 10%.
The number of years is 5.
The number where the row and column meet is 1.63862. This is the compound interest factor.

Compound Interest Table

Compounded Annually

	11%	12%	13%	14%	15%	16%	17%
1 year	1.11000	1.12000	1.13000	1.14000	1.15000	1.16000	1.17000
5 years	1.68506	1.76234	1.84244	1.92542	2.01136	2.10034	2.19245
10 years	2.83942	3.10585	3.39457	3.70722	4.04556	4.41144	4.80683
15 years	4.78459	5.47357	6.25427	7.13794	8.13706	9.26552	10.53872
20 years	8.06239	9.64629	11.52309	13.74349	16.36654	19.46076	23.10560

Compounded Semiannually

	11%	12%	13%	14%	15%	16%	17%
1 year	1.11303	1.12360	1.13423	1.14490	1.15563	1.16640	1.17723
5 years	1.70814	1.79085	1.87714	1.96715	2.06103	2.15893	2.26098
10 years	2.91776	3.20714	3.52365	3.86968	4.24785	4.66096	5.11205
15 years	4.98395	5.74349	6.61437	7.61226	8.75496	10.06266	11.55825
20 years	8.51331	10.28572	12.41607	14.97446	18.04424	21.72452	26.13302

Compounded Quarterly

	11%	12%	13%	14%	15%	16%	17%
1 year	1.11462	1.12551	1.13648	1.14752	1.15865	1.16986	1.18115
5 years	1.72043	1.80611	1.89584	1.98979	2.08815	2.19112	2.29891
10 years	2.95987	3.26204	3.59420	3.95926	4.36038	4.80102	5.28497
15 years	5.09225	5.89160	6.81402	7.87809	9.10513	10.51963	12.14965
20 years	8.76085	10.64089	12.91828	15.67574	19.01290	23.04980	27.93091

Compounded Monthly

	11%	12%	13%	14%	15%	16%	17%
1 year	1.115719	1.126825	1.138032	1.149342	1.160755	1.172271	1.183892
5 years	1.728916	1.816697	1.908857	2.005610	2.107181	2.213807	2.325733
10 years	2.989150	3.300387	3.643733	4.022471	4.440213	4.900941	5.409036
15 years	5.167988	5.995802	6.955364	8.067507	9.356334	10.849737	12.579975
20 years	8.935015	10.892554	13.276792	16.180270	19.715494	24.019222	29.257669

Compounded Daily

	11%	12%	13%	14%	15%	16%	17%
1 year	1.11626	1.12747	1.13880	1.15024	1.16180	1.17347	1.18526
5 years	1.73311	1.82194	1.91532	2.01348	2.11667	2.22515	2.33918
10 years	3.00367	3.31946	3.66845	4.05411	4.48031	4.95130	5.47178
15 years	5.20569	6.04786	7.02625	8.16288	9.48335	11.01738	12.79950
20 years	9.02203	11.01883	13.45751	16.43582	20.07316	24.51534	29.94039

Monthly Payment Table

	4%	*5%*	*6%*	*7%*	*8%*	*9%*	
1 year	0.0851499	0.0856075	0.0860664	0.0865267	0.0869884	0.0874515	1
2 years	0.0434249	0.0438714	0.0443206	0.0447726	0.0452273	0.0456847	2
3 years	0.0295240	0.0299709	0.0304219	0.0308771	0.0313364	0.0317997	3
4 years	0.0225791	0.0230293	0.0234850	0.0239462	0.0244129 ←	0.0248850	4
5 years	0.0184165	0.0188712	0.0193328	0.0198012	0.0202764	0.0207584	5
15 years	0.0073969	0.0079079	0.0084386	0.0089883	0.0095565	0.0101427	15
20 years	0.0060598	0.0065996	0.0071643	0.0077530	0.0083644	0.0089973	20
25 years	0.0052784	0.0058459	0.0064430	0.0070678	0.0077182	0.0083920	25
30 years	0.0047742	0.0053682	0.0059955	0.0066530	0.0073376	0.0080462	30

	10%	*11%*	*12%*	*13%*
1 year	0.0879159	0.0883817	0.0888488	0.0893173
2 years	0.0461449	0.0466078	0.0470735	0.0475418
3 years	0.0322672	0.0327387	0.0332143	0.0336940
4 years	0.0253626	0.0258455	0.0263338	0.0268275
5 years	0.0212470	0.0217424	0.0222445	0.0227531
15 years	0.0107461	0.0113660	0.0120017	0.0126524
20 years	0.0096502	0.0103219	0.0110109	0.0117158
25 years	0.0090870	0.0098011	0.0105322	0.0112784
30 years	0.0087757	0.0095232	0.0102861	0.0110620

To use this table:

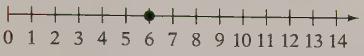
1. Locate the desired interest rate in the top row.
2. Locate the number of years in the left-hand column.
3. Locate the number where the interest-rate column and the number-of-years row meet. This is the monthly payment factor.

Example A home has a 30-year mortgage at an annual interest rate of 12%.
The interest rate is 12%.
The number of years is 30.
The number where the row and column meet is 0.0102861. This is the monthly payment factor.

Solutions to Chapter 1 "You Try It"

SECTION 1.1

You Try It 1



You Try It 2 **a.** $45 > 29$ **b.** $27 > 0$

You Try It 3 Thirty-six million four hundred sixty-two thousand seventy-five

You Try It 4 452,007

You Try It 5 60,000 + 8000 + 200 + 80 + 1

You Try It 6 100,000 + 9000 + 200 + 7

You Try It 7

Given place value
368,492
$8 > 5$

368,492 rounded to the nearest ten-thousand is 370,000.

You Try It 8

Given place value
3962
$6 > 5$

3962 rounded to the nearest hundred is 4000.

SECTION 1.2

You Try It 1

$$\begin{array}{r} {\scriptstyle 1\,1}\\ 347\\ +\ 12{,}453\\ \hline 12{,}800 \end{array}$$

347 increased by 12,453 is 12,800.

You Try It 2

$$\begin{array}{r} {\scriptstyle 2}\\ 95\\ 88\\ +\ 67\\ \hline 250 \end{array}$$

You Try It 3

$$\begin{array}{r} {\scriptstyle 1\,1\;\;2\,1}\\ 392\\ 4{,}079\\ 89{,}035\\ +\ \ 4{,}992\\ \hline 98{,}498 \end{array}$$

You Try It 4

Strategy To find the total amount spent on these items, add the four amounts (30, 22, 16, and 11).

Solution

$$\begin{array}{r} 30\\ 22\\ 16\\ +\ 11\\ \hline 79 \end{array}$$

The total amount spent on these items is $79.

SECTION 1.3

You Try It 1

$$\begin{array}{r} 8925\\ -\ 6413\\ \hline 2512 \end{array} \qquad \textit{Check:}\quad \begin{array}{r} 6413\\ +\ 2512\\ \hline 8925 \end{array}$$

You Try It 2

$$\begin{array}{r} 17{,}504\\ -\ \ 9{,}302\\ \hline 8{,}202 \end{array} \qquad \textit{Check:}\quad \begin{array}{r} 9{,}302\\ +\ \ 8{,}202\\ \hline 17{,}504 \end{array}$$

You Try It 3

$$\begin{array}{rcccc} & 2 & 14 & 7 & 11\\ & \cancel{3} & \cancel{4} & \cancel{8} & \cancel{1}\\ - & & 8 & 6 & 5\\ \hline & 2 & 6 & 1 & 6 \end{array} \qquad \textit{Check:}\quad \begin{array}{r} 865\\ +\ 2616\\ \hline 3481 \end{array}$$

You Try It 4

$$\begin{array}{rccccc} & & & & 15 & \\ & & & 4 & \cancel{5} & 12\\ & 5 & 4, & \cancel{5} & \cancel{6} & \cancel{2}\\ - & 1 & 4, & 4 & 8 & 5\\ \hline & 4 & 0, & 0 & 7 & 7 \end{array} \qquad \textit{Check:}\quad \begin{array}{r} 14{,}485\\ +\ 40{,}077\\ \hline 54{,}562 \end{array}$$

You Try It 5

$$\begin{array}{rccccc} & & 13 & 9 & 9 & \\ & 5 & \cancel{3} & \cancel{10} & \cancel{10} & 13\\ & \cancel{6} & \cancel{4}, & \cancel{0} & \cancel{0} & \cancel{3}\\ - & 5 & 4, & 9 & 3 & 6\\ \hline & & 9, & 0 & 6 & 7 \end{array} \qquad \textit{Check:}\quad \begin{array}{r} 54{,}936\\ +\ \ 9{,}067\\ \hline 64{,}003 \end{array}$$

You Try It 6

Strategy To find the difference, subtract the number of personnel on active duty in the Air Force in 1945 (2,282,259) from the number of personnel on active duty in the Navy in 1945 (3,380,817).

Solution

$$\begin{array}{r} 3{,}380{,}817\\ -\ 2{,}282{,}259\\ \hline 1{,}098{,}558 \end{array}$$

The difference between the number of personnel on active duty in the Navy and in the Air Force in 1945 was 1,098,558.

You Try It 7

Strategy To find your take-home pay:

- Add to find the total of the deductions (127 + 18 + 35).
- Subtract the total of the deductions from your total salary (638).

Solution

$$\begin{array}{r} 127\\ 18\\ +\ 35\\ \hline 180 \end{array} \text{ in deductions} \qquad \begin{array}{r} 638\\ -\ 180\\ \hline 458 \end{array}$$

Your take-home pay is \$458.

SECTION 1.4

You Try It 1

$$\begin{array}{r} {}^{3\,5}\\ 648\\ \times\quad 7\\ \hline 4536 \end{array}$$

You Try It 2

$$\begin{array}{r} 756\\ \times\ 305\\ \hline 3780\\ 22680\\ \hline 230{,}580 \end{array}$$

You Try It 3

Strategy To find the number of cars the dealer will receive in 12 months, multiply the number of months (12) by the number of cars received each month (37).

Solution

$$\begin{array}{r} 37\\ \times\ 12\\ \hline 74\\ 37\\ \hline 444 \end{array}$$

The dealer will receive 444 cars in 12 months.

You Try It 4

Strategy To find the total cost of the order:

- Find the cost of the sports jackets by multiplying the number of jackets (25) by the cost for each jacket (23).
- Add the product to the cost for the suits (4800).

Solution

$$\begin{array}{r} 23\\ \times\ 25\\ \hline 115\\ 46\\ \hline 575 \end{array} \text{ cost for jackets} \qquad \begin{array}{r} 4800\\ +\quad 575\\ \hline 5375 \end{array}$$

The total cost of the order is \$5375.

SECTION 1.5

You Try It 1

$$\begin{array}{r} 7\\ 9\overline{)63} \end{array}$$

Check: 7 × 9 = 63

You Try It 2

$$\begin{array}{r} 453\\ 9\overline{)\ 4077}\\ \underline{-36}\\ 47\\ \underline{-45}\\ 27\\ \underline{-27}\\ 0 \end{array}$$

Check: 453 × 9 = 4077

You Try It 3

$$\begin{array}{r} 705\\ 9\overline{)\ 6345}\\ \underline{-63}\\ 04\\ \underline{-\ \ 0}\\ 45\\ \underline{-45}\\ 0 \end{array}$$

Check: 705 × 9 = 6345

You Try It 4

$$\begin{array}{r} 870 \text{ r5}\\ 6\overline{)\ 5225}\phantom{\text{ r5}}\\ \underline{-48}\phantom{\text{ r5}}\\ 42\phantom{\text{ r5}}\\ \underline{-42}\phantom{\text{ r5}}\\ 05\phantom{\text{ r5}}\\ \underline{-\ \ 0}\phantom{\text{ r5}}\\ 5\phantom{\text{ r5}} \end{array}$$

Check: (870 × 6) + 5 =
5220 + 5 = 5225

You Try It 5

$$\begin{array}{r} 3{,}058 \text{ r3}\\ 7\overline{)\ 21{,}409}\phantom{\text{ r3}}\\ \underline{-21}\phantom{\text{ r3}}\\ 0\ 4\phantom{\text{ r3}}\\ \underline{-\ \ 0}\phantom{\text{ r3}}\\ 40\phantom{\text{ r3}}\\ \underline{-35}\phantom{\text{ r3}}\\ 59\phantom{\text{ r3}}\\ \underline{-56}\phantom{\text{ r3}}\\ 3\phantom{\text{ r3}} \end{array}$$

Check: (3058 × 7) + 3 =
21,406 + 3 = 21,409

You Try It 6

$$\begin{array}{r} 109 \\ 42\overline{)\,4578} \\ -42 \\ \hline 37 \\ -\ 0 \\ \hline 378 \\ -378 \\ \hline 0 \end{array}$$

Check: $109 \times 42 = 4578$

You Try It 7

$$\begin{array}{r} 470 \text{ r29} \\ 39\overline{)\,18{,}359} \\ -15\ 6 \\ \hline 2\ 75 \\ -2\ 73 \\ \hline 29 \\ -\ 0 \\ \hline 29 \end{array}$$

Check: $(470 \times 39) + 29 =$
$18{,}330 + 29 = 18{,}359$

You Try It 8

$$\begin{array}{r} 62 \text{ r111} \\ 534\overline{)\,33{,}219} \\ -32\ 04 \\ \hline 1\ 179 \\ -1\ 068 \\ \hline 111 \end{array}$$

Check: $(62 \times 534) + 111 =$
$33{,}108 + 111 = 33{,}219$

You Try It 9

$$\begin{array}{r} 421 \text{ r33} \\ 515\overline{)\,216{,}848} \\ -206\ 0 \\ \hline 10\ 84 \\ -10\ 30 \\ \hline 548 \\ -515 \\ \hline 33 \end{array}$$

Check: $(421 \times 515) + 33 =$
$216{,}815 + 33 = 216{,}848$

You Try It 10

Strategy To find the number of tires that can be stored on each shelf, divide the number of tires (270) by the number of shelves (15).

Solution

$$\begin{array}{r} 18 \\ 15\overline{)\,270} \\ -15 \\ \hline 120 \\ -120 \\ \hline 0 \end{array}$$

Each shelf can store 18 tires.

You Try It 11

Strategy To find the number of cases produced in 8 hours:

- Find the number of cases produced in 1 hour by dividing the number of cans produced (12,600) by the number of cans to a case (24).
- Multiply the number of cases produced in 1 hour by 8.

Solution

$$\begin{array}{r} 525 \\ 24\overline{)\,12{,}600} \\ -12\ 0 \\ \hline 60 \\ -48 \\ \hline 120 \\ -120 \\ \hline 0 \end{array}$$

cases produced in 1 hour

$$\begin{array}{r} 525 \\ \times\quad 8 \\ \hline 4200 \end{array}$$

In 8 hours, 4200 cases are produced.

SECTION 1.6

You Try It 1 $2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 = 2^4 \cdot 3^3$

You Try It 2 $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 10^7$

You Try It 3 $2^3 \cdot 5^2 = (2 \cdot 2 \cdot 2) \cdot (5 \cdot 5) = 8 \cdot 25$
$= 200$

You Try It 4 $5 \cdot (8 - 4)^2 \div 4 - 2$
$= 5 \cdot 4^2 \div 4 - 2$
$= 5 \cdot 16 \div 4 - 2$
$= 80 \div 4 - 2$
$= 20 - 2$
$= 18$

SECTION 1.7

You Try It 1
$40 \div 1 = 40$
$40 \div 2 = 20$
$40 \div 3$ Will not divide evenly
$40 \div 4 = 10$
$40 \div 5 = 8$
$40 \div 6$ Will not divide evenly
$40 \div 7$ Will not divide evenly
$40 \div 8 = 5$

1, 2, 4, 5, 8, 10, 20, and 40 are factors of 40.

You Try It 2

$$\begin{array}{r|l} \multicolumn{2}{c}{44} \\ \hline 2 & 22 \\ 2 & 11 \\ 11 & 1 \end{array}$$

$44 = 2 \cdot 2 \cdot 11$

You Try It 3

$$\begin{array}{r|l} \multicolumn{2}{c}{177} \\ \hline 3 & 59 \\ 59 & 1 \end{array}$$

$177 = 3 \cdot 59$

Solutions to Chapter 2 "You Try It"

SECTION 2.1

You Try It 1

	2	3	5	7
50 =	2		$\enclose{circle}{5 \cdot 5}$	
84 =	$\enclose{circle}{2 \cdot 2}$	3		$\enclose{circle}{7}$
135 =		$\enclose{circle}{3 \cdot 3 \cdot 3}$	5	

The LCM $= 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 7$
$= 18{,}900$

You Try It 2

	2	3	5
36 =	$\enclose{circle}{2 \cdot 2}$	$3 \cdot 3$	
60 =	$2 \cdot 2$	$\enclose{circle}{3}$	5
72 =	$2 \cdot 2 \cdot 2$	$3 \cdot 3$	

The GCF $= 2 \cdot 2 \cdot 3 = 12$.

You Try It 3

	2	3	5	11
11 =				11
24 =	$2 \cdot 2 \cdot 2$	3		
30 =	2	3	5	

Because no numbers are circled, the GCF = 1.

SECTION 2.2

You Try It 1 $4\frac{1}{4}$

You Try It 2 $\frac{17}{4}$

You Try It 3

$$\begin{array}{r} 4 \\ 5\overline{)22} \\ \underline{-20} \\ 2 \end{array} \qquad \frac{22}{5} = 4\frac{2}{5}$$

You Try It 4

$$\begin{array}{r} 4 \\ 7\overline{)28} \\ \underline{-28} \\ 0 \end{array} \qquad \frac{28}{7} = 4$$

You Try It 5 $\underset{\times}{\overset{+}{14\frac{5}{8}}} = \frac{112 + 5}{8} = \frac{117}{8}$

SECTION 2.3

You Try It 1 $45 \div 5 = 9 \qquad \frac{3}{5} = \frac{3 \cdot 9}{5 \cdot 9} = \frac{27}{45}$

$\frac{27}{45}$ is equivalent to $\frac{3}{5}$.

You Try It 2 Write 6 as $\frac{6}{1}$.

$18 \div 1 = 18 \qquad 6 = \frac{6 \cdot 18}{1 \cdot 18} = \frac{108}{18}$

$\frac{108}{18}$ is equivalent to 6.

You Try It 3

$$\frac{16}{24} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 2}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot 3} = \frac{2}{3}$$

You Try It 4

$$\frac{8}{56} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}}}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot 7} = \frac{1}{7}$$

You Try It 5

$$\frac{15}{32} = \frac{3 \cdot 5}{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} = \frac{15}{32}$$

You Try It 6

$$\frac{48}{36} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 2 \cdot 2 \cdot \overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{3}} \cdot 3} = \frac{4}{3} = 1\frac{1}{3}$$

SECTION 2.4

You Try It 1

$$\begin{array}{r} \frac{3}{8} \\ +\ \frac{7}{8} \\ \hline \frac{10}{8} = \frac{5}{4} = 1\frac{1}{4} \end{array}$$

You Try It 2

$$\begin{array}{rcl} \frac{5}{12} & = & \frac{20}{48} \\ +\ \frac{9}{16} & = & \frac{27}{48} \\ \hline & & \frac{47}{48} \end{array}$$

- The LCM of 12 and 16 is 48.

You Try It 3

$$\begin{array}{rcl} \frac{7}{8} & = & \frac{105}{120} \\ +\ \frac{11}{15} & = & \frac{88}{120} \\ \hline & & \frac{193}{120} = 1\frac{73}{120} \end{array}$$

- The LCM of 8 and 15 is 120.

You Try It 4

$$\begin{array}{rcl} \frac{3}{4} & = & \frac{30}{40} \\ \frac{4}{5} & = & \frac{32}{40} \\ +\ \frac{5}{8} & = & \frac{25}{40} \\ \hline & & \frac{87}{40} = 2\frac{7}{40} \end{array}$$

- The LCM of 4, 5, and 8 is 40.

You Try It 5

$$7 + \frac{6}{11} = 7\frac{6}{11}$$

You Try It 6

$$\begin{array}{r} 29 \\ +\ 17\frac{5}{12} \\ \hline 46\frac{5}{12} \end{array}$$

You Try It 7

$$\begin{array}{rcl} 7\frac{4}{5} & = & 7\frac{24}{30} \\ 6\frac{7}{10} & = & 6\frac{21}{30} \\ +\ 13\frac{11}{15} & = & 13\frac{22}{30} \\ \hline & & 26\frac{67}{30} = 28\frac{7}{30} \end{array}$$

- LCM = 30

You Try It 8

$$\begin{array}{rcl} 9\frac{3}{8} & = & 9\frac{45}{120} \\ 17\frac{7}{12} & = & 17\frac{70}{120} \\ +\ 10\frac{14}{15} & = & 10\frac{112}{120} \\ \hline & & 36\frac{227}{120} = 37\frac{107}{120} \end{array}$$

- LCM = 120

You Try It 9

Strategy To find the total time spent on the activities, add the three times $\left(4\frac{1}{2}, 3\frac{3}{4}, 1\frac{1}{3}\right)$.

Solution

$$\begin{array}{rcl} 4\frac{1}{2} & = & 4\frac{6}{12} \\ 3\frac{3}{4} & = & 3\frac{9}{12} \\ +\ 1\frac{1}{3} & = & 1\frac{4}{12} \\ \hline & & 8\frac{19}{12} = 9\frac{7}{12} \end{array}$$

The total time spent on the three activities was $9\frac{7}{12}$ hours.

You Try It 10

Strategy To find the overtime pay:

- Find the total number of overtime hours $\left(1\frac{2}{3} + 3\frac{1}{3} + 2\right)$.
- Multiply the total number of hours by the overtime hourly wage (36).

Solution

$$\begin{array}{r} 1\frac{2}{3} \\ 3\frac{1}{3} \\ +\ 2 \\ \hline 6\frac{3}{3} = 7 \text{ hours} \end{array} \qquad \begin{array}{r} 36 \\ \times\ \ 7 \\ \hline 252 \end{array}$$

Jeff earned \$252 in overtime pay.

SECTION 2.5

You Try It 1

$$\begin{array}{r} \frac{16}{27} \\ -\ \frac{7}{27} \\ \hline \frac{9}{27} = \frac{1}{3} \end{array}$$

You Try It 2

$$\begin{array}{rcl} \frac{13}{18} & = & \frac{52}{72} \\ -\ \frac{7}{24} & = & \frac{21}{72} \\ \hline & & \frac{31}{72} \end{array}$$

- LCM = 72

You Try It 3

$$\begin{array}{rcl} 17\frac{5}{9} & = & 17\frac{20}{36} \\ -\ 11\frac{5}{12} & = & 11\frac{15}{36} \\ \hline & & 6\frac{5}{36} \end{array}$$

• LCM = 36

You Try It 4

$$\begin{array}{rcl} 8 & = & 7\frac{13}{13} \\ -\ 2\frac{4}{13} & = & 2\frac{4}{13} \\ \hline & & 5\frac{9}{13} \end{array}$$

• LCM = 13

You Try It 5

$$\begin{array}{rcccl} 21\frac{7}{9} & = & 21\frac{28}{36} & = & 20\frac{64}{36} \\ -\ 7\frac{11}{12} & = & 7\frac{33}{36} & = & 7\frac{33}{36} \\ \hline & & & & 13\frac{31}{36} \end{array}$$

• LCM = 36

You Try It 6

Strategy To find the time remaining before the plane lands, subtract the number of hours already in the air $\left(2\frac{3}{4}\right)$ from the total time of the trip $\left(5\frac{1}{2}\right)$.

Solution

$$\begin{array}{rcccl} 5\frac{1}{2} & = & 5\frac{2}{4} & = & 4\frac{6}{4} \\ -\ 2\frac{3}{4} & = & 2\frac{3}{4} & = & 2\frac{3}{4} \\ \hline & & & & 2\frac{3}{4} \text{ hours} \end{array}$$

The plane will land in $2\frac{3}{4}$ hours.

You Try It 7

Strategy To find the amount of weight to be lost during the third month:

- Find the total weight loss during the first two months $\left(7\frac{1}{2} + 5\frac{3}{4}\right)$.
- Subtract the total weight loss from the goal (24 pounds).

Solution

$$\begin{array}{rcl} 7\frac{1}{2} & = & 7\frac{2}{4} \\ +\ 5\frac{3}{4} & = & 5\frac{3}{4} \\ \hline & & 12\frac{5}{4} = 13\frac{1}{4} \text{ pounds lost} \end{array}$$

$$\begin{array}{rcl} 24 & = & 23\frac{4}{4} \\ -\ 13\frac{1}{4} & = & 13\frac{1}{4} \\ \hline & & 10\frac{3}{4} \text{ pounds} \end{array}$$

The patient must lose $10\frac{3}{4}$ pounds to achieve the goal.

SECTION 2.6

You Try It 1

$$\frac{4}{21} \times \frac{7}{44} = \frac{4 \cdot 7}{21 \cdot 44}$$

$$= \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{7}}}{3 \cdot \underset{1}{\cancel{7}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot 11} = \frac{1}{33}$$

You Try It 2

$$\frac{2}{21} \times \frac{10}{33} = \frac{2 \cdot 10}{21 \cdot 33}$$

$$= \frac{2 \cdot 2 \cdot 5}{3 \cdot 7 \cdot 3 \cdot 11} = \frac{20}{693}$$

You Try It 3

$$\frac{16}{5} \times \frac{15}{24} = \frac{16 \cdot 15}{5 \cdot 24}$$

$$= \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 2 \cdot \overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{5}}}{\underset{1}{\cancel{5}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{3}}} = 2$$

You Try It 4

$$5\frac{2}{5} \times \frac{5}{9} = \frac{27}{5} \times \frac{5}{9} = \frac{27 \cdot 5}{5 \cdot 9}$$

$$= \frac{\overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{3}} \cdot 3 \cdot \overset{1}{\cancel{5}}}{\underset{1}{\cancel{5}} \cdot \underset{1}{\cancel{3}} \cdot \underset{1}{\cancel{3}}} = 3$$

You Try It 5

$$3\frac{2}{5} \times 6\frac{1}{4} = \frac{17}{5} \times \frac{25}{4} = \frac{17 \cdot 25}{5 \cdot 4}$$

$$= \frac{17 \cdot \overset{1}{\cancel{5}} \cdot 5}{\underset{1}{\cancel{5}} \cdot 2 \cdot 2} = \frac{85}{4} = 21\frac{1}{4}$$

You Try It 6

$$3\frac{2}{7} \times 6 = \frac{23}{7} \times \frac{6}{1} = \frac{23 \cdot 6}{7 \cdot 1}$$

$$= \frac{23 \cdot 2 \cdot 3}{7 \cdot 1} = \frac{138}{7} = 19\frac{5}{7}$$

You Try It 7

Strategy To find the value of the house today, multiply the old value of the house (70,000) by $2\frac{1}{2}$.

Solution

$$\begin{aligned}70{,}000 \times 2\frac{1}{2} &= \frac{70{,}000}{1} \times \frac{5}{2}\\ &= \frac{70{,}000 \cdot 5}{1 \cdot 2}\\ &= 175{,}000\end{aligned}$$

The value of the house today is $175,000.

You Try It 8

Strategy To find the cost of the air compressor:

- Multiply to find the value of the drying chamber $\left(\frac{4}{5} \times 60{,}000\right)$.
- Subtract the value of the drying chamber from the total value of the two items (60,000).

Solution

$$\begin{aligned}\frac{4}{5} \times \frac{60{,}000}{1} &= \frac{240{,}000}{5}\\ &= 48{,}000\end{aligned}$$

- Value of the drying chamber

$$\begin{array}{r} 60{,}000\\ -48{,}000\\ \hline 12{,}000\end{array}$$

The cost of the air compressor was $12,000.

SECTION 2.7

You Try It 1

$$\frac{3}{7} \div \frac{2}{3} = \frac{3}{7} \times \frac{3}{2} = \frac{3 \cdot 3}{7 \cdot 2} = \frac{9}{14}$$

You Try It 2

$$\begin{aligned}\frac{3}{4} \div \frac{9}{10} &= \frac{3}{4} \times \frac{10}{9}\\ &= \frac{3 \cdot 10}{4 \cdot 9} = \frac{\overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{2}} \cdot 5}{\underset{1}{\cancel{2}} \cdot 2 \cdot \underset{1}{\cancel{3}} \cdot 3} = \frac{5}{6}\end{aligned}$$

You Try It 3

$$\begin{aligned}\frac{5}{7} \div 6 &= \frac{5}{7} \div \frac{6}{1}\\ &= \frac{5}{7} \times \frac{1}{6} = \frac{5 \cdot 1}{7 \cdot 6}\\ &= \frac{5}{7 \cdot 2 \cdot 3} = \frac{5}{42}\end{aligned}$$

You Try It 4

$$\begin{aligned}12\frac{3}{5} \div 7 &= \frac{63}{5} \div \frac{7}{1} = \frac{63}{5} \times \frac{1}{7}\\ &= \frac{63 \cdot 1}{5 \cdot 7} = \frac{3 \cdot 3 \cdot \overset{1}{\cancel{7}}}{5 \cdot \underset{1}{\cancel{7}}} = \frac{9}{5} = 1\frac{4}{5}\end{aligned}$$

You Try It 5

$$\begin{aligned}3\frac{2}{3} \div 2\frac{2}{5} &= \frac{11}{3} \div \frac{12}{5}\\ &= \frac{11}{3} \times \frac{5}{12} = \frac{11 \cdot 5}{3 \cdot 12}\\ &= \frac{11 \cdot 5}{3 \cdot 2 \cdot 2 \cdot 3} = \frac{55}{36} = 1\frac{19}{36}\end{aligned}$$

You Try it 6

$$\begin{aligned}2\frac{5}{6} \div 8\frac{1}{2} &= \frac{17}{6} \div \frac{17}{2}\\ &= \frac{17}{6} \times \frac{2}{17} = \frac{17 \cdot 2}{6 \cdot 17}\\ &= \frac{\overset{1}{\cancel{17}} \cdot \cancel{2}}{\cancel{2} \cdot 3 \cdot \underset{1}{\cancel{17}}} = \frac{1}{3}\end{aligned}$$

You Try It 7

$$\begin{aligned}6\frac{2}{5} \div 4 &= \frac{32}{5} \div \frac{4}{1}\\ &= \frac{32}{5} \times \frac{1}{4} = \frac{32 \cdot 1}{5 \cdot 4}\\ &= \frac{2 \cdot 2 \cdot 2 \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}}}{5 \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}}} = \frac{8}{5} = 1\frac{3}{5}\end{aligned}$$

You Try It 8

Strategy To find the cost of a similar coin weighing 1 ounce, divide the price of the coin (195) by the number of ounces $\left(\frac{1}{2}\right)$.

Solution

$$\begin{aligned}195 \div \frac{1}{2} &= \frac{195}{1} \div \frac{1}{2}\\ &= \frac{195}{1} \times \frac{2}{1} = \frac{195 \cdot 2}{1 \cdot 1} = 390\end{aligned}$$

The cost of a similar coin weighing 1 ounce would be $390.

You Try It 9

Strategy To find the length of the remaining piece:

- Divide the total length of the board (16) by the length of each shelf $\left(3\frac{1}{3}\right)$. This will give you the number of shelves cut, with a certain fraction of a shelf left over.
- Multiply the fractional part of the result in Step 1 by the length of one shelf to determine the length of the remaining piece.

Solution

$$\begin{aligned} 16 \div 3\tfrac{1}{3} &= 16 \div \frac{10}{3} \\ &= \frac{16}{1} \times \frac{3}{10} = \frac{16 \cdot 3}{1 \cdot 10} \\ &= \frac{\overset{1}{\cancel{2}} \cdot 2 \cdot 2 \cdot 2 \cdot 3}{\underset{1}{\cancel{2}} \cdot 5} = \frac{24}{5} \\ &= 4\tfrac{4}{5} \end{aligned}$$

There are 4 pieces $3\frac{1}{3}$ feet long. There is 1 piece that is $\frac{4}{5}$ of $3\frac{1}{3}$ feet long.

$$\begin{aligned} \frac{4}{5} \times 3\tfrac{1}{3} &= \frac{4}{5} \times \frac{10}{3} \\ &= \frac{4 \cdot 10}{5 \cdot 3} = \frac{2 \cdot 2 \cdot 2 \cdot \overset{1}{\cancel{5}}}{\underset{1}{\cancel{5}} \cdot 3} \\ &= \frac{8}{3} = 2\tfrac{2}{3} \end{aligned}$$

The length of the piece remaining is $2\frac{2}{3}$ feet.

SECTION 2.8

You Try It 1 $\frac{9}{14} = \frac{27}{42}$ $\frac{13}{21} = \frac{26}{42}$ $\frac{9}{14} > \frac{13}{21}$

You Try It 2

$$\begin{aligned} \left(\frac{7}{11}\right)^2 \cdot \left(\frac{2}{7}\right) &= \left(\frac{7}{11} \cdot \frac{7}{11}\right) \cdot \left(\frac{2}{7}\right) \\ &= \frac{\overset{1}{\cancel{7}} \cdot 7 \cdot 2}{11 \cdot 11 \cdot \underset{1}{\cancel{7}}} = \frac{14}{121} \end{aligned}$$

You Try It 3

$$\left(\frac{1}{13}\right)^2 \cdot \left(\frac{1}{4} + \frac{1}{6}\right) \div \frac{5}{13} =$$

$$\left(\frac{1}{13}\right)^2 \cdot \left(\frac{5}{12}\right) \div \frac{5}{13} =$$

$$\left(\frac{1}{169}\right) \cdot \left(\frac{5}{12}\right) \div \frac{5}{13} =$$

$$\left(\frac{1 \cdot 5}{13 \cdot 13 \cdot 12}\right) \div \frac{5}{13} =$$

$$\left(\frac{1 \cdot 5}{13 \cdot 13 \cdot 12}\right) \times \frac{13}{5} =$$

$$\frac{1 \cdot \overset{1}{\cancel{5}} \cdot \overset{1}{\cancel{13}}}{\underset{1}{\cancel{13}} \cdot 13 \cdot 12 \cdot \underset{1}{\cancel{5}}} = \frac{1}{156}$$

Solutions to Chapter 3 "You Try It"

SECTION 3.1

You Try It 1 Two hundred nine and five thousand eight hundred thirty-eight hundred-thousandths

You Try It 2 42,000.000207

You Try It 3

Given place value

4.349254

9 > 5

4.349254 rounded to the nearest hundredth is 4.35.

You Try It 4

Given place value

3.290532

2 < 5

3.290532 rounded to the nearest hundred-thousandth is 3.29053.

SECTION 3.2

You Try It 1

$$\begin{array}{r} {\scriptstyle 1\,2} \\ 4.62 \\ 27.9 \\ +\ \ 0.62054 \\ \hline 33.14054 \end{array}$$

You Try It 2

$$\begin{array}{r} {\scriptstyle 1} \\ 6.05 \\ 12. \\ +\ 0.374 \\ \hline 18.424 \end{array}$$

You Try It 3

Strategy To find the amount he would be paid, add the amounts in the column headed "Boys."

Solution $2.61 + 1.64 + 2.86 + 1.29 + 9.02 + 2.56 + 1.63 + 2.06 + 2.76 = 26.43$

For performing each one of the chores listed, a boy would be paid, on average, $26.43.

You Try It 4

Strategy To find the total income, add the four commissions (985.80, 791.46, 829.75, and 635.42) to the salary (875).

Solution $875 + 985.80 + 791.46 + 829.75 + 635.42 = 4117.43$

Anita's total income was $4117.43.

SECTION 3.3

You Try It 1

$$\begin{array}{ccccccc} & & {\scriptstyle 11} & & {\scriptstyle 9} & & \\ & {\scriptstyle 6} & {\scriptstyle \cancel{1}} & & {\scriptstyle \cancel{10}} & {\scriptstyle 13} & \\ & \cancel{7} & \cancel{2} & . & \cancel{0} & \cancel{3} & 9 \\ - & & 8 & . & 4 & 7 & \\ \hline & 6 & 3 & . & 5 & 6 & 9 \end{array}$$

Check:
$$\begin{array}{r} {\scriptstyle 1\ 1\ \ 1} \\ 8.47 \\ +\ 63.569 \\ \hline 72.039 \end{array}$$

You Try It 2

$$\begin{array}{cccccc} & & {\scriptstyle 14} & & {\scriptstyle 9} & \\ & {\scriptstyle 2} & {\scriptstyle \cancel{4}} & & {\scriptstyle \cancel{10}} & {\scriptstyle 10} \\ & \cancel{3} & \cancel{5} & . & \cancel{0} & \cancel{0} \\ - & & 9 & . & 6 & 7 \\ \hline & 2 & 5 & . & 3 & 3 \end{array}$$

Check:
$$\begin{array}{r} {\scriptstyle 1\ 1\ \ 1} \\ 9.67 \\ +\ 25.33 \\ \hline 35.00 \end{array}$$

You Try It 3

$$\begin{array}{ccccccc} & & & {\scriptstyle 16} & {\scriptstyle 9} & {\scriptstyle 9} & \\ & {\scriptstyle 2} & & {\scriptstyle \cancel{6}} & {\scriptstyle \cancel{10}} & {\scriptstyle \cancel{10}} & {\scriptstyle 10} \\ & \cancel{3} & . & \cancel{7} & \cancel{0} & \cancel{0} & \cancel{0} \\ - & 1 & . & 9 & 7 & 1 & 5 \\ \hline & 1 & . & 7 & 2 & 8 & 5 \end{array}$$

Check:
$$\begin{array}{r} {\scriptstyle 1\ \ 1\ 1\ 1} \\ 1.9715 \\ +\ 1.7285 \\ \hline 3.7000 \end{array}$$

You Try It 4

Strategy To find the amount of change, subtract the amount paid (6.85) from 10.00.

Solution
$$\begin{array}{r} 10.00 \\ -\ 6.85 \\ \hline 3.15 \end{array}$$

Your change was $3.15.

You Try It 5

Strategy To find the new balance:

- Add to find the total of the three checks (1025.60 + 79.85 + 162.47).
- Subtract the total from the previous balance (2472.69).

Solution
$$\begin{array}{r} 1025.60 \\ 79.85 \\ +\ 162.47 \\ \hline 1267.92 \end{array} \qquad \begin{array}{r} 2472.69 \\ -\ 1267.92 \\ \hline 1204.77 \end{array}$$

The new balance is $1204.77.

SECTION 3.4

You Try It 1

$$\begin{array}{r} 870 \\ \times\quad 4.6 \\ \hline 522\ 0 \\ 3480 \\ \hline 4002.0 \end{array}$$

You Try It 2

$$\begin{array}{r} 0.000086 \\ \times\qquad 0.057 \\ \hline 602 \\ 430 \\ \hline 0.000004902 \end{array}$$

You Try It 3

$$\begin{array}{r} 4.68 \\ \times\ 6.03 \\ \hline 1404 \\ 28\ 080 \\ \hline 28.2204 \end{array}$$

You Try It 4 $6.9 \times 1000 = 6900$

You Try It 5 $4.0273 \times 10^2 = 402.73$

You Try It 6

Strategy To find the total bill:

- Find the number of gallons of water used by multiplying the number of gallons used per day (5000) by the number of days (62).
- Find the cost of water by multiplying the cost per 1000 gallons (1.39) by the number of 1000-gallon units used.
- Add the cost of the water to the meter fee (133.70).

Solution

Number of gallons = 5000(62) = 310,000

$$\text{Cost of water} = \frac{310{,}000}{1000} \times 1.39 = 430.90$$

Total cost = 430.90 + 133.70 = 564.60

The total bill is $564.60.

You Try It 7

Strategy To find the cost of running the freezer for 210 hours, multiply the hourly cost (0.035) by the number of hours the freezer has run (210).

Solution

$$\begin{array}{r} 0.035 \\ \underline{\times\quad 210} \\ 350 \\ \underline{70} \\ 7.350 \end{array}$$

The cost of running the freezer for 210 hours is $7.35.

You Try It 8

Strategy To find the total cost of the stereo:

- Multiply the monthly payment (37.18) by the number of months (18).
- Add that product to the down payment (175.00).

Solution

$$\begin{array}{r} 37.18 \\ \underline{\times\quad 18} \\ 297\,44 \\ \underline{371\,8} \\ 669.24 \end{array} \qquad \begin{array}{r} 175.00 \\ \underline{+\ 669.24} \\ 844.24 \end{array}$$

The total cost of the stereo is $844.24.

SECTION 3.5

You Try It 1

$$\begin{array}{r} 2.7 \\ 0.\underset{\curvearrowright}{052.}\overline{)0.\underset{\curvearrowright}{140.}4} \\ \underline{-104} \\ 36\,4 \\ \underline{-36\,4} \\ 0 \end{array}$$

You Try It 2

$$\begin{array}{r} 0.4873 \approx 0.487 \\ 76\overline{)37.0420} \\ \underline{-30\,4} \\ 6\,64 \\ \underline{-6\,08} \\ 562 \\ \underline{-532} \\ 300 \\ \underline{-228} \end{array}$$

You Try It 3

$$\begin{array}{r} 72.73 \approx 72.7 \\ 5.\underset{\curvearrowright}{09.}\overline{)370.\underset{\curvearrowright}{20.}00} \\ \underline{-356\,3} \\ 13\,90 \\ \underline{10\,18} \\ 3\,720 \\ \underline{-3\,563} \\ 1570 \\ \underline{-1527} \end{array}$$

You Try It 4 $309.21 \div 10{,}000 = 0.030921$

You Try It 5 $42.93 \div 10^4 = 0.004293$

You Try It 6

Strategy To find how many times greater the average hourly earnings were, divide the 1990 average hourly earnings (10.02) by the 1970 average hourly earnings (3.23).

Solution $10.02 \div 3.23 \approx 3.1$

The average hourly earnings in 1990 were about 3.1 times greater than in 1970.

You Try It 7

Strategy To find the average number of people watching TV:

- Add the number of people watching each day of the week.
- Divide the total number of people watching by 7.

Solution

$$91.9 + 89.8 + 90.6 + 93.9 + 78.0 + 77.1 + 87.7 = 609$$

$$\frac{609}{7} = 87$$

An average of 87 million people watch television per day.

SECTION 3.6

You Try It 1

$$\begin{array}{r} 0.56 \approx 0.6 \\ 16\overline{)9.00} \end{array}$$

You Try It 2

$$4\frac{1}{6} = \frac{25}{6}$$

$$\begin{array}{r} 4.166 \approx 4.17 \\ 6\overline{)25.000} \end{array}$$

You Try It 3

$$0.56 = \frac{56}{100} = \frac{14}{25}$$

$$5.35 = 5\frac{35}{100} = 5\frac{7}{20}$$

You Try It 4

$$0.12\frac{7}{8} = \frac{12\frac{7}{8}}{100} = 12\frac{7}{8} \div 100$$
$$= \frac{103}{8} \times \frac{1}{100} = \frac{103}{800}$$

You Try It 5

$$\frac{5}{8} = 0.625$$
$$0.63 > 0.625$$
$$0.63 > \frac{5}{8}$$

Solutions to Chapter 4 "You Try It"

SECTION 4.1

You Try It 1

$$\frac{20 \text{ pounds}}{24 \text{ pounds}} = \frac{20}{24} = \frac{5}{6}$$

20 pounds : 24 pounds = 20 : 24 = 5 : 6

20 pounds to 24 pounds = 20 to 24
= 5 to 6

You Try It 2

$$\frac{64 \text{ miles}}{8 \text{ miles}} = \frac{64}{8} = \frac{8}{1}$$

64 miles : 8 miles = 64 : 8 = 8 : 1

64 miles to 8 miles = 64 to 8 = 8 to 1

You Try It 3

Strategy To find the ratio, write the ratio of board feet of cedar (12,000) to board feet of ash (18,000) in simplest form.

Solution

$$\frac{12{,}000}{18{,}000} = \frac{2}{3}$$

The ratio is $\frac{2}{3}$.

You Try It 4

Strategy To find the ratio, write the ratio of the amount spent on radio advertising (15,000) to the amount spent on radio and television advertising (15,000 + 20,000) in simplest form.

Solution

$$\frac{\$15{,}000}{\$15{,}000 + \$20{,}000} = \frac{15{,}000}{35{,}000} = \frac{3}{7}$$

The ratio is $\frac{3}{7}$.

SECTION 4.2

You Try It 1

$$\frac{15 \text{ pounds}}{12 \text{ trees}} = \frac{5 \text{ pounds}}{4 \text{ trees}}$$

You Try It 2

$$\frac{260 \text{ miles}}{8 \text{ hours}}$$

$$8\overline{)260.0}\ \ 32.5$$

32.5 miles/hour

You Try It 3

Strategy To find Erik's profit per ounce:

- Find the total profit by subtracting the cost (1625) from the selling price (1720).
- Divide the total profit by the number of ounces (5).

Solution 1720 − 1625 = 95

95 ÷ 5 = 19

The profit was $19/ounce.

SECTION 4.3

You Try It 1

$$\frac{6}{10} \asymp \frac{9}{15} \begin{array}{l} \rightarrow 10 \times 9 = 90 \\ \rightarrow 6 \times 15 = 90 \end{array}$$

The cross products are equal. The proportion is true.

You Try It 2

$$\frac{32}{6} \asymp \frac{90}{8} \begin{array}{l} \rightarrow 6 \times 90 = 540 \\ \rightarrow 32 \times 8 = 256 \end{array}$$

The cross products are not equal. The proportion is not true.

You Try It 3

$$\frac{n}{14} = \frac{3}{7}$$

$$n \times 7 = 14 \times 3$$
$$n \times 7 = 42$$
$$n = 42 \div 7$$
$$n = 6$$

Check:

$$\frac{6}{14} \asymp \frac{3}{7} \begin{array}{l} \rightarrow 14 \times 3 = 42 \\ \rightarrow 6 \times 7 = 42 \end{array}$$

You Try It 4 $\frac{5}{7} = \frac{n}{20}$

$$\begin{aligned} 5 \times 20 &= 7 \times n \\ 100 &= 7 \times n \\ 100 \div 7 &= n \\ 14.3 &\approx n \end{aligned}$$

You Try It 5 $\frac{15}{20} = \frac{12}{n}$

$$\begin{aligned} 15 \times n &= 20 \times 12 \\ 15 \times n &= 240 \\ n &= 240 \div 15 \\ n &= 16 \end{aligned}$$

Check: $$\frac{15}{20} \times \frac{12}{16} \quad \begin{matrix} \rightarrow 20 \times 12 = 240 \\ \rightarrow 15 \times 16 = 240 \end{matrix}$$

You Try It 6 $\frac{12}{n} = \frac{7}{4}$

$$\begin{aligned} 12 \times 4 &= n \times 7 \\ 48 &= n \times 7 \\ 48 \div 7 &= n \\ 6.86 &\approx n \end{aligned}$$

You Try It 7 $\frac{n}{12} = \frac{4}{1}$

$$\begin{aligned} n \times 1 &= 12 \times 4 \\ n \times 1 &= 48 \\ n &= 48 \div 1 \\ n &= 48 \end{aligned}$$

Check: $$\frac{48}{12} \times \frac{4}{1} \quad \begin{matrix} \rightarrow 12 \times 4 = 48 \\ \rightarrow 48 \times 1 = 48 \end{matrix}$$

You Try It 8 $\frac{3}{8} = \frac{12}{n}$

$$\begin{aligned} 3 \times n &= 8 \times 12 \\ 3 \times n &= 96 \\ n &= 96 \div 3 \\ n &= 32 \end{aligned}$$

Check: $$\frac{3}{8} \times \frac{12}{32} \quad \begin{matrix} \rightarrow 8 \times 12 = 96 \\ \rightarrow 3 \times 32 = 96 \end{matrix}$$

You Try It 9

Strategy To find the number of jars that can be packed in 15 boxes, write and solve a proportion using *n* to represent the number of jars.

Solution $$\frac{24 \text{ jars}}{6 \text{ boxes}} = \frac{n \text{ jars}}{15 \text{ boxes}}$$

$$\begin{aligned} 24 \times 15 &= 6 \times n \\ 360 &= 6 \times n \\ 360 \div 6 &= n \\ 60 &= n \end{aligned}$$

60 jars can be packed in 15 boxes.

You Try It 10

Strategy To find the number of tablespoons of fertilizer needed, write and solve a proportion using *n* to represent the number of tablespoons of fertilizer.

Solution $$\frac{3 \text{ tablespoons}}{4 \text{ gallons}} = \frac{n \text{ tablespoons}}{10 \text{ gallons}}$$

$$\begin{aligned} 3 \times 10 &= 4 \times n \\ 30 &= 4 \times n \\ 30 \div 4 &= n \\ 7.5 &= n \end{aligned}$$

For 10 gallons of water, 7.5 tablespoons of fertilizer are required.

Solutions to Chapter 5 "You Try It"

SECTION 5.1

You Try It 1 $125\% = 125 \times \frac{1}{100} = \frac{125}{100} = 1\frac{1}{4}$

$125\% = 125 \times 0.01 = 1.25$

You Try It 2 $$\begin{aligned} 33\tfrac{1}{3}\% &= 33\tfrac{1}{3} \times \frac{1}{100} \\ &= \frac{100}{3} \times \frac{1}{100} \\ &= \frac{100}{300} = \frac{1}{3} \end{aligned}$$

You Try It 3 $0.25\% = 0.25 \times 0.01 = 0.0025$

You Try It 4 $0.048 = 0.048 \times 100\% = 4.8\%$

You Try It 5 $3.67 = 3.67 \times 100\% = 367\%$

You Try It 6

$$0.62\frac{1}{2} = 0.62\frac{1}{2} \times 100\% = 62\frac{1}{2}\%$$

You Try It 7

$$\frac{5}{6} = \frac{5}{6} \times 100\% = \frac{500}{6}\% = 83\frac{1}{3}\%$$

You Try It 8

$$1\frac{4}{9} = \frac{13}{9} = \frac{13}{9} \times 100\% = \frac{1300}{9}\% \approx 144.4\%$$

SECTION 5.2

You Try It 1

$$\begin{aligned} n &= 0.063 \times 150 \\ n &= 9.45 \end{aligned}$$

You Try It 2

$$\begin{aligned} n &= \frac{1}{6} \times 66 \\ n &= 11 \end{aligned}$$

You Try It 3

Strategy To find the amount donated, write and solve the basic percent equation using n to represent the amount. The percent is 14%. The base is $190 billion.

Solution

$$\begin{aligned} 14\% \times 190 &= n \\ 0.14 \times 190 &= n \\ 26.6 &= n \end{aligned}$$

$26.6 billion was donated to educational organizations.

You Try It 4

Strategy To find the new hourly wage:

- Find the amount of the raise. Write and solve the basic percent equation, using n to represent the amount of the raise (amount). The percent is 8%. The base is $23.50.
- Add the amount of the raise to the old wage (23.50).

Solution

$$\begin{aligned} 8\% \times 23.50 &= n \\ 0.08 \times 23.50 &= n \\ 1.88 &= n \end{aligned} \qquad \begin{array}{r} 23.50 \\ +\ 1.88 \\ \hline 25.38 \end{array}$$

The new hourly wage is $25.38.

SECTION 5.3

You Try It 1

$$\begin{aligned} n \times 32 &= 16 \\ n &= 16 \div 32 \\ n &= 0.50 \\ n &= 50\% \end{aligned}$$

You Try It 2

$$\begin{aligned} n \times 15 &= 48 \\ n &= 48 \div 15 \\ n &= 3.20 \\ n &= 320\% \end{aligned}$$

You Try It 3

$$\begin{aligned} 30 &= n \times 45 \\ 30 \div 45 &= n \\ \frac{2}{3} &= n \\ 66\frac{2}{3}\% &= n \end{aligned}$$

You Try It 4

Strategy To find what percent of the income the income tax is, write and solve the basic percent equation, using n to represent the percent. The base is $33,500 and the amount is $5025.

Solution

$$\begin{aligned} n \times 33{,}500 &= 5025 \\ n &= 5025 \div 33{,}500 \\ n &= 0.15 = 15\% \end{aligned}$$

The income tax is 15% of the income.

You Try It 5

Strategy To find the percent who were women:

- Subtract to find the number of enlisted personnel who were women (518,921 − 512,370).
- Write and solve the basic percent equation, using n to represent the percent. The base is 518,921, and the amount is the number of enlisted personnel who were women.

Solution

$$518{,}921 - 512{,}370 = 6551$$

$$\begin{aligned} n \times 518{,}921 &= 6551 \\ n &= 6551 \div 518{,}921 \\ n &\approx 0.013 \end{aligned}$$

In 1950, 1.3% of the enlisted personnel in the U.S. Army were women.

SECTION 5.4

You Try It 1

$$\begin{aligned} 0.86 \times n &= 215 \\ n &= 215 \div 0.86 \\ n &= 250 \end{aligned}$$

You Try It 2

$$\begin{aligned} 15 &= 0.025 \times n \\ 15 \div 0.025 &= n \\ 600 &= n \end{aligned}$$

You Try It 3

$$\begin{aligned} \frac{1}{6} \times n &= 5 \\ n &= 5 \div \frac{1}{6} \\ n &= 30 \end{aligned}$$

You Try It 4

Strategy To find the original value of the car, write and solve the basic percent equation, using n to represent the original value (base). The percent is 42%. The amount is $10,458.

Solution

$$\begin{aligned} 42\% \times n &= 10{,}458 \\ 0.42 \times n &= 10{,}458 \\ n &= 10{,}458 \div 0.42 \\ n &= 24{,}900 \end{aligned}$$

The original value of the car was $24,900.

You Try It 5

Strategy To find the difference between the original price and the sale price:

- Find the original price. Write and solve the basic percent equation, using n to represent the original price (base). The percent is 80%. The amount is $89.60.
- Subtract the sale price (89.60) from the original price.

Solution

$$\begin{aligned} 80\% \times n &= 89.60 \\ 0.80 \times n &= 89.60 \\ n &= 89.60 \div 0.80 \\ n &= 112.00 \quad \text{original price} \end{aligned}$$

$$112.00 - 89.60 = 22.40$$

The difference between the original price and the sale price is $22.40.

SECTION 5.5

You Try It 1

$$\begin{aligned} \frac{26}{100} &= \frac{22}{n} \\ 26 \times n &= 100 \times 22 \\ 26 \times n &= 2200 \\ n &= 2200 \div 26 \\ n &\approx 84.62 \end{aligned}$$

You Try It 2

$$\begin{aligned} \frac{16}{100} &= \frac{n}{132} \\ 16 \times 132 &= 100 \times n \\ 2112 &= 100 \times n \\ 2112 \div 100 &= n \\ 21.12 &= n \end{aligned}$$

You Try It 3

Strategy To find the number of days it snowed, write and solve a proportion, using n to represent the number of days it snowed (amount). The percent is 64%. The base is 150.

Solution

$$\begin{aligned} \frac{64}{100} &= \frac{n}{150} \\ 64 \times 150 &= 100 \times n \\ 9600 &= 100 \times n \\ 9600 \div 100 &= n \\ 96 &= n \end{aligned}$$

It snowed 96 days.

You Try It 4

Strategy To find the percent of pens that were not defective:

- Subtract to find the number of pens that were not defective (200 − 5).
- Write and solve a proportion, using n to represent the percent of pens that were not defective. The base is 200, and the amount is the number of pens not defective.

Solution

$$200 - 5 = 195 \quad \text{number of pens not defective}$$

$$\begin{aligned} \frac{n}{100} &= \frac{195}{200} \\ n \times 200 &= 100 \times 195 \\ n \times 200 &= 19{,}500 \\ n &= 19{,}500 \div 200 \\ n &= 97.5\% \end{aligned}$$

97.5% of the pens were not defective.

Solutions to Chapter 6 "You Try It"

SECTION 6.1

You Try It 1

Strategy To find the unit cost, divide the total cost by the number of units.

Solution

a. $7.67 \div 8 = 0.95875$

$.959 for each battery

b. $2.29 \div 15 \approx 0.153$

$.153 per ounce

You Try It 2

Strategy To find the more economical purchase, compare the unit costs.

Solution

$$\begin{aligned} 5.70 \div 6 &= 0.95 \\ 3.96 \div 4 &= 0.99 \\ \$.95 &< \$.99 \end{aligned}$$

The more economical purchase is 6 cans for $5.70.

You Try It 3

Strategy To find the total cost, multiply the unit cost (9.96) by the number of units (7).

Solution $9.96 \times 7 = 69.72$

The total cost is $69.72.

SECTION 6.2

You Try It 1

Strategy To find the percent increase:

- Find the amount of the increase.
- Solve the basic percent equation for *percent*.

Solution $1.83 - 1.46 = 0.37$

$$\begin{aligned} n \times 1.46 &= 0.37 \\ n &= 0.37 \div 1.46 \\ n &\approx 0.25 = 25\% \end{aligned}$$

The percent increase was 25%.

You Try It 2

Strategy To find the new hourly wage:

- Solve the basic percent equation for *amount*.
- Add the amount of the increase to the original wage.

Solution

$$\begin{aligned} 0.14 \times 12.50 &= n \\ 1.75 &= n \end{aligned}$$

$12.50 + 1.75 = 14.25$

The new hourly wage is $14.25.

You Try It 3

Strategy To find the markup, solve the basic percent equation for *amount*.

Solution

$$\begin{aligned} 0.20 \times 8 &= n \\ 1.60 &= n \end{aligned}$$

The markup is $1.60.

You Try It 4

Strategy To find the selling price:

- Find the markup by solving the basic percent equation for *amount*.
- Add the markup to the cost.

Solution

$$\begin{aligned} 0.55 \times 72 &= n \\ 39.60 &= n \end{aligned}$$

$72 + 39.60 = 111.60$

The selling price is $111.60.

You Try It 5

Strategy To find the percent decrease:

- Find the amount of the decrease.
- Solve the basic percent equation for *percent*.

Solution $261{,}000 - 215{,}000 = 46{,}000$

$$\begin{aligned} n \times 261{,}000 &= 46{,}000 \\ n &= 46{,}000 \div 261{,}000 \\ n &\approx 0.176 \end{aligned}$$

The percent decrease is 17.6%.

You Try It 6

Strategy To find the visibility:

- Find the amount of decrease by solving the basic percent equation for *amount*.
- Subtract the amount of decrease from the original visibility.

Solution

$$\begin{aligned} 0.40 \times 5 &= n \\ 2 &= n \end{aligned}$$

$5 - 2 = 3$

The visibility was 3 miles.

You Try It 7

Strategy To find the discount rate:

- Find the discount.
- Solve the basic percent equation for *percent*.

Solution $12.50 - 10.99 = 1.51$

$$n \times 12.50 = 1.51$$
$$n = 1.51 \div 12.50$$
$$n = 0.1208$$

The discount rate is 12.1%.

You Try It 8

Strategy To find the sale price:

- Find the discount by solving the basic percent equation for *amount*.
- Subtract to find the sale price.

Solution $0.15 \times 110 = n$

$$16.5 = n$$

$110 - 16.5 = 93.5$

The sale price is $93.50.

SECTION 6.3

You Try It 1

Strategy To find the simple interest due, multiply the principal (15,000) times the annual interest rate (8% = 0.08) times the time, in years ($18 \text{ months} = \frac{18}{12} \text{ years} = 1.5 \text{ years}$).

Solution $15{,}000 \times 0.08 \times 1.5 = 1800$

The interest due is $1800.

You Try It 2

Strategy To find the maturity value:

- Use the simple interest formula to find the simple interest due.
- Find the maturity value by adding the principal and the interest.

Solution $3800 \times 0.06 \times \frac{90}{365} \approx 56.22$

$3800 + 56.22 = 3856.22$

The maturity value is $3856.22.

You Try It 3

Strategy To find the monthly payment:

- Find the maturity value by adding the principal and the interest.
- Divide the maturity value by the length of the loan in months (12).

Solution $1900 + 152 = 2052$

$2052 \div 12 = 171$

The monthly payment is $171.

You Try It 4

Strategy To find the finance charge, multiply the principal, or unpaid balance (1250), times the monthly interest rate (1.6%) times the number of months (1).

Solution $1250 \times 0.016 \times 1 = 20$

The simple interest due is $20.

You Try It 5

Strategy To find the interest earned:

- Find the new principal by multiplying the original principal (1000) by the factor found in the Compound Interest Table (3.29066).
- Subtract the original principal from the new principal.

Solution $1000 \times 3.29066 = 3290.66$

The new principal is $3290.66.

$3290.66 - 1000 = 2290.66$

The interest earned is $2290.66.

SECTION 6.4

You Try It 1

Strategy To find the mortgage:

- Find the down payment by solving the basic percent equation for *amount*.
- Subtract the down payment from the purchase price.

Solution $0.25 \times 216{,}000 = n$

$$54{,}000 = n$$

The down payment is $54,000.

$216{,}000 - 54{,}000 = 162{,}000$

The mortgage is $162,000.

You Try It 2

Strategy To find the loan origination fee, solve the basic percent equation for *amount*.

Solution $0.045 \times 80{,}000 = n$

$$3600 = n$$

The loan origination fee was $3600.

You Try It 3

Strategy To find the monthly mortgage payment:

- Subtract the down payment from the purchase price to find the mortgage.
- Multiply the mortgage by the factor found in the Monthly Payment Table.

Solution $75{,}000 - 15{,}000 = 60{,}000$

The mortgage is \$60,000.

$60{,}000 \times 0.0089973 = 539.838$

The monthly mortgage payment is \$539.84.

You Try It 4

Strategy To find the interest:

- Multiply the mortgage by the factor found in the Monthly Payment Table to find the monthly mortgage payment.
- Subtract the principal from the monthly mortgage payment.

Solution $125{,}000 \times 0.0083920 = 1049$

The monthly mortgage payment is \$1049.

$1049 - 492.65 = 556.35$

The interest on the mortgage is \$556.35.

You Try It 5

Strategy To find the monthly payment:

- Divide the annual property tax by 12 to find the monthly property tax.
- Add the monthly property tax to the monthly mortgage payment.

Solution $744 \div 12 = 62$

The monthly property tax is \$62.

$415.20 + 62 = 477.20$

The total monthly payment is \$477.20.

SECTION 6.5

You Try It 1

Strategy To find the amount financed:

- Find the down payment by solving the basic percent equation for *amount*.
- Subtract the down payment from the purchase price.

Solution

$$\begin{aligned} 0.20 \times 19{,}200 &= n \\ 3840 &= n \end{aligned}$$

The down payment is \$3840.

$19{,}200 - 3840 = 15{,}360$

The amount financed is \$15,360.

You Try It 2

Strategy To find the license fee, solve the basic percent equation for *amount*.

Solution

$$\begin{aligned} 0.015 \times 17{,}350 &= n \\ 260.25 &= n \end{aligned}$$

The license fee is \$260.25.

You Try It 3

Strategy To find the cost, multiply the cost per mile (0.31) by the number of miles driven (23,000).

Solution $23{,}000 \times 0.31 = 7130$

The cost is \$7130.

You Try It 4

Strategy To find the cost per mile for car insurance, divide the cost for insurance (360) by the number of miles driven (15,000).

Solution $360 \div 15{,}000 = 0.024$

The cost per mile for insurance is \$.024.

You Try It 5

Strategy To find the monthly payment:

- Subtract the down payment from the purchase price to find the amount financed.
- Multiply the amount financed by the factor found in the Monthly Payment Table.

Solution $25{,}900 - 6475 = 19{,}425$

$19{,}425 \times 0.0244129 \approx 474.22$

The monthly payment is \$474.22.

SECTION 6.6

You Try It 1

Strategy To find the worker's earnings:

- Find the worker's overtime wage by multiplying the hourly wage by 2.

- Multiply the number of overtime hours worked by the overtime wage.

Solution $18.50 \times 2 = 37$

The hourly wage for overtime is $37.

$37 \times 8 = 296$

The construction worker earns $296.

You Try It 2

Strategy To find the salary per month, divide the annual salary by the number of months in a year (12).

Solution $48{,}228 \div 12 = 4019$

The contractor's monthly salary is $4019.

You Try It 3

Strategy To find the total earnings:

- Find the sales over $50,000.
- Multiply the commission rate by sales over $50,000.
- Add the commission to the annual salary.

Solution $175{,}000 - 50{,}000 = 125{,}000$

Sales over $50,000 totaled $125,000.

$125{,}000 \times 0.095 = 11{,}875$

Earnings from commissions totaled $11,875.

$27{,}000 + 11{,}875 = 38{,}875$

The insurance agent earned $38,875.

SECTION 6.7

You Try It 1

Strategy To find the current balance:

- Subtract the amount of the check from the old balance.
- Add the amount of each deposit.

Solution

$$\begin{array}{rl} 302.46 & \\ -\ \ 20.59 & \text{check} \\ \hline 281.87 & \\ 176.86 & \text{first deposit} \\ +\ \ 94.73 & \text{second deposit} \\ \hline 553.46 & \end{array}$$

The current checking account balance is $553.46.

You Try It 2

$$\begin{array}{lr} \text{Current checkbook balance:} & 623.41 \\ \text{Check: 237} & +\ \ 78.73 \\ \hline & 702.14 \\ \text{Interest:} & +\ \ \ 2.11 \\ \hline & 704.25 \\ \text{Deposit:} & -523.84 \\ \hline & 180.41 \end{array}$$

Closing bank balance from bank statement: $180.41.

Checkbook balance: $180.41.

The bank statement and the checkbook balance.

Solutions to Chapter 7 "You Try It"

SECTION 7.1

You Try It 1

Strategy To find what percent of the total number of cellular phones purchased the number purchased in March represents:

- Read the pictograph to determine the number of cellular phones purchased for each month.
- Find the total cellular phone purchases for the 4-month period.
- Solve the basic percent equation for percent (n). Amount = 3000; the base is the total sales for the 4-month period.

Solution

$$\begin{array}{r} 4{,}500 \\ 3{,}500 \\ 3{,}000 \\ 1{,}500 \\ \hline 12{,}500 \end{array}$$

$$n \times 12{,}500 = 3000$$
$$n = 3000 \div 12{,}500$$
$$n = 0.24$$

The number of cellular phones purchased in March represents 24% of the total number of cellular phones purchased.

You Try It 2

Strategy To find the ratio of the annual cost of fuel to the annual cost of maintenance:

- Locate the annual fuel cost and the annual maintenance cost in the circle graph.
- Write the ratio of the annual fuel cost to the annual maintenance cost in simplest form.

Solution Annual fuel cost: $700
Annual maintenance cost: $500

$$\frac{700}{500} = \frac{7}{5}$$

The ratio is $\frac{7}{5}$.

You Try It 3

Strategy To find the amount paid for medical/dental insurance:

- Locate the percent of the distribution that is medical/dental insurance.
- Solve the basic percent equation for amount.

Solution The percent of the distribution that is medical/dental insurance: 6%

$$\text{Percent} \times \text{base} = \text{amount}$$
$$0.06 \times 2900 = n$$
$$174 = n$$

The amount paid for medical/dental insurance was $174.

SECTION 7.2

You Try It 1

Strategy To write the ratio:

- Read the graph to find the lung capacity of an inactive female and of an athletic female.
- Write the ratio in simplest form.

Solution Lung capacity of an inactive female: 25

Lung capacity of an athletic female: 55

$$\frac{25}{55} = \frac{5}{11}$$

The ratio is $\frac{5}{11}$.

You Try It 2

Strategy To determine between which two years the net income of Math Associates increased the most:

- Read the line graph to determine the net income of Math Associates for each of the years shown.
- Subtract to find the difference between each two years.
- Find the greatest difference.

Solution 2000: $1 million
2001: $3 million
2002: $5 million
2003: $8 million
2004: $12 million

Between 2000 and 2001: $3 - 1 = 2$
Between 2001 and 2002: $5 - 3 = 2$
Between 2002 and 2003: $8 - 5 = 3$
Between 2003 and 2004: $12 - 8 = 4$

$4 > 3 > 2$

The net income of Math Associates increased the most between the years 2003 and 2004.

SECTION 7.3

You Try It 1

Strategy To find the number of employees:

- Read the histogram to find the number of employees whose hourly wage is between $10 and $12 and the number whose hourly wage is between $12 and $14.
- Add the two numbers.

Solution Number whose wage is between $10 and $12: 7;
between $12 and $14: 15.

$7 + 15 = 22$

22 employees earn between $10 and $14.

You Try It 2

Strategy To write the ratio:

- Read the graph to find the number of states with a per capita income between \$18,000 and \$22,000 and the number with a per capita income between \$26,000 and \$30,000.
- Write the ratio in simplest form.

Solution Number of states with a per capita income between \$18,000 and \$22,000: 13

Number of states with a per capita income between \$26,000 and \$30,000: 13

$$\frac{13}{13} = \frac{1}{1}$$

The ratio is $\frac{1}{1}$.

SECTION 7.4

You Try It 1

Strategy To find the mean amount spent by the 12 customers:

- Find the sum of the numbers.
- Divide the sum by the number of customers (12).

Solution $6.26 + 8.23 + 5.09 + 8.11 + 7.50 + 6.69 + 5.66 + 4.89 + 5.25 + 9.36 + 6.75 + 7.05 = 80.84$

$$\bar{x} = \frac{80.84}{12} \approx 6.74$$

The mean amount spent by the 12 customers was \$6.74.

You Try It 2

Strategy To find the median weight loss:

- Arrange the weight losses from smallest to largest.
- Because there is an even number of values, the median is the mean of the middle two numbers.

Solution 10, 14, 16, 16, 22, 27, 29, 31, 31, 40

$$\text{Median} = \frac{22 + 27}{2} = \frac{49}{2} = 24.5$$

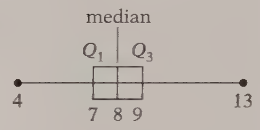
The median weight loss was 24.5 pounds.

You Try It 3

Strategy To draw the box-and-whiskers plot:

- Find the median, Q_1, and Q_3.
- Use the smallest value, Q_1, the median, Q_3, and the largest value to draw the box-and-whiskers plot.

Solution $4 \quad \underset{Q_1}{\underset{\uparrow}{7}} \quad 7 \quad \underset{\text{Median}}{\underset{\uparrow}{8}} \quad 8 \quad \underset{Q_3}{\underset{\uparrow}{9}} \quad 13$



Answers about the spread of the data will vary. For example, in You Try It 3, the values in the interquartile range are all very close to the median. They are not so close to the median in Example 3. The whiskers are long with respect to the box in You Try It 3, whereas they are short with respect to the box in Example 3, showing that the data values outside the interquartile range are closer together in Example 3 than in You Try It 3.

SECTION 7.5

You Try It 1

Strategy To find the probability:

- List the outcomes of the experiment in a systematic way. We will use a table.
- Use the table to count the number of possible outcomes of the experiment.
- Count the number of outcomes of the experiment that are favorable to the event of 2 true questions and 1 false question.
- Use the probability formula.

Solution

Question 1	Question 2	Question 3
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

There are 8 possible outcomes:

$S = \{\text{TTT, TTF, TFT, TFF, FTT, FTF, FFT, FFF}\}$

There are 3 outcomes favorable to the event:

$\{\text{TTF, TFT, FTT}\}$

Probability of an event $= \dfrac{\text{number of favorable outcomes}}{\text{number of possible outcomes}} = \dfrac{3}{8}$

The probability of 2 true questions and 1 false question is $\frac{3}{8}$.

Solutions to Chapter 8 "You Try It"

SECTION 8.1

You Try It 1

$$14 \text{ ft} = 14 \cancel{\text{ft}} \times \frac{1 \text{ yd}}{3 \cancel{\text{ft}}} = \frac{14 \text{ yd}}{3} = 4\frac{2}{3} \text{ yd}$$

You Try It 2

$$9240 \text{ ft} = 9240 \cancel{\text{ft}} \times \frac{1 \text{ mi}}{5280 \cancel{\text{ft}}} = \frac{9240 \text{ mi}}{5280} = 1\frac{3}{4} \text{ mi}$$

You Try It 3

$$\begin{array}{rl} & 3 \text{ ft } 6 \text{ in.} \\ 12\,\big) & 42 \\ & \underline{-36} \\ & 6 \end{array}$$

- **12 in. = 1 ft**

42 in. = 3 ft 6 in.

You Try It 4

$$\begin{array}{rl} & 4 \text{ yd } 2 \text{ ft} \\ 3\,\big) & 14 \\ & \underline{-12} \\ & 2 \end{array}$$

- **3 ft = 1 yd**

14 ft = 4 yd 2 ft

You Try It 5

$$\begin{array}{rr} & 3 \text{ ft} \quad 5 \text{ in.} \\ + & 4 \text{ ft} \quad 9 \text{ in.} \\ \hline & 7 \text{ ft } 14 \text{ in.} \end{array}$$

- **14 in. = 1 ft 2 in.**

7 ft 14 in. = 8 ft 2 in.

You Try It 6

$$\begin{array}{rr} & \overset{3 \text{ ft}}{\cancel{4 \text{ ft}}}\ \overset{14 \text{ in.}}{\cancel{2 \text{ in.}}} \\ - & 1 \text{ ft } 8 \text{ in.} \\ \hline & 2 \text{ ft } 6 \text{ in.} \end{array}$$

- **Borrow 1 ft (12 in.) from 4 ft and add it to 2 in.**

You Try It 7

$$\begin{array}{rr} & 4 \text{ yd } 1 \text{ ft} \\ \times & 8 \\ \hline & 32 \text{ yd } 8 \text{ ft} \end{array}$$

- **8 ft = 2 yd 2 ft**

32 yd 8 ft = 34 yd 2 ft

You Try It 8

$$\begin{array}{rrr} & 3 \text{ yd} & 2 \text{ ft} \\ 2\,\big) & 7 \text{ yd} & 1 \text{ ft} \\ & \underline{-6 \text{ yd}} & \\ & 1 \text{ yd} = & \underline{3 \text{ ft}} \\ & & 4 \text{ ft} \\ & & \underline{-4 \text{ ft}} \\ & & 0 \end{array}$$

You Try It 9

$$\begin{array}{rcrcr} 6\frac{1}{4} \text{ ft} & = & 6\frac{3}{12} \text{ ft} & = & 5\frac{15}{12} \text{ ft} \\ -3\frac{2}{3} \text{ ft} & = & 3\frac{8}{12} \text{ ft} & = & 3\frac{8}{12} \text{ ft} \\ \hline & & & & 2\frac{7}{12} \text{ ft} \end{array}$$

You Try It 10

Strategy To find the width of the storage room:

- Multiply the number of tiles (8) by the width of each tile (9 in.).
- Divide the result by the number of inches in 1 foot (12) to find the width in feet.

Solution 9 in. × 8 = 72 in.

72 ÷ 12 = 6

The width is 6 ft.

You Try It 11

Strategy To find the length of each piece, divide the total length (9 ft 8 in.) by the number of pieces (4).

Solution

$$\begin{array}{rrr} & 2 \text{ ft} & 5 \text{ in.} \\ 4\,\big) & 9 \text{ ft} & 8 \text{ in.} \\ & \underline{-8 \text{ ft}} & \\ & 1 \text{ ft} = & \underline{12 \text{ in.}} \\ & & 20 \text{ in.} \\ & & \underline{-20 \text{ in.}} \\ & & 0 \text{ in.} \end{array}$$

Each piece is 2 ft 5 in. long.

SECTION 8.2

You Try It 1 $3\text{ lb} = 3\ \cancel{\text{lb}} \times \frac{16\text{ oz}}{1\ \cancel{\text{lb}}} = 48\text{ oz}$

You Try It 2

$$\begin{aligned} 4200\text{ lb} &= 4200\ \cancel{\text{lb}} \times \frac{1\text{ ton}}{2000\ \cancel{\text{lb}}} \\ &= \frac{4200\text{ tons}}{2000} = 2\frac{1}{10}\text{ tons} \end{aligned}$$

You Try It 3

$$\begin{array}{rr} \overset{6\text{ lb}}{\cancel{7\text{ lb}}} & \overset{17\text{ oz}}{\cancel{1\text{ oz}}} \\ -\ 3\text{ lb} & 4\text{ oz} \\ \hline 3\text{ lb} & 13\text{ oz} \end{array}$$

- **Borrow 1 lb (16 oz) from 7 lb and add it to 1 oz.**

You Try It 4

$$\begin{array}{r} 3\text{ lb }6\text{ oz} \\ \times\qquad 4 \\ \hline 12\text{ lb }24\text{ oz} = 13\text{ lb }8\text{ oz} \end{array}$$

You Try It 5

Strategy To find the weight of 12 bars of soap:

- Multiply the number of bars (12) by the weight of each bar (7 oz).
- Convert the number of ounces to pounds.

Solution

$$\begin{array}{r} 12 \\ \times\ \ 7\text{ oz} \\ \hline 84\text{ oz} \end{array} \qquad 84\ \cancel{\text{oz}} \times \frac{1\text{ lb}}{16\ \cancel{\text{oz}}} = 5\frac{1}{4}\text{ lb}$$

The 12 bars of soap weigh $5\frac{1}{4}$ lb.

SECTION 8.3

You Try It 1

$$\begin{aligned} 18\text{ pt} &= 18\ \cancel{\text{pt}} \times \frac{1\ \cancel{\text{qt}}}{2\ \cancel{\text{pt}}} \times \frac{1\text{ gal}}{4\ \cancel{\text{qt}}} \\ &= \frac{18\text{ gal}}{8} = 2\frac{1}{4}\text{ gal} \end{aligned}$$

You Try It 2

$$\begin{array}{rrr} & 1\text{ gal} & 2\text{ qt} \\ \cline{2-3} 3) & 4\text{ gal} & 2\text{ qt} \\ & -3\text{ gal} & \\ \cline{2-2} & 1\text{ gal} = & 4\text{ qt} \\ \cline{3-3} & & 6\text{ qt} \\ & & -6\text{ qt} \\ \cline{3-3} & & 0 \end{array}$$

You Try It 3

Strategy To find the number of gallons of water needed:

- Find the number of quarts required by multiplying the number of quarts one student needs per day (3) by the number of students (5) by the number of days (3).
- Convert the number of quarts to gallons.

Solution $3 \times 5 \times 3 = 45\text{ qt}$

$$45\ \cancel{\text{qt}} \cdot \frac{1\text{ gal}}{4\ \cancel{\text{qt}}} = \frac{45\text{ gal}}{4} = 11\frac{1}{4}\text{ gal}$$

The students should take $11\frac{1}{4}$ gal of water.

SECTION 8.4

You Try It 1

$$\begin{aligned} 4.5\text{ Btu} &= 4.5\ \cancel{\text{Btu}} \times \frac{778\text{ ft}\cdot\text{lb}}{1\ \cancel{\text{Btu}}} \\ &= 3501\text{ ft}\cdot\text{lb} \end{aligned}$$

You Try It 2 $800\text{ lb} \times 16\text{ ft} = 12{,}800\text{ ft}\cdot\text{lb}$

You Try It 3

$$\begin{aligned} &56{,}000\text{ Btu} = \\ &56{,}000\ \cancel{\text{Btu}} \times \frac{778\text{ ft}\cdot\text{lb}}{1\ \cancel{\text{Btu}}} = \\ &43{,}568{,}000\text{ ft}\cdot\text{lb} \end{aligned}$$

You Try It 4

$$\begin{aligned} \text{Power} &= \frac{90\text{ ft} \times 1200\text{ lb}}{24\text{ s}} \\ &= 4500\ \frac{\text{ft}\cdot\text{lb}}{\text{s}} \end{aligned}$$

You Try It 5 $\frac{3300}{550} = 6\text{ hp}$

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